INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

14th September 2016

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (10.30 – 13.30 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*

2. *Mark allocations are shown in brackets.*

3. *Attempt all questions, beginning your answer to each question on a separate sheet.*

4. *Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

Q. 1) In each of following circumstances, state with reasons whether the calculations should use money or real rate of interest.

    i) A man looking to make investments to cover for his new born child's college fees.   (1)

    ii) A company giving a wrist watch and cash amount of INR 10,000 to its employees on completion of 10 years with them.   (1)

    iii) An aged woman purchasing a fixed annuity to pay off her care home rent in future.   (1)
    [3]

Q. 2) If an investment fund offers to increase INR 20,000 to INR 26,000 in 17 months, calculate the following:

    i) The nominal rate of interest per annum convertible quarterly   (1)

    ii) The nominal rate of discount per annum convertible half yearly   (1)

    iii) Simple rate of interest per annum   (1)

    iv) Comment on the results.   (1)
    [4]

Q. 3) **i)** Describe how cash flows are exchanged in an interest rate swap   (2)

    ii) The Liabilities of an insurance company involves a series of fixed cash payments, evenly spread over the next ten years. All its investments are currently held in cash. Describe the structure of a fixed-interest swap that could help the insurance company immunize its liabilities.   (2)
    [4]

Q. 4) **i)** List reasons for the returns achieved over a period up to the maturity date of a fixed interest bond to be uncertain.   (2)

    ii) A loan of INR 200,000 nominal bears an interest of 5.5% per annum payable quarterly in arrear and is redeemable in 7 years time at INR 110 per 100 nominal. The term of the loan is to be altered such that the repayment would be by a single sum of INR 297,000 at a certain future time. Calculate this time assuming a rate of interest at 6% p.a. effective while ignoring taxation.   (3)
    [5]

Q. 5) An investment bank models the expected performance of its assets over a five day period. Over that period, the return on the bank's portfolio, say $i,$ has a mean value of 0.1% and standard deviation of 0.2%.

If $(1 + i)$ is log-normally distributed, then calculate the value of j such that the

probability that *i* less than or equal to *j is* 0.05. **[4]**

Q. 6) Explain with the help of a graph, for an effective interest rate of 8% p.a, the relationship between equivalent nominal rates of interest convertible pthly (i^p) and p. **[5]**

Q. 7) **i)** A life insurance company is liable to make four payments at five yearly intervals, the first payment being due five years from now. The amount of the rth payment is (1000+100 r).

Calculate, using an effective annual interest rate of 5%, the present value, volatility and convexity of the liability. (6)

ii) The assets of a fund consist of a single zero coupon bonds that will mature 11.5 years from now with a redemption payment of INR. 4,823. Explain using Redington's theory of immunization, the portfolio details (3)

[9]

Q. 8) An Insurance company has a liability of INR 100,000 due in eight years' time. The company which has exactly sufficient money to cover the liability using a constant force of interest at 5% p.a., now wishes to invest this money in a combination of securities below

1. Zero coupon bond redeemable at par in 20 years' time
2. Very short term deposits equivalent to interest bearing cash

i) The insurance company requires that on the basis of the constant force of interest at 5% p.a., the discounted mean of the assets equal to that of the liability. Find the amounts to be invested in each of the above securities 1 and 2 (6)

ii) Assuming investments are made in securities 1 and 2, find the present value of the profit to the company on the basis of the constant force of interest at 3% p.a. (2)

[8]

Q. 9) Investors A and B are both liable to capital gain tax at the rate of 40% per annum but neither is liable to Income Tax.

Investor A bought a bond of nominal worth INR 1,000 bearing an interest of 6% p.a. payable half-yearly in arrear. The bond was to be redeemed at par ten years after the date of purchase, and the price paid was such that, if the investor A had held the bond until it was redeemed, he would have obtained a net yield on his investment at 10% p.a.

Five years after purchasing the bond and immediately after receiving the interest payment, the investor A sold it to investor B, paying capital gain tax. The bond was held by investor B until redemption.

i) Calculate the purchase price paid by the Investor B so that he obtains a net yield on his investment at 10% p.a., Also determine the net annual yield

obtained by the Investor A over his completed transaction.   (6)

ii) If the purchase price paid by Investor B was such that Investor A obtains a 10% p.a.net yield over his completed transaction, then find the net annual yield obtained by the Investor B on his investment.   (5)

[11]

Q. 10) i) The annual effective forward rate applicable over the period from t to t + r is defined as $f_{t,r}$ where t and r are measured in years. If $f_{0,1} = 8\%$, $f_{1,1} = 7\%$, $f_{2,1} = 6\%$ and $f_{3,1} = 5\%$, calculate the gross redemption yield at the issue date from a 4-year bond, redeemable at par, with a 5% coupon payable annually in arrears.   (7)

ii) Explain why the gross redemption yield from the 4-year bond is higher than the 4-year forward rate $f_{3,1}$.   (2)

[9]

Q. 11) Mr. Sahil is looking to evaluate cheapest internet option from the following:

a) Landline phone – monthly rental of INR 750 which will give him unlimited internet access; with installation charges at start amounting to INR 500.

b) Mobile data – a monthly data pack of INR 600 to get 2 GB of internet. Excess data is charged at INR 1/MB

c) Internet Dongle – monthly rental of INR 650 with initial purchase cost at INR 1,500. This will give 3 GB of internet. Excess data is charged at INR 1.5/MB

Assume evaluation between options is only on the basis of the above information and the customer remains neutral on all other aspects.

Mr. Sahil's current data usage is 1 GB, and is expected to increase at compounding rate of 2% p.m. Considering effective interest rate of 8% p.a., evaluate the three options using net present value with a time horizon of 5 years.

Consider 1 GB = 1,024 MB   **[13]**

Q. 12) Mr. Brown has approached a bank for a 20 year home loan for his new home, payable as monthly installments in arrears. The loan has three levels of monthly installments:

a) Pre-Possession – Before the possession of the house, Mr. Brown has to repay equal monthly installments containing only the interest component of the outstanding loan.

b) Post possession first 5 years – level monthly installment of INR 55,000.

c) Post possession after 5 years – level monthly installments to close the loan over

the total loan period.

Mr. Brown would like to take a loan of total INR 70 lakhs of which 50% will be taken at time 0 and the rest, at the time of the possession. The Bank has offered an interest rate of 9.5% p.a. for the first 10 years and 9% p.a. thereafter. The home will be ready for possession in 3 years' time.

You are required to prepare a home loan schedule for Mr. Brown which includes Monthly Instalment, Interest due paid, Capital repaid and Loan outstanding for the 6th, 51st, 101st and 151st repayments.   **[13]**

Q. 13) A long term investor has made investments in a unit linked balanced fund which invests in equity and bonds with 60%: 40% proportion respectively. The table below shows the amount of investment in Equity and bonds against specific dates

Time	Total cash flow	Equity fund value just before investment	Bond fund value just before investment
1st Jan 2010	100,000	-	-
1st Jan 2011	50,000	57,500	43,500
1st Jan 2012	-50,000	92,100	65,200
1st Jan 2014	100,000	67,200	53,600
1st Jan 2015	-	142,700	104,000

i) Calculate the effective money weighted rate of return (MWRR) and time weighted rate of return (TWRR) on total investments.   (5)

ii) Explain the reason for difference between the two measures.   (1)

iii) Compare the effective time weighted rate of return (TWRR) for investments in equity and bonds.   (6)
                                                                                       [12]

****