Actuarial Society of India

EXAMINATIONS

08th November 2006

Subject ST6 – Finance and Investment B

Time allowed: Three Hours (2.15* - 5.30 pm)

INSTRUCTIONS TO THE CANDIDATE

1. ***You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only but notes may be made. You then have three hours to complete the paper.***
2. ***You must not start writing your answers until instructed to do so by the supervisor.***
3. ***The answers are not expected to be any country or jurisdiction specific. However, if examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.***
4. ***Mark allocations are shown in brackets.***
5. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
6. ***Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.***

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION
Hand in BOTH your answer script and this question paper to the supervisor.

Q.1) The Black-Scholes formula for a call option with strike price K and maturity T is:

$$f(t, s_t) = S_t \Phi(d_1) - Ke^{-r(T-t)} \Phi(d_2)$$

Where Φ denotes the standard normal cumulative distribution function.

Derive an expression for $-d_2$, the lower limit of the range of values of X for which $(S_T - K)$ is positive. **[5]**

Q.2) "Evergreen" is a commercial bank which issues a one-year bond which entitles the holder to the return on the S&P 500 share index up to a maximum level of 25% growth over the year. The bond has a guaranteed minimum level of return so that investors will receive at least x% of their initial investment back. Investors cannot redeem their bonds prior to the end of the year.

(i) Explain how the bank can use a combination of call and put options to prevent making a loss. (4)

(ii) The volatility of the S&P 500 index is 25% per annum and the continuously-compounded risk-free rate of return is 2.5% per annum. Use the Black-Scholes pricing formulate to determine the value of x that the bank should choose to make neither a profit nor a loss. (4)

[8]

Q.3) Consider an option on a non-dividend-paying stock when the stock price in Rs.30, the exercise price is Rs.29, the continuously-compounded risk-free rate of interest is 5% per annum, the volatility is 25% (in annual units) and the time to maturity 4 months.

(i) Using the Black-Scholes model, calculate the price of the option if it is:

(a) an European call

(b) an American call

(c) an European put

The stock is now assumed to be dividend-paying and will go ex-dividend in 1½ months' time. The expected dividend is Re. 0.50. (5)

(ii) Without doing any further calculations, explain how you would modify each of your calculation in (i) to allow for the dividend, and state, with reasons, what effect this would have on your answers. (5)

[10]

Q.4)

(a) Define hedge ratio (1)

(b) Derive the formula for hedge ratio from spot price and future price (2)

(c) What is meant by 'rolling the hedge forward'? What are the risks associated in using this technique. (2)

(d) Define 'basis risk' and briefly outline three main causes of basis risk. (2)

[7]

Q.5)

(a) Suppose that a = 0.1, µ = 0.1, and s = 0.02 in Vasicek's model with the initial value of short rate being 10% per annum. Consider a bond that has a principal of Rs. 1000 and pays a coupon of Rs. 50 every six months. The bond will mature in two years. What is the current price of the bond given by the Vasicek model? (5)

(b) Explain why a bank (acting as intermediary) is subject to the credit risk when it enters into two offsetting swap contracts. (3)

(c) Define:

(i) Aggregation Risk
(ii) Concentration Risk
(iii) Operational Risk (3)

[11]

Q.6) It is June 25, 2006. The futures price for the June 2006 CBOT (Chicago Board of Trade) bond futures contract is Rs.118.75.

(a) Compute the conversion factor for a bond maturing on January 1, 2022, paying a semi-annual coupon of 10% per annum. (3)

(b) Compute the conversion factor for a bond maturing on October 1, 2027, paying a semi-annual coupon of 7% per annum. (3)

(c) Suppose that the quoted prices of the bonds in (a) and (b) are 170 and 140, respectively. Which bond is cheaper to deliver?
The quoted price is for a bond with a face value of Rs. 100. (1)

[7]

Q.7) Suppose that the LIBOR yield curve is flat at 6% per annum with continuous compounding. Consider a swaption that gives the holder the right to enter into a three-year swap starting in four years where a fixed rate of 6% per annum (with semi-annual compounding) is received and LIBOR is paid. The volatility of the swap rate is 20% per annum. Payments are made semi-annually and the swap principal is $10 million. What is the value of the swaption? **[5]**

Q.8) Suppose that the par yields (with annual compounding) of bonds with various maturities are given in the following table. The face value of each bond is Rs. 1000.

Maturity (Years)	Par Yield (% Per Annum)
1	5
2	5.2
3	6
4	7
5	7

(a) Calculate zero rates (with continuous compounding) for maturities of 1 year, 2 years, 3 years, 4 years and 5 years. (3)

(b) What are the forward rates (with continuous compounding) for the periods: 1 year to 2 years, 2 years to 3 years, 3 years to 4 years, 4 years to 5 years? (2)

(c) How could you construct a one-year forward loan beginning at the end of year 3? Confirm that the rate on that loan equals the forward rate. Assume that the bonds can be bought and sold in fractions also. (3)

(d) Estimate the price of a three-year bond providing annual coupon of 6% per annum. (2)

(e) If you forecast that the yield curve in one year will be flat at 7% (with continuous compounding), what is your forecast for the expected rate of return on the coupon bond [mentioned in (d)] for the one year holding period? (2)

[12]

Q.9)

(a) Explain the expression $V_o = F\ S_o + ?$ as used in a simple one-period binomial model. (3)

(b) A stock price is currently Rs. 500. Over each of the next two quarters it is expected to go up by 6% or down by 5%. The risk free interest rate is 5% per annum with continuous compounding.

(i) What is the value of a six month European call option with a strike price of Rs. 510 (3)

(ii) What is the value of a six month European put option with a strike price of Rs. 510 (2)

(iii) Verify that the European call and European put prices satisfy put-call parity (2)

(iv) If the put option were American, would it ever be optimal to exercise it early at any of the nodes on the tree (3)

(v) With respect to the binomial tree explain what is meant by a “filtration” (2)

[15]

Q.10)

(a) Explain how the derivation of futures price from the spot price and other observable variables is different for silver and sugar. (2)

(b) Explain the terms “convenience yield” and “cost of carry”. State the relationship between the futures price, the spot price, the convenience yield and the cost of carry. (3)

(c) Show that when the risk free rate is constant and the same for all maturities, the forward price for a contract with a certain delivery date is the same as the futures price for a contract with that delivery date. (8)

[13]

Q.11)

(a) When is the process $W = (W_t : t = 0)$ said to be a Brownian motion under probability measure P (2)

(b) If Z is a normal N (0, 1), then the process $X_t = vt\ Z$ is continuous and marginally distributed as a normal N(0,t). Explain whether X is a Brownian motion. (3)

(c) State the Martingale Representation Theorem. (2)

[7]

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