INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

16th September 2016

Subject CT8 – Financial Economics

Time allowed: Three Hours (10.30 – 13.30 Hrs.)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION
Please return your answer book and this question paper to the supervisor separately.

Q. 1) Describe the evidence for and against all the three forms of Efficient Market Hypothesis covering detailed aspects related to informational asymmetry and volatility tests. **[9]**

Q. 2) A market has 2 uncorrelated risky securities A and B. The mean and variance of returns of these securities are given as follows:

Security	Mean return	Standard deviation of returns
A	6%	10%
B	8%	12%

i) Determine the equation of efficient frontier if these are the only assets in the market. (5)

ii) State the risk characteristics of the investors for whom this portfolio is 'efficient'. (1)

iii) Determine the risk free rate 'r' and hence the equation of new efficient frontier if there exists a risk free asset and x_A=1.5x_B where x_i is the proportion of security I in the efficient portfolio. (7)

iv) Express the above equation as a Capital Market Line. (2)

v) Determine the beta for both the securities A and B assuming that the CAPM holds for the market. (3)

[18]

Q. 3) **i)** Suppose X_1 and X_2 are two generalized Weiner processes with drift and variances (μ_1, μ_2) and $({\sigma_1}^2, {\sigma_2}^2)$ respectively.

State the process followed by $X_1 + X_2$ if

a) The changes in X_1 and X_2 are uncorrelated in a short interval of time (2)

b) There is a correlation between changes in X_1 and X_2 in any short interval of time (2)

ii) Stock X_1 and X_2 both follow Geometric Brownian motion with drift and volatility given by μ_i and σ_i ,respectively, where i = either X_1 or X_2. As stated in part (i) Assume changes in any short interval of time are uncorrelated with each other. Explain whether the value of the portfolio consisting of stock X_1 and stock X_2 will follow the same distribution (3)

iii) A company's cash position follows X_1 as in part (i) such that drift rate = 0.5 per quarter and variance = 9 per quarter. Determine the initial cash position of the company to have less than 5% chance of a negative cash position by the end of the year? (4)

iv) What initial cash position is required so that the probability of the cash flow being less than 1 by the end of year is 5%, assuming the initial cash flow follows the distribution in part (ii)? (4)

[15]

Q. 4) Suppose that price of a bond at time T is given by $F(y_T)$ where y is the forward bond yield which is assumed to follow a Geometric Brownian motion in a risk neutral world. Suppose that the growth rate of the forward bond yield is α and its volatility is σ.

i) Determine the process followed by the forward bond price in terms of α, σ, y and F(y) (3)

ii) Determine the value of α assuming that the forward bond price follows a martingale in the world stated. (3)

iii) Explain if the bond yield can be negative. (2)

iv) Explain if stock price and return on stock price can be negative given that the underlying stock price follows Geometric Brownian motion (2)

v) Define convexity and modified duration and establish the relation between the two. Express α in terms of modified duration (5)

[15]

Q. 5) **i)** State the price of a forward contract at time t on a non dividend paying stock with price S_t Sat time t and show that it satisfies Black Scholes differential equation (3)

ii) Derive the price of a derivative at time t that pays off $1/S_T$ at time T and show that it satisfies Black Scholes differential equation. (7)

[10]

Q. 6) **i)** Derive the price of an option in one step binominal model using risk neural valuations. (5)

ii) What does the price of an option derived above say about the probabilities of the underlying stock going up or down? (2)

iii) Show that the underlying stock price grows on average at risk free rate (3)

iv) Interpret (i) and (iii) in terms of risk neutral world (2)

[12]

Q. 7) Black-Derman-Toy model for short rate 'r' is given by $d(\ln(r)) = \theta(t)dt + \sigma dW$. Compare and contrast this model with Vasicek Model of short rates. **[4]**

Q. 8) A three state Jarrow-Lando-Turnbull model has following one year transition probability:

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{pmatrix} = \begin{pmatrix} 0.7 & 0.2 & 0.1 & 0 \\ 0.1 & 0.6 & 0.2 & 0.1 \\ 0 & 0.3 & 0.3 & 0.4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Where state 1, 2 and 3 corresponds to the following states:

1	AAA rating	3	Junk Rating
2	BB rating	4	Default

i) What is the probability that a credit event occurs till the maturity of the bond. (2)

ii) Determine the credit spread on a 3 - year zero coupon bond issued by the company that is redeemable at par if the company currently has AAA rating and the risk free rate is 7% p.a. (6)

[8]

Q. 9) State and explain the conditions for First Order and Second Order Stochastic Dominance between two securities A and B. **[4]**

Q. 10) There are 100 investment contracts currently priced at INR 200 per contract that provide returns, on the current price, as per following distribution

Returns	**Probability**
6.00%	65%
8.5%	35%

Behaviour of each contract is independent of the other. Determine the values of the following at the end of one time period:

i) Expected value of the portfolio (1)

ii) Standard deviation of the value of the portfolio (1.5)

iii) Value at risk at 95% confidence level. Explain in words the significance of the calculated number. (2.5)

[5]

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