# INSTITUTE OF ACTUARIES OF INDIA

# EXAMINATIONS

## 19th September 2016

## Subject ST6 – Finance and Investment B

## Time allowed: Three Hours (10.15* – 13.30 Hrs)

## Total Marks: 100

### INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***
2. **** You have 15 minutes at the start of the examination in which you are required to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You have then three hours to complete the paper.***
3. ***You must not start writing your answers in the answer sheet unless instructed to do so by the supervisor.***
4. ***The answers are not expected to be any country or jurisdication specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.***
5. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
6. ***Mark allocations are shown in brackets.***
7. ***Please check if you have received complete Question paper and no page is missing. If so, kindly get a new set of Question paper from the Invigilator.***

**AT THE END OF THE EXAMINATION**

**Please return your answer book and this question paper to the supervisor separately.**

**Q. 1)** The new solvency regulations for a country have significantly increased the contribution of the longevity risk to the capital requirement for annuity companies. As an investment manager of an annuity company you are tasked to provide a report to the board to manage this risk.

**i)** Describe the main types of derivative instruments which can be used to reduce the longevity risk. (5)

The company had decided to enter into a longevity swap. The swap is not the traditional zero start swap. This is due to the fact that company is required to hedge the risk at the regulatory basis which has a 10% margin for adverse deviation. This means that to remove the longevity risk from the balance sheet the company needs to receive 10% more cashflows than the best estimate cashflows of a traditional swap on the date of calculation.

**ii)** Find out the initial value of the swap with 5 year duration payment in arrear of Rs100,000 per annum at best estimate assumptions. Assume the risk-free interest rate is 5% with continuous compounding. (5)

**iii)** Estimate the reduction in the capital requirement due to the longevity swap. The capital requirement for the longevity risk is the difference between base regulatory reserves and reserves assuming the regulatory cashflows increased by 50%. (4)

The regulator has rejected the proposal of the longevity swap, suggesting an issue with the exposure to counterparty risk.

**iv)** Discuss the concerns of the regulators (assuming the transaction is over the counter). State different ways by which the company can mitigate it (4)

One of the solutions recommended by the regulator is to have sufficient collateral to reduce the counterparty risk.

**v)** Estimate the collateral the counterparty should post such that counterparty risk is minimum. Assume that collateral can cover a 1-in 200 annuity payout scenario where in 1-in-200 scenario cashflow is 50% higher than the base regulatory basis cashflow. (6)

Moody's credit rating of the counterparty is A with the following default assumptions.

| Year | Default Rate |
|---|---|
| 1 | 2% |
| 2 | 4% |
| 3 | 6% |
| 4 | 8% |
| 5 | 10% |

**[24]**

**Q. 2)** Define and describe the following traded derivative contracts.

**i)** Warrants (2)

**ii)** Convertibles (2)

**iii)** Property derivatives (2)

**[6]**

**Q. 3)** A bank is launching a bespoke lending product aimed at property developers called CLINK. The loans to the developers have the following characteristics:

The loan is for 5 years of amount 1,000,000. The builder is expected to pay the interest at the end of each year along with 10% of the original principal. The final payment includes the residual principal along with the last year's interest. The bank underwriting department has done a risk analysis and suggested a spread of 2.5% over the risk free rate when determining the interest rate for the loan.

**i)** Discuss the key factors which would have been considered by the bank underwriting department to arrive at the spread over risk free (4)

**ii)** Calculate the theoretical value of the bond and state the amount of allowance the bank has for default. Assume the risk-free interest rate is 6% with continuous compounding. (6)

Bank has put a "spense clause" in the contract document to protect itself from prepayment risk. This clause states that if the loan is prepaid then

- Prepayment can only happen at the anniversary of the loan.
- There is a requirement for the property developer to pay a repayment charge estimated as the difference between
  - The value of the future cashflows of the loan left at prepayment discounted at the rate of interest at the date of inception
  - The value of an "equivalent" interest only loan (with the same spread, i.e., total interest = 2.5% + risk free yield) that can be issued at the date of prepayment with the principal received discounted by the rate of interest at the date of inception.

At the end of 2 years, the risk free interest rate has fallen 2% from 6% and the property developer has decided to repay the CLINK loan.

**iii)** Estimate the loss to the bank if there is no "spense clause" (4)

**iv)** Calculate the prepayment fine due to the spense clause and estimate the loss/gain to the bank after the fine. (6)

**[20]**

**Q. 4)** **i)** Explain the use and significance of Ito's lemma for the valuation of derivatives based on stochastic processes. (1)

**ii)** Explain what it means for a stochastic process $X_t$ to be a martingale under a probability measure **P**. (1)

**iii)** Consider a standard ***P-Brownian*** motion $W_t$. Obtain the differential equation for the stochastic process

$$Y_t = W_t^2 - t,$$

and state with reasons whether it is a martingale. (2)

**iv)** Show for what value(s) of the constant a, if any, the stochastic process

$$Z_t = W_t^4 - 6W_t^2 t + at^2$$

is a martingale. (3)

[For parts (iii) and (iv) you may assume that both processes are sufficiently bounded.] **[7]**

**Q. 5)** Answer the following:

**i)** Without using the Black Scholes formula, show that European call and put options with the same strike and expiry have the same gamma. (2)

**ii)** How does the price of a discrete barrier option compare to the price of a vanilla option and the price of a continuous barrier option? (3)

**iii)** How does the price of an Asian option compare to the price of a vanilla option with the same strike and maturity? (2)

**iv)** Suppose an asset follows a Brownian motion and there are no interest rates. What can be said about the relative prices of out – of – the money American and European digital calls? (2)

**v)** What happens to the price of a derivative if it has a negative vega and the volatility increases? (3)

**vi)** Show that if the current spot price is $S_0$, and the continuous compounding rate is "*r*" then a call and a put both struck at $S_0e^{rT}$ and expiring at time "T" are of equal value. (3)

**vii)** Show that the Put – Call parity becomes $C = P + S\, e^{-qT} – PV(X)$ under the continuous model. (3)

**viii)** A binary call pays off \$1 if the underlying asset finished above the strike price and nothing otherwise. Show that its price equals $e^{-rT}\, N(x - \sigma\sqrt{T})$; where N (.) has the usual meaning in terms of the cumulative normal density function. (5)

**[23]**

**Q. 6)** A stock is currently priced at \$160 and pays no dividends. The price at time "i+1" is given by $S_{i+1}$ and can be written as $S_{i+1} = S_i + 0.5\, S_i$ for the up move or $S_{i+1} = S_i – 0.5\, S_i$ for the down move. The risk free interest rate is 18.232% per period and the strike price is \$150.

Use this information to consider a European Call option on this stock with a strike price of \$150 with three periods to expiration.

**i)** Draw a three period binomial tree. (4)

**ii)** Using the tree drawn in part (i), price a European Call Option using the information given in the question (rounded to the nearest integer). (4)

**iii)** Repeat part (ii) but now price the corresponding American Call Option. (4)

**iv)** Finally, using the tree drawn in part (i) and the information given in the question, what will be the price of the American Put Option? (3)

**[15]**

**Q. 7)** Why is it not optimal to exercise an American put immediately before an ex-dividend date? **[5]**

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