Actuarial Society of India

EXAMINATIONS

1st November 2006

Subject CT4 (II) – Survival Model (104 Part)

Time allowed: One and a Half Hours (10.30 am – 12.00 noon)

INSTRUCTIONS TO THE CANDIDATES

1. ***Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
4. ***Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.***
5. ***In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.***

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor

B1)

a) Define complete and curtate expectation of life and derive from first principles their algebraic expressions. (4)

b) For a particular population e_{50}= 25.35 and e_{51}=24.65 .Calculate q_{50} (2)

[6]

B2)

a) Define Type I and Type II censoring (2)

b) The following data relate to 12 patients who had an operation which was intended to correct a life threatening condition, where time 0 is the start of the period of investigation

Patient Number	Time of Operation (in weeks)	Time Observation ended (in weeks)	Reason Observation ended
1	0	120	Censored
2	0	68	Death
3	0	40	Death
4	4	120	Censored
5	5	35	Censored
6	10	40	Death
7	20	120	Censored
8	44	115	Death
9	50	90	Death
10	63	98	Death
11	70	120	Death
12	80	110	Death

You can assume that censoring was non-informative with regard to the survival of any individual patient.

(i) Compute the Nelson-Aalen estimate of the cumulative hazard function ? (t), where *t* is the time since having the operation (6)

(ii) Using the results of part (i), deduce an estimate of the survival function for patients who have had this operation (2)

(iii) Estimate the probability of a patient surviving for at least 70 weeks after undergoing the operation (1)

[11]

B3) A large life office is investigating the recent mortality experience of its term assurance policyholders. It has been decided to graduate the data by reference to a standard table using the formula:

$$q_x / q^s_x = ax+b$$

Where q^s_x is the rate for standard table.

a) Explain why it is necessary to graduate crude rates of mortality for practical use? (4)

b) Describe briefly how you would estimate a and b in the formula using

i.) A weighted least squares method

ii.) A maximum likelihood method (8)

[12]

B4)

(i) Given that $p_x = 0.9$, calculate ${}_{0.5}p_x$ and ${}_{0.5}p_{x+0.5}$ using the following assumptions about mortality between ages x and x+1:
   (a) Uniform distribution of deaths
   (b) Balducci assumption (4)

(ii) Comment on how appropriate you think each of these assumptions is (2)

[6]

B5) A mortality investigation has been carried out over the three calendar years: 2000, 2001 and 2002.

Deaths during the Period of Investigation , ?x have been classified by age *x* at the date of death , where
x = calendar year of death – calendar year of birth.

(i) State the principle of correspondence (1)

(ii) State the rate year implied by this classification and give the age range of the lives at the beginning of the rate year. (2)

(iii) Censuses of the numbers alive on 1 July 2000, 1 July 2001 and 1 July 2002 have been tabulated and denoted by Px(½), Px(1½) and Px(2½) respectively, where x is the age determined at the date of each census.

The force of mortality at age $x+f$ is to be estimated using the formula

$$\mu_{x+f} = \quad ?x/\ (Px(½)+Px(1½) + Px(2½)$$

Where ?x is the number of deaths.

a.) Determine the appropriate age definition *x* in $P_x(t)$, t =½,1½,2½ if this formula is correct. (3)

b.) Determine the value of f stating clearly all the assumptions you have made. (3)

[9]

B6) A mortality table has been estimated for the ages 4 to 100 inclusive. The rates have been graduated fitting a mathematical formula to the crude estimates. The deviations of the observed number of deaths from the expected number of deaths at each age using the graduated mortality rates have been calculated. The results are:

Positive Deviations	57
Negative Deviations	40

Test this graduation using the Signs Test by:

a) stating the Null Hypothesis
b) stating the sampling distribution of the test statistics if the Null Hypothesis is true
c) completing the test and stating your conclusions **[6]**

****