Actuarial Society of India

Examinations

November 2006

CT4 (104 PART) –Survival Models

Indicative Solution

1 (a): The complete expectation of life at age x is defined as E[Tx], i.e., the expected future lifetime after age x. It is denoted by $\mathring{e}_x$.

$$\mathring{e}_x = \int_0^{w-x} {}_tp_x \mu_{x+t}\, dt$$
$$= \int_0^{w-x} t.(-d/dt({}_tp_x))\, dt$$
$$= -[t.\,{}_tp_x]_0^{w-x} + \int_0^{w-x} {}_tp_x\, dt$$
$$= \int_0^{w-x} {}_tp_x\, dt$$ since the term in the square brackets is zero for both t=0 and t=w-x (2)

The curtate expectation of a life age x is: Kx = [Tx] where the square brackets indicate the integer part. In other words Kx is equal to Tx rounded down to the integer below. It is represented by e_x

$$e_x = E[Kx]$$
$$= \sum_{0}^{w-x} k.\,{}_kp_x\,.\,q_{x+k}$$
$$= {}_1p_x\,.\,q_{x+1}$$
$$+ 2p_x\,.\,q_{x+2+2}p_x\,.\,q_{x+2}$$
$$+ \ldots\ldots$$
$$= \sum_{k=1}^{[w-x]} \sum_{j=k}^{[w-x]} {}_jp_x\,.\,q_{x+j} \quad \text{(Summing Columns)}$$
$$= \sum_{k=1}^{w-x} {}_kp_x$$

The last step follows since

$\sum^{[w-x]} {}_jp_x\,.\,q_{x+j}$ represents the probability of dying at any time ager age x+k, which can be written simply as ${}_kp_x$.

1(b) e_{50}= 25.35 e_{51}=24.65

$$e_x=p_x(1+e_{x+1})$$

$$25.3= p_x\,(1+24.65)$$
$$p_x = 25.35/25.65 = 0.98830$$

Therefore $\mathbf{q_x}$ **= 1- 0.98830 =0.0117**

[6]

2(a) Type I Censoring:

If the censoring times $\{c_j\}$ are known in advance then the mechanism is called Type I censoring. (1)

If observation is continued until a predetermined number of deaths has occurred, then "Type-II Censoring" is said to be present. (1)

1	2	3	4	5	6
T	n_j	d_j deaths	d_j/ n_j	$\sum d_j/ n_j$	$S(t)_j = \exp(-\Lambda(t))$
$0 \le t < 30$	12	0	0	0	1
$30 \le t < 35$	12	2	2/12	0.16667	0.8464
$35 \le t < 40$	9	1	1/9	0.2778	0.7575
$40 \le t < 50$	8	2	2/8	0.5278	0.5899
$50 \le t < 68$	6	1	1/6	0.6945	0.4993
$68 \le t < 71$	5	1	1/5	0.8945	0.4088
$71 \le t < 120$	4	1	1/4	1.1444	0.3184

Correct Answers till column 5 shall be awarded 6 marks.

Correct answers in column 6 should be awarded 2 marks which is the answer to 2 b(ii).

(iii) S (70) = 0.4088.

[11]

3 (a)

- To make them fit for the purpose for which they are intended
- To remove random sampling errors, thus better estimating the true underlying mortality rates
- To allow the rate at a particular age to be set with reference to the rates at adjacent ages
- To produce a set of mortality rates that progress smoothly from age to age which allows a practical smooth set of premium rates to be produced.

3(b) (i) Weighted lowst squares method:

Answer: We need to look at the function: $\sum\left[\hat{q}_x - q_n^s(ax+b)^2\right]^2 \times \dfrac{Ex}{\hat{q}_x}$

And minimise this function by the choice of parameters a & b. This is a weighted sum of squares, where the wieght are inversely proportional to the approximate variance of

(ii) Maximum likelihood method:

We have On = obseved deaths at age x
En = initial enposed to risked at age x
And the graduated ratios are to be calculated from the relationship;

$$q_n^0 = q_n^s\left(ax+b\right)$$

Let L be the likelihood of obtaining the actual observed set of on values based on given parameter values a and b. These

$$L = \pi_x \binom{E_x}{\theta_x} \left(\overset{0}{q}_x\right)^{\theta_x} \left(1-\overset{0}{q}_x\right)^{E_x-\theta_x}$$

$$= \pi_x \binom{E_x}{\theta_x} \left(q_x^s\right)^{\theta_x} \left(1-q_n^s\left(ax+b\right)\right)^{E_x-\theta_x}$$

The maximum likelihood estamates of a and b are those values of the paramaters that maximise L, or equavalently log L, which is :

$$Log_e L - \sum_x \left\{ \log\binom{E_x}{\theta_n} + \theta_n \log_e\left(q_x^s\left(ax+b\right)\right) + \left(E_x-\theta_n\right)\log_e\left(1-q_x^s\left(ax+b\right)\right) \right\}$$

The first term is the summation is fixed and hence we can reduce the Log likelihood to:

$$Log_e L - \sum_x \left\{ \theta_n \log_e\left(q_x^s\left(ax+b\right)\right) + \left(E_x-\theta_n\right)\log_e\left(1-q_x^s\left(ax+b\right)\right) \right\}$$

We can obtain simultanous equations for the maximum likelihood estimates of a and b by solving simultaneous equations $\frac{d}{da}\log_e L = 0$ *and* $\frac{d}{db}\log_e L = 0$

[12]

(4) (i)(a) Uniform distribution of deaths:
Under uniform distribution of deaths

$${}_t v_n = t - v_n$$

$$Pn = 0.9$$

$$\therefore\ {}_{0.5}v_n = 0.5\times(1-0.9) = 0.05$$

$\therefore {}_{0.5}P_n = 1 - {}_{0.5}v_n = 1 - 0.05 - 0.95$

${}_{0.5}P_{x+0.5} = P_x / {}_{0.5}P_x = 0.9/0.95 = 0.9474$

(b) Balducci assumption :

${}_{1-t}v_{x+t} = (1-t)q_x$

${}_{0.5}P_{x+0.5} = 1 - \left[(1-0.5)q_x\right]$

$= 1 - 0.5q_n$

$= 1 - 0.5[1-0.9] = 0.9$

$P_x = {}_{0.5}P_{x+0.5} \times 0.5P_x$

$\therefore {}_{0.5}P_x = 0.9/0.95 = 0.9474$

(ii) Under UDD assumption, 0.5 Pn+ 0.5 < 0.5 Pn, to the force of mortality is increasing between x and x+1. Conversely under Balducci is asssumption, the force of mortality is decreasing. So UDD assumption seams more appropriate for most ages. The Balducci assumption would be appropriate at very young ages or the back of the accident Lump.

[6]

(5) (i) Principle of Correspondance: -

Principle of Correspondance states that if a life would heve been included in the deaths figure were it to die on a particular day, then the life should contribute to the exposed to risk for that day.

(ii) X = CY of death – CY of birth
= age, x , on birth day is CY of death
= age next, x, on 1 January in CY of death
= age next, x, on 1 January before date of death

So calcudor year Rate Interval starting, for lives classified x, on 1 January on which the life is aged x next birthday.

Age range at the start of the calender year x-1 to x.

(iii) (a) Pn(t) couses at t of those x next on previns 1 january would correspond to
the classification of deaths.
But agrs in the couseses used are ages on 1 July.
So (x-1,x) on 1 January

Is (x-y ; x+y) on 1 July = date of couses

So requested x in Pn (1/2), Pn(1½), Pn(2½) is x nearest birthday at date of couses.

(b) Need Birthdays uniform over the calender year to get average age at start of rate interval, $x - \frac{1}{2}$

Need force constant over (n-½ , x + ½)

So $\hat{\mu}_{x+f}$ will be $x + 0,\ f = 0$

[9]

Q (6) (a) Ho : The observed rates are a sample from a population in which the graduated rates are the true rates.

(b) Of the null hypothesis in ture, then the observed number of positive deviations (where deviation = observed number of deaths exposed number of deaths if Ho is ture), P will be such that

$$P \sim Binomial\ (97, \tfrac{1}{2})$$

(c) vising normal opprocemate to Bi nomial because "n" parameter is large enorgh
to use contral limit there observed value of

$$\frac{57 - 97 \times \frac{1}{2}}{\sqrt{97 \times \frac{1}{2} \times \frac{1}{2}}}$$
$$= \frac{8.5}{4.92} = 1.73$$

Which would N (0,1) if Ho is true :

$Now\ P\left[-1.73 \prec z \prec 1.73\right] \prec 0.95$ or $P\left[|z| \succ 1.73\right] \succ 0.1$
so no reason to reject will hypothesis and graduate appear acceptable.

[6]
[Total 50]