INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

8th September 2016

Subject CT4 – Models

Time allowed: Three Hours (10.30 – 13.30 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*
2. *Mark allocations are shown in brackets.*
3. *Attempt all questions, beginning your answer to each question on a separate sheet.*
4. *Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) You have been commissioned to develop a model to project the assets and liabilities of an insurer after one year. This has been requested following a change in the regulatory capital requirement. Sufficient capital must now be held such that there is less than a 0.5% chance of liabilities exceeding assets after one year.

The company does not have any existing stochastic models, but estimates have been made in the planning process of the worst case scenarios.

i) Set out the steps you would take in the development of the model. (4)

ii) Write down The difference between Stochastic and Deterministic Model. (4)
[8]

Q. 2) Prove that

For any $0 < a < b < 1$,

i) Under the assumption of uniform distribution of deaths,
$_{b-a}q_{x+a} = (b-a)q_x / 1 - aq_x$ (2)

ii) Under the Balducci assumption, (which is defined as $_{1-t}q_{x+t} = (1-t)q_x$ for $0 <= t <= 1$)
$_{b-a}q_{x+a} = (b-a)q_x / (1-(1-b)q_x)$ (2)

iii) Under the constant force of mortality assumption,

$_{b-a}q_{x+a} = (1-q_x)^a - (1-q_x)^b / (1-q_x)^a$ (2)

iv) Comment on the relationship between the force of mortality and duration under the above 3 assumptions (2)
[8]

Q. 3) **i)** In the context of a Stochastic process denoted by $\{X_t : t\ J\}$, define:

a) State space
b) Time set
c) Sample path
d) Jump Chains (2)

ii) Stochastic process models can be placed in one of four categories according to whether the state space is continuous or discrete, and whether the time set is continuous or discrete. For each of the four categories:

a) State a Stochastic process model of that type. (2)

b) Give an example of a problem an actuary may wish to study using a model from that category. (4)
[8]

Q. 4) A study was undertaken on the duration that the drug users were able to avoid (use of drug) following a treatment programme. It was assumed that the following factors influenced the duration

- Age of the drug addict
- Gender of the drug addict
- Type of drug taken – heroin, cocaine, others
- Number of Previous treatments undertaken

i) Suggest, with reasons, a suitable model for analyzing the above (1)

ii) Give an expression for the 'hazard' function, explaining all the terms used (2)

iii) What are the assumptions underlying the model proposed? (1)

iv) In a particular part of the investigation, interest focused on the impact of gender on relapse, so the other covariates were not considered. Ten lives (five men and five women) were observed following the end of the treatment programme until either they resumed using the drug or were lost to follow-up for other reasons. The observed times until resuming drug use or loss to follow-up are given below (where times with a '+' denotes a loss to follow-up and times without a '+' are times at which drug use was observed to resume).

Females: 4+, 6, 8+, 9, 10+
Males: 2, 5, 5+, 6, 9+

Derive the partial likelihood contribution for the observed deaths. (4)
[8]

Q. 5) **i)** State the principle of correspondence (1)

ii) Different definitions of d_x are given below. The mortality analysis is carried over a period of one year from 1st January to 31st December. For each definition, give corresponding definitions and suitable approximations for E_x^c (central exposed to risk), giving the assumptions used and explain what may be estimated by d_x/E^c_x

a) d_x = deaths with age x nearest birthday at death (2)

b) d_x = deaths in calendar year of xth birthday (2)

c) d_x = deaths with age x last birthday on the last policy anniversary (2)

iii) Explain what is meant by initial exposed to risk and state how it differs from E_x^c. (1)

iv) State how the 'initial exposed to risk' can be derived from E_x^c, in the above examples, if a) the exact age at death is known and b) exact age at deaths is not known (2)
[10]

Q. 6) The mortality analysis of insured lives aged 45 was undertaken in which the lives aged 45 were observed exactly for one year or till earlier death. The investigation was done during the period from 1st January 2015 to 31st December 2015. Lives went out of the observation on attaining age 46 or if the policy was lapsed/surrendered.

Data pertaining to 6 lives is given below

Life	**Date of birth**	**Status on 31st December 2015**	**Date of death/lapse/surrender**
A	1st January 1970	In-force	
B	1st November 1969	Lapsed	1st May 2015
C	1st May 1970	In-force	
D	1st October 1969	Died	1st May 2015
E	1st February 1969	Lapsed	1st November 2014
F	1st April 1970	Died	1st October 2015

Assume that the survival function follows a Weibull distribution with parameters **α** and **β.**

i) Derive the likelihood function of observing the data as given in the table above and express this in terms of **α** and **β.** (6)

ii) Explain how you would estimate the parameters **α** and **β.** (1)

iii) Derive the likelihood function of observing the above data, if instead of Weibull distribution an exponential function represented the survival function with force of mortality μ (1.5)

iv) Maximise the likelihood function and derive the estimate of μ (1.5)

v) Explain the kind of censoring present in the above analysis (2)

[12]

Q. 7) In a tailor made game of wrestling in India, when the two players have equal score called "Identical", the player winning the next point holds "Edge". If a player holding "Edge" wins the following point that player wins the game, but if that point is won by the other player the score returns to "Identical".

When Hanu plays wrestling against Bheem, the probability of Hanu winning any point is 0.6. Consider a particular game when the score is at "Identical".

i) Show that the subsequent score in game can be modelled as a Markov Chain, specifying both the state space; and the transition matrix (3)

ii) State, with reasons, whether the chain is:

a) irreducible; and

b) aperiodic (2)

iii) Calculate the number of points which must be played before there is more than a 90% game having been completed. (3)

iv) a) Calculate the probability Hanu ultimately wins the game. (4)

b) Comment on your answer. (1)

[13]

Q. 8) i) Explain what is meant by graduation and the aims of graduation (2)

ii) Explain the three desirable features of graduation (1)

iii) Describe what is meant by ‘over-graduation’ and ‘under-graduation’ (2)

iv) What are the methods of graduation – Give the advantages, disadvantages of each method and also give examples of situations where each would be most appropriate (5)

v) In one of the mortality studies, the crude mortality rates have been fitted by using the below equation

$$q_x = A \exp\left(-B * x\right) + \frac{CD^x}{1 + CD^x}$$

Where q_x is the probability of a person aged x dying before age x+1 and A, B, C, D are parameters that were estimated by least squares.

The parameters were estimated separately for males and females and are as given below

Parameter	**Male**	**Female**
A	0.00119	0.00012
B	0.86895	1.83524
C	0.00012	0.00007
D	1.08145	1.08175

Below are the crude mortality rates observed at certain ages

Age	**Male**	**Female**
20	0.00569	0.00337
25	0.00794	0.00468
28	0.01106	0.00651
33	0.01541	0.00904
66	0.02151	0.01255
71	0.02998	0.01744

For the above ages, derive the graduated rates using the formula given above (4)

vi) Perform an overall test of the graduation process and comment on the appropriateness of the graduation. (3)

[17]

Q. 9) The Candy maker has a single machine that is used to prepare different shapes of candies in children's amusement park.

The machine has a tendency to break down, at which point it must be repaired. The time until breakdown and the time required to effect repairs both follow the exponential distribution.

Let $P1i(t)$, $i = 0, 1$, be the probability that at time t ($t > 0$) there are i candy machines working, given that the candy maker machine is working at time $t = 0$.

i) Derive the Kolmogorov forward differential equations for $P1i(t)$ $i = 0, 1$ in terms of:

σ where $1/\sigma$ is the mean time to breakdown for a machine; and

ρ, where $1/\rho$ is the mean time to repair a machine (3)

ii) Show that $P10\ (t) = \sigma /(\sigma + \rho)^* \ (1- \exp - (\sigma + \rho)\ ^*t)$ deduce the value of $P11(t)$. (5)

iii) The candy maker is considering adding a second identical machine, though there is only one repair team to work on the machines in the event that both are out of action simultaneously. Assuming that a second machine is added and operates independently of the first one:

a) Write down the generator matrix of the Markov jump process Xt which counts the number of working candy maker machines at time t. (2)

b) Derive the Kolmogorov forward differential equations for $pi(t)$, $i = 0, 1, 2$, (the probability that i ticket machines are working) (3)

c) Given that, for some t,

$P0\ (t) = 2\sigma^2/(2\sigma^2 + 2\rho\sigma + 2\rho^2)$
$P1\ (t) = 2\rho\sigma/(2\sigma^2 + 2\rho\sigma + 2\rho^2)$
$P2\ (t) = \rho^2)/(2\sigma^2 + 2\rho\sigma + 2\rho^2)$

Show that $d/dt\ P_i(t) = 0$ for $i = 0,1,2$ (1)

d) State what conclusions you draw from part (c). (2)

[16]

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