Actuarial Society of India

Examinations

November 2006

CT5 –General Insurance, Life and Health Contingencies

Indicative Solution

1.(a) The graph has the following features, which are typical of life tables based on human mortality in modern times:

- Mortality just after birth ("infant mortality") is very high.
- Mortality falls during the first few years of life.
- There is a distinct "hump" in the function at ages around 18–25. This is often attributed to a rise in accidental deaths during young adulthood, and is called the "accident hump".
- From middle age onwards there is a steep increase in mortality, reaching a peak at about age 80.
- The probability of death at higher ages falls again (even though q_x continues to increase) since the probabilities of surviving to these ages are small.

Total [3]

1.(b) ${}_t p_x = e^{-\int_0^t \mu_{x+s}^m ds}$ for males and ${}_t p_x = e^{-\int_0^t \mu_{x+s}^f ds}$ for females.

Where μ_x^m and μ_x^f are the constant force of mortality for males and females respectively.

Also, ${}_t p_x = \dfrac{\ell_{x+t}}{l_x}$. Thus $\ell_{60} = \ell_0 * {}_{60}p_0$

Let ℓ_0 be the same number of males and females at birth.

Then $\ell_{60} = \ell_0 * e^{-60*\mu_m}$ for males $\ell_{60} = \ell_0 * e^{-60*\mu_f}$ for females

Similarly $\ell_{61} = \ell_0 * e^{-61*\mu_m}$ for males $\ell_{61} = \ell_0 * e^{-61*\mu_f}$ for females

Therefore for the total population

$\ell_{60} = \ell_0 * (e^{-60*\mu_m} + e^{-60*\mu_f})$

$\ell_{61} = \ell_0 * (e^{-61*\mu_m} + e^{-61*\mu_f})$

$q_{60} = 1 - \dfrac{\ell_{61}}{l_{60}}$

Substituting the values of μ_x^m and μ_x^f as 0.10 and 0.08, we get

$$q_{60} = 1 - \frac{\ell_0 * (e^{-61*0.1} + e^{-61*0.08})}{\ell_0 * (e^{-60*0.1} + e^{-60*0.08})}$$

$$q_{60} = 1 - \frac{(e^{-6.1} + e^{-4.88})}{(e^{-6.0} + e^{-4.8})} = \quad 1\text{- } 0.9188852 \ = 0.0811147$$

Total [4]

2. Let 'I' be effective the rate of interest per annum and v = 1/(1+I)
Let 'S_t 'denote the state occupied by the policyholder at age 30+t, so that S_0 = a and 'S_t '= a, i or d for t = 1, 2, 3 etc.

Where 'a' is the healthy or able state, 'i' is the ill state and d is the dead state

the transition probability can be then be expressed as

$$p_{30+t}{}^{jk} = P(S_{t+1} = k \mid S_t = j)$$

the required expression can then be expressed as

$$3000 \int_0^{29} v^t \, {}_tp_{30}{}^{aa} \, \sigma_{30+t} \left(\int_1^{30-t} v^s \, {}_sp_{30+t}{}^{\overline{ii}} \, ds \right) dt$$

Total [3]

3. As the decrements γ and δ operate on 1st April and 1st October, the probabilities $(aq)_x{}^{\alpha}$ and $(aq)_x{}^{\beta}$ have to be expressed as integrals for the first quarter, next 2 quarters and last quarter as given below:

$$(aq)_x{}^{\alpha} = \int_0^{1/4} {}_t(ap)_x \, \mu_{x+t}{}^{\alpha} \, dt + (1 - q_x{}^{\gamma}) \int_{1/4}^{3/4} {}_t(ap)_x \, \mu_{x+t}{}^{\alpha} \, dt$$

$$+ (1 - q_x{}^{\gamma}) (1 - q_x{}^{\delta}) \int_{3/4}^{1} {}_t(ap)_x \, \mu_{x+t}{}^{\alpha} \, dt$$

where ${}_t(ap)_x = {}_tp_x{}^{\alpha} \, {}_tp_x{}^{\beta}$

Now ${}_tp_x{}^{\alpha} \, \mu_{x+t}{}^{\alpha}$ is constant and equal to $q_x{}^{\alpha}$ since the decrements are assumed to be evenly spread in the single life tables.

Hence

$$(aq)_x^{\alpha} = q_x^{\alpha} \left[\int_0^{1/4} {}_tp_x^{\beta}\, dt + (1 - q_x^{\gamma}) \int_{1/4}^{3/4} {}_tp_x^{\beta}\, dt + (1 - q_x^{\gamma})(1 - q_x^{\delta}) \int_{3/4}^{1} {}_tp_x^{\beta}\, dt \right]$$

and $$\int_0^k {}_tp_x^{\beta}\, dt = \int_0^k (1 - tq_x^{\beta})\, dt = k - (k^2/2)\, q_x^{\beta}$$

Therefore inserting figures, we have

$$(aq)_x^{\alpha} = (8/29) [((1/4) - (1/32) q_x^{\beta}) + (8/10) ((1/2) - (1/4) q_x^{\beta})$$
$$+ (6/10) ((1/4) - (7/32) q_x^{\beta})]$$
$$= (8/29) [(8/10) - (29/80)(4/29)] = 6/29$$

Similarly

$$(aq)_x^{\beta} = (4/29) [((1/4) - (1/32) q_x^{\alpha}) + (8/10) ((1/2) - (1/4) q_x^{\alpha})$$
$$+ (6/10) ((1/4) - (7/32) q_x^{\alpha})]$$
$$= (4/29) [(8/10) - (29/80)(8/29)] = 28/290$$

Total [10]

4. a)

   i. Time selection arises when the mortality of a homogeneous group of lives changes over time.

      For instance, the mortality of a life company's policyholders in 2005 would probably be different to that of equivalently aged policyholders in 1985, due to general improvements in mortality between these dates.

   ii. Spurious selection arises when:
      - we compare 2 groups of lives and observe different mortality
      - we then ascribe the variation to some apparent difference between the groups
      - but the mortality difference has arisen (at least partly) because the groups are heterogeneous with respect to some subtle factor which influences mortality
      - and the two groups have different proportions of these heterogeneous sub-groups.

An example would be measuring mortality of different age groups and ascribing all difference to age (i.e. class selection) when some of the difference in mortality is due to the older age groups containing a higher proportions of smokers than the younger age groups.

Total [5]

b)

	Region A		Country		CMR
Age	No. in force	Deaths	No. in force	Deaths	Region
-39	650	400	13,725	8,378	0.00062
40 – 59	500	2,430	8,145	45,385	0.00486
60+	385	27,500	6,390	489,889	0.07143

Standardised mortality rate =

$$= (13725 * 0.00062 + 8145 * 0.00486 + 6390 * 0.07143)/ (13725 + 8145 + 6390)$$

$$= 0.01785$$

Total [2]

c) Occupation can have several direct effects on mortality and morbidity. Occupation determines a person's environment for 40 or more hours each week. The environment may be rural or urban, the occupation may involve exposure to harmful substances e.g. chemicals, or to potentially dangerous situations e.g. working at heights. Much of this is moderated by health and safety at work regulations.

Some occupations are more healthy by their very nature e.g. bus drivers have a sedentary and stressful occupation while bus conductors are more active and less stressed. Some work environments e.g. pubs, give exposure to less healthy lifestyle.

Some occupations by their nature attract more healthy workers. This may be accentuated by health checks made on appointment or by the need to pass regular health checks e.g. airline pilots.

Some occupations can attract less healthy workers e.g. former miners who have left the mining industry as a result of ill-health and then chosen to sell newspapers. This will affect the mortality rates of newspaper sellers.

A persons' occupation largely determines their income, which permits them to adopt a particular lifestyle e.g. content and pattern of diet, quality of housing. This effect can be positive or negative e.g. over-indulgence.

Total [3]

5. For a Pure Endowment policy,

E[G] = $v^n * {}_n p_x$

Var[G] $= v_j^n * {}_n p_x - (v^n * {}_n p_x)^2$ ------------ -(A)

Where $j = (1+i)^2 - 1$.

Thus $v_j^n = v^{2n}$

Given, n=15, v=0.9 and Var[G] = 0.065 * E[G] --------(B)

We have, from (A),

Var[G] = $(0.9)^{30} * {}_{15}p_x - \{(0.9)^{15} * {}_{15}p_x\}^2 = 0.065 * (0.9)^{15} * {}_{15}p_x$

$= (0.9)^{30} * {}_{15}p_x * (1 - {}_{15}p_x)$

From (B), Var[G] = = $0.065 * (0.9)^{15} * {}_{15}p_x$

$$\Rightarrow (1 - {}_{15}p_x) = \frac{0.065 * (0.9)^{15} * {}_{15}p_x}{(0.9)^{30} * {}_{15}p_x}$$

$$\Rightarrow (1 - {}_{15}p_x) = 0.3157008$$

$$\Rightarrow {}_{15}p_x = 1 - 0.3157008 = 0.6842991$$

Since the force of mortality is constant, ${}_{15}p_x = (p_x)^{15}$

$$\Rightarrow (p_x)^{15} = 0.6842991$$

$$\Rightarrow p_x = (0.6842991)^{1/15} = 0.9750264$$

$$\Rightarrow q_x = 1 - p_x = 1 - 0.9750264 = 0.0249735 \cong 0.025$$

Total [5]

6. $({}_tV + P) * (1+i) = q_{x+t} * (1000 + {}_{t+1}V) + p_{x+t} * {}_{t+1}V$

$= q_{x+t} * (1000 + {}_{t+1}V) + (1 - q_{x+t}) * {}_{t+1}V$

$= q_{x+t} * (1000 + {}_{t+1}V - {}_{t+1}V) + {}_{t+1}V$

$$= 1000 * q_{x+t} + {}_{t+1}V$$

$${}_{t+1}V = ({}_tV + P) * (1+i) - 1000 * q_{x+t}$$

is the relationship between the reserves at time t and t+1.

The reserve at the beginning of the contract i.e ${}_0V = 0$ and the reserve required at the end of 3rd year is the maturity value, i.e ${}_3V = 1000$

Thus ${}_1V = ({}_0V + P) * (1.04) - 1000 * q_{[65]}$

$${}_1V = 1.04 * P - 1000 * 0.009864$$
$${}_1V = 1.04 * P - 9.864$$

$${}_2V = ({}_1V + P) * (1.04) - 1000 * q_{[65]+1}$$
$${}_2V = (1.04 * P - 9.864 + P) * 1.04 - 1000 * 0.014873$$
$${}_2V = (1.04)^2 * P - 10.25856 + 1.04 * P - 14.873$$
$${}_2V = (1.04)^2 * P + 1.04 * P - 25.13156$$

$${}_3V = ({}_2V + P) * (1.04) - 1000 * q_{67}$$
$${}_3V = \{(1.04)^2 * P + 1.04 * P - 25.13156 + P)\} * 1.04 - 1000 * 0.017824$$
$${}_3V = (1.04)^3 * P + (1.04)^2 * P + 1.04 * P - 26.136822 - 17.824$$
$${}_3V = P * (1.124864 + 1.0816 + 1.04) - 43.960822$$
$$1000 = 3.246464*P - 43.960822$$
$$3.246464 *P = 1000 + 43.960822$$
$$\Rightarrow P = 1043.960822 / 3.246464 = 321.56$$

Total [5]

7.

a) The profit vector is the vector of the expected end-year profits for policies which are still in force at the start of each year.

The profit signature is the vector of the expected year-end profits allowing for survivorship from the start of the contract.

Total [2]

b)

Age	qx
55	0.004469
56	0.005025
57	0.005650

		Unit-Fund				
	C1	C2=Annual Premium*Allocation rate	C3=C2*5%	C4=(C1+C2-C3)*8%	C5=(C1+C2-C3+C4)*1.25%	C6=C1+C2-C3+C4-C5
Year	**Value of Units at start**	**Allocation**	**Bid-Offer spread**	**interest**	**AMC**	**value of units at end**
1	0.00	8500.00	425.00	646.00	109.01	8611.99
2	8611.99	8500.00	425.00	1334.96	225.27	17796.67
3	17796.67	8500.00	425.00	2069.73	349.27	27592.14

Non-Unit Fund

			C1	C2	C3	C4=(C1+C2-C3)*4%	C5
Year	**Year**	**Age**	**Unallocated premium**	**Bid-Offer spread**	**Expenses**	**interest**	**AMC**
1	1	55	1500.00	425.00	600	53.000	109.01
2	2	56	1500.00	425.00	100	73.0000	225.27
3	3	57	1500.00	425.00	100	73.0000	349.27

	C6=max(unit value, 20000)*qx+t	C7 = Bid Value*10%	C8=C1+C2-C3+C4+C5-C6-C7
Year	Extra Death Benefit	Extra Maturity Cost	End of year cash flow
1	50.8930	0	1436.119
2	11.0717	0	2112.203
3	0.0000	2759.214	-511.950

Provision required at the start of the 3rd year 477.2696547
(511.95 – 2759.214 *(1- p57)) / 1.04
Revised Profit at the end of 2nd year 1637.331234
2112.203 – 477.269*p56

Year	Revised profit vector	Probablility in force	Discount factor	Profit Signature	PV of Premium
1	1436.119	1.000000	0.869565	1248.7995	10000
2	1637.331	0.995531	0.756144	1232.5248	8656.7913
3	0.000	0.990528	0.657516	0.0000	7489.8182

NPV	2481.32
PV of premium	26146.610
Profit Margin	9.49%

Total 15

8.

a) $(2/3)(30{,}000)(s_{40}/s_{39})({}^{z}M_{40}^{ia}/{}^{s}D_{40})$

$= (2/3)(30{,}000)(7.814/7.623)(58094/25059)$

$= 47{,}527.51$

b) $(30{,}000/80)(s_{40}/s_{39})\ [(18\ {}^{z}M_{40}^{ra} + {}^{z}\overline{R}_{40}^{ra} - {}^{z}\overline{R}_{62}^{ra})/\ {}^{s}D_{40}]$
$= (30{,}000/80)\ (7.814/7.623)\ [((18)(128{,}026) + 2{,}884{,}260 - 159{,}030)/25059]$

$= 77{,}153.73$

c) $50{,}000\ [(M_{40}^{i} + M_{40}^{r})/D_{40}]$

$= 50{,}000\ [(369 + 782)/3207]$

$= 17{,}945.12$

Total [9]

9. Overhead expenses are those that in the short term do not vary with the amount of business.

An example of an overhead expense is the cost of the company's premises (as the sale of an extra policy now will have no impact on these costs).

Direct expenses are those that do vary with the amount of business.

An example of a direct expense is commission payment to a direct salesman (as the sale of an extra policy now will have an impact on these costs).

Total [4]

10.(i) Premium calculated using equivalence principle

The premium equation is:

$P * \ddot{a}_{[35]} = 100{,}000\ A_{[35]} + 0.05 * P * \ddot{a}_{[35]}$

So:

15.993 * P = 100,000 * 0.09475 + 0.05* P * 15.993

$\Rightarrow 15.19335 * P = 9475$
$\Rightarrow P = 9475 / 15.19335 = 623.6281$
P= Rs.623.63

Total [4]

(ii) ***Minimum premium***

Let K denote the curtate future lifetime of a new policyholder. Then the insurer's loss random variable for the policy is:

$$L = 100{,}000 * v^{K+1} + 0.05 * P * \ddot{a}_{\overline{K+1}|} - P * \ddot{a}_{\overline{K+1}|}$$

$$= 100{,}000 * v^{K+1} - 0.95 * P * \ddot{a}_{\overline{K+1}|}$$

L will be positive if the policyholder dies "too soon". We want to find the value of t such that:

$$P(L > 0) = P(T < t) = 0.01$$

where T represents the policyholder's complete future lifetime.

In other words, we want to find t such that:

$$P(T \geq t) = {}_t p_{[35]} = 0.99$$

In terms of life table functions, we have:

$$\frac{l_{[35]+t}}{l_{[35]}} \geq 0.99$$

$$\Rightarrow l_{[35]+t} \geq 0.99 * l_{[35]} = 0.99 * 9892.9151 = 9793.99$$

From the *Tables*: l_{45} = 9,801.3123 and l_{45} = 9,786.9534
So t lies somewhere between 10 and 11, and we set K =10

So we need to find the "break even" premium P, assuming the benefit is paid at the end of year 11 and using 6% interest. This is given by the equation:

$$0.95 * P * \ddot{a}_{\overline{11}|} = 100{,}000 * v^{11}$$

$$\Rightarrow P = \frac{100{,}000 * v^{11}}{0.95 * \ddot{a}_{\overline{11}|}} = \frac{100{,}000}{0.95 * \ddot{s}_{\overline{11}|}} = \frac{100{,}000}{0.95 * 15.86994} = Rs.6{,}632.86$$

Total [6]

11. Expected Present Value (EPV) of Premiums:

$$12*P*\ddot{a}^{(12)}_{[45]:\overline{20}|} = 12*P*\left(\ddot{a}_{[45]:\overline{20}|} - \frac{11}{24}(1 - {}_{20}p_{[45]} * v^{20})\right)$$

$$= 12*P\left(11.888 - \frac{11}{24}\left(1 - (1.06)^{-20} * \frac{8821.2612}{9798.0837}\right)\right)$$

$$= 12*P*11.5583296 = 138.69995*P$$

Expected Present Value of Benefits:

$$\frac{100{,}000}{(1+b)} * (1.06)^{1/2} * A^{1}_{[45]:\overline{20}|} \ @\ j + 100{,}000 * \frac{D_{65}}{D_{[45]}} \ @\ j$$

where $j = \frac{i-b}{1+b} = \frac{0.06 - 0.0192308}{1 + 0.0192308} = \frac{0.0407692}{1.0192308} = 0.04$

$$A^{1}_{[45]:\overline{20}|} = A_{[45]:20} - \frac{D_{65}}{D_{[45]}} = 0.46982 - \frac{689.23}{1677.42} = 0.058933$$

$$\frac{D_{65}}{D_{[45]}} = \frac{689.23}{1677.42} = 0.41088695$$

$$= \frac{100{,}000}{(1 + 0.0192308)} * (1.06)^{1/2} * 0.058933 + 100{,}000 * 0.41088695$$

$$= 5953.0463 + 41088.695 = 47041.741$$

Expected Present Value of Expenses:

$$0.35*12*P + 800 + 0.05*12*P*\left(\ddot{a}^{(12)}_{[45]:\overline{20}|} - \ddot{a}^{(12)}_{[45]:\overline{1}|}\right) + 150*\left(\ddot{a}_{[45]:\overline{20}|} - 1\right)$$

$$\ddot{a}^{(12)}_{[45]:\overline{20}|} = 11.5583296$$

$$\ddot{a}^{(12)}_{[45]:\overline{1}|} = \ddot{a}_{[45]:\overline{1}|} - \frac{11}{24}(1 - {}_{1}p_{[45]} * v^{1})$$

$$= 1 - \frac{11}{24}\left(1 - \frac{l_{[45]+1}}{l_{[45]}} * (1.06)^{-1}\right)$$

$$= 1 - \frac{11}{24}\left(1 - \frac{9786.3162}{9798.0837} * (1.06)^{-1}\right) = 0.973537$$

$$= 0.35 * 12 * P + 800 + 0.05 * 12 * P * (11.5583296 - 0.973537) + 150 * (11.888 - 1)$$

=4.2*P + 800 +6.3508753*P + 1633.2 = 10.550875*P + 2433.2

Claim Expenses are 2.5% of the EPV of Benefits = 0.025* 47041.74 = 1176.04

By the principle of Equivalence, the EPV of Premiums must be equal to the sum of the EPVs of Benefits and Expenses.

$$138.7 * P = 47041.74 + 10.550875 * P + 2433.2 + 1176.04 = 50650.98 + 10.550875 * P$$
$$\Rightarrow P * (138.7 - 10.550875) = 50650.98$$

$$\Rightarrow P = \frac{50650.98}{(138.7 - 10.550875)} = \frac{50650.98}{128.14913} = 395.2503$$

The Premium for the policy is Rs.395.25 **Total [10]**

11 (ii) The Prospective Reserve:

$$12 * P' * \ddot{a}^{(12)}_{45:\overline{20}|} = 100{,}000 * \overline{A}_{45:\overline{20}|} = 100{,}000 * \left((1.04)^{1/2} * A^{1}_{45:\overline{20}|} + \frac{D_{65}}{D_{45}}\right)$$

$$\ddot{a}^{(12)}_{45:\overline{20}|} = \ddot{a}_{45:\overline{20}|} - \frac{11}{24}(1 - {}_{20}p_{45} * v^{20})$$

$$= 13.780 - \frac{11}{24}\left(1 - \frac{l_{65}}{l_{45}} * (1.04)^{-20}\right)$$

$$= 13.780 - \frac{11}{24}\left(1 - \frac{8821.2612}{9801.3123} * (1.04)^{-20}\right) = 13.780 - 0.270072$$
$$= 13.509928$$

$$12 * P' * 13.509928$$
$$=100{,}000 * \left((1.04)^{1/2} * (0.46998 - 0.41075) + (1.04)^{-20} * \frac{8821.2612}{9801.3123} \right)$$

= 100,000*(0.0604029 + 0.41075) = 47115.29

$$\Rightarrow P' = \frac{47115.29}{12 * 13.509928} = \frac{47115.29}{162.11914} = 290.62$$

$\therefore$ The net premium for the policy is Rs.290.62.

The reserve at the end of the 13th year is calculated as follows:

$${}_{13}V = (100{,}000 + 25{,}000) * \overline{A}_{58:\overline{7|}} - 12 * P' * \ddot{a}^{(12)}_{58:\overline{7|}}$$

$\overline{A}_{58:\overline{7|}}$ = 0.766211 (Calculated in the same way as above)

$\ddot{a}^{(12)}_{58:\overline{7|}}$ = 5.9740403 (Calculated in the same way as above)

$${}_{13}V = 125{,}000 * 0.766211 - 12 * 290.62 * 5.974043 = Rs.74{,}942.27$$

Total [6]

12. The definition of l_x tells us that for every 1,000 lives age 40, we would expect 5 to die at each age 40, 41, *etc*.

So the value of the endowment assurance is:

$$A_{40:\overline{10|}} = \frac{5v + 5v^2 + 5v^3 + \ldots.. + 5v^{10} + 950v^{10}}{1000} \quad @6\%$$

$$A_{40:\overline{10|}} = \frac{5a_{\overline{10|}} + 950v^{10}}{1000} @ 6\%$$

$$= \frac{5 * 7.3601 + 950 * 0.55839}{1000} = 0.5673$$

Total [4]

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