Institute of Actuaries of India

Subject SA3 – General Insurance

October 2015 Examination

INDICATIVE SOLUTION

Solution 1:

i)

Value at Risk (VaR) a statistical technique used to measure or quantify the maximum loss not exceeded with a given probability (the confidence level), over a given period of time. As per IRDAI guidelines, VaR is estimated with a 99.5% level of confidence over a one – year time horizon on the insurer's 'basic own funds' as defined.

Whereas VaR estimates the probable maximum loss, Tail VaR estimates the expected loss given the loss exceeds VaR.

[4]

ii)

a) The formula used for calculating the EC for the combined premium and reserve risk is as follows:

$$= V_o * \left[\frac{\exp\left(\Phi^{-1}(0.995) * \sqrt{\log\left(\varphi^2 + 1\right)}\right)}{\sqrt{\left(\varphi^2 + 1\right)}} - 1 \right]$$

$= V_o * VaR_{0.995}^{mean}(\psi)$

Where

Ψ = log normally distributed random variable with E (Ψ) =1 and Variance (Ψ) = ϕ^2

$VaR_{0.995}^{mean}(\psi)$ =99.5% value at risk of [Ψ - E (Ψ)]

[4]

b) Approach 1: If Max (A,B,C) is taken (assuming prudence)

Amounts in INR Crore

Formula	Notation	Measure / LoB	Fire	Health	Marks
Given	A	NWP for year ended 31 March X	100	200	
Given	B	NEP Projected for year ending 31 March X+1	130	240	
Given	C	NEP Projected for year ending 31 March X+2	150	300	
Given	D	Net UPR as on 31 March X	30	60	
Given	E	Net IBNR+Outstanding as on 31 March X	65	40	
Given	F	NIC for year ended 31 March X	95	125	
Max(A,B,C)	G	P[i]	150	300	0.25
D + E	H	t[i]	95	100	0.25
G + H	I	v[i]	245	400	0.25
G^2	J	p[i]^2	22500	90000	0.25
H^2	K	t[i]^2	9025	10000	0.25
I^2	L	v[i]^2	60025	160000	0.25
Given	M	PR	12%	4%	
Given	N	RR	12%	10%	
M^2	O	PR^2	0.0144	0.0016	0.5
N^2	P	RR^2	0.0144	0.01	0.5
Given	Q	Correlation between PR & RR	0.5	0.5	
[J*O+K*P+2*G*H*M*N*Q]/L	R	Combined Variance of Premium & Reserve Risk for each LoB	0.010981424	0.002275	1.5
sqrt(R)	S	Combined SD of Premium & Reserve Risk for each LoB	10.48%	4.77%	0.5
Given	T	Correlation between Health and Fire	0.25	0.25	
	U	Combined Variance of Premium & Reserve Risk for both LoB	0.003048075		1
SQRT(U)	V	Combined SD of Premium & Reserve Risk for both LoB	5.52094%		1
	W	Φ-1 (0.995)	2.58		0.5
exp(W*V)	X	exponential term	115.27%		0.5
	Y	denominator term	1.001522878		0.5
	Z	VaR (mean) @ 99.5%	15.09%		1
		Value at Risk (INR Crore)	97.36		1
			TOTAL MARKS		10

Approach 2: if Max (A,B) is only taken (assuming EC does not consider X+2 and NWP for X+1 is not given)

Amounts in INR Crore

Formula	Notation	Measure / LoB	Fire	Health	Marks
Given	A	NWP for year ended 31 March X	100	200	
Given	B	NEP Projected for year ending 31 March X+1	130	240	
Given	C	NEP Projected for year ending 31 March X+2	150	300	
Given	D	Net UPR as on 31 March X	30	60	
Given	E	Net IBNR+Outstanding as on 31 March X	65	40	
Given	F	NIC for year ended 31 March X	95	125	
Max(A,B,C)	G	P[i]	130	240	0.25
D + E	H	t[i]	95	100	0.25
G + H	I	v[i]	225	340	0.25
G^2	J	p[i]^2	16900	57600	0.25
H^2	K	t[i]^2	9025	10000	0.25
I^2	L	v[i]^2	50625	115600	0.25
Given	M	PR	12%	4%	
Given	N	RR	12%	10%	
M^2	O	PR^2	0.0144	0.0016	0.5
N^2	P	RR^2	0.0144	0.01	0.5
Given	Q	Correlation between PR & RR	0.5	0.5	
[J*O+K*P+2*G*H*M*N*Q]/L	R	Combined Variance of Premium & Reserve Risk for each LoB	0.010887111	0.002492734	1.5
sqrt(R)	S	Combined SD of Premium & Reserve Risk for each LoB	10.43%	4.99%	0.5
Given	T	Correlation between Health and Fire	0.25	0.25	
	U	Combined Variance of Premium & Reserve Risk for both LoB	0.00325345		1
SQRT(U)	V	Combined SD of Premium & Reserve Risk for both LoB	5.70390%		1
	W	Φ-1 (0.995)	2.58		0.5
exp(W*V)	X	exponential term	115.81%		0.5
	Y	denominator term	1.001625404		0.5
	Z	VaR (mean) @ 99.5%	15.62%		1
		Value at Risk (INR Crore)	88.28		1
				TOTAL MARKS	10

[10]

c)

VaR is not a coherent risk measure in all cases as it fails to comply with the sub-additivity property. This is particularly the case where the underlying distribution is asymmetrical or has fat tails or are heavily skewed. However where the underlying distribution is normal, VaR has been seen to be coherent.

As per the sub-additivity rule, the sum of VaRs should be greater than or equal to the combined VaR. Suppose there are two portfolios X & Y and if ρ is a risk measure then the sub-additivity rule states:

$$\rho(X+Y) <= \rho(X) + \rho(Y)$$

i.e. combining two portfolios should not create more risk. One would expect the combination of two portfolios to reduce the risk, due to diversification.

Under normal distribution assumptions, intuitively we may expect that if the lines of business are perfectly correlated (i.e. correlation = 1) then sum of VaRs would be equal to the combined VaR. However, so long as the correlation measure is less than 1, the diversification benefit should set in and therefore the combined VaR would be less than the individual VaRs added together. Hence the above inequality.

To demonstrate with the given case, we see that:

Approach 1 Fire:

Variance = 0.01098

Sqrt(ln(1+variance)) = 10.45%

Sqrt(1+variance) = 1.005476
VaR_{Fire} = 245 * exp (Φ-1(.995)*0.1045)/1.005476
= 318.94
VaR_{Fire} (mean) = 318.94 – 245 = INR 73.94 Crore

OR

Approach 2 Fire
Variance = 0.01088
Sqrt(ln(1+variance)) = 10.405%
Sqrt(1+variance) = 1.00542
VaR_{Fire} = 225 * exp (Φ-1(.995)*0.10445)/1.00542
= 292.56
VaR_{Fire} (mean) = 292.56 – 225 = INR 67.58 Crore

Approach 1 Health:
Variance = 0.002075
Sqrt(ln(1+variance)) = 4.767%
Sqrt(1+variance) = 1.001137
VaR_{Health} = 400 * exp (Φ-1(.995)*0.0477)/1.001137
= 451.74
VaR_{Health} (mean) = 451.74 – 400 = INR 51.74 Crore

OR

Approach 2 Health:
Variance = 0.0020492
Sqrt(ln(1+variance)) = 4.989%
Sqrt(1+variance) = 1.001245
VaR_{Health} = 340 * exp (Φ-1(.995)*0.04989)/1.001245
= 386.1496
VaR_{Health} (mean) = 386.1496 – 340 = INR 46.15 Crore

VaR_{Fire} (mean) + VaR_{Health} (mean) = INR 125.68 Crore > INR 97.36 Crore [$VaR_{Combined}$ (mean)]
OR
VaR_{Fire} (mean) + VaR_{Health} (mean) = INR 113.73 Crore > INR 88.28 Crore [$VaR_{Combined}$ (mean)]

Therefore, under the normality assumption, the sub-additivity property of a coherent risk measure holds good for VaR as demonstrated.

[8]

d)
Four Methods of capital allocation
The main allocation methods (any four of these six may be awarded marks – half mark for each)

- Proportional

- marginal last in
- game theory (Shapley's)
- equalise relative risk
- co-measures
- option pricing framework.

[2]

[28 marks]

Solution 2:

i) General process for Pricing a treaty:

a) Data requirements and analysis
- Historical claim and policy data for the previous year(s) with trade off between relevance and quantum
- Data coherence to be ensured between data supplied by the ceding insurer and data required for pricing the treaty for that line of business
- For new business / first time pricing, comparable data from existing book may be used

b) Data adjustments
- Adjustments to be made to historical data to be consistent with the ensuing treaty period.
- Typical adjustments include rate changes, loss inflation, volume projections, mix analysis, ultimate loss development, terms and deductibles and so on.
- Particularly the claims development triangle need to be adjusted for the most recent development year (across the diagonal)

c) Fitting loss distribution
- A loss model needs to be fit to the adjusted data based on the underlying loss characteristics (Pareto, lognormal, etc.). Typically a Gross Loss model is used.
- Where the losses are expected to be heavy tailed, it could be necessary to use a mixture of distributions – say use a log normal till a particular quantile and then use a Pareto beyond that
- This also involves projecting extreme adverse losses that might not have been observed till date but could arise in the future
- The distribution should also be able to model catastrophic outcomes
- Parameters are estimated at this stage based on the *a priori* loss model

d) Sensitivity tests / Stress tests/ Reasonableness checks
- Having fit a model it is essential to evaluate the applicability of the model to the given loss profile – one way this can be done is by varying the parameter estimates and assessing the impact
- Reasonableness checks may be performed by calibrating the model using past data and proposed reinsurance structure
- At this stage it is also essential to de-trend as existence of trends may indicate that the model has not captured the changes in business over time

e) Apply the proposed reinsurance structure to the modelled losses
- Relevant treaty features are applied on the loss samples for cash flow projections.
- Monte Carlo simulation methods may be used to draw loss samples
- Typical treaty features to be considered include limits and deductibles, premium features, expense rates
- An NPV of the projected cash flows is evaluated under each of the simulations

f) Internal expenses and capital costs
- All expenses and costs need to be built in to assess the profitability of the contract

[15]

ii) Specific considerations:

1. Any retrocessions / pool arrangements / common account protections
2. Potential facultative reinsurance by the ceding insurer with the reinsurer
3. Total exposure (Total Sum Insured, Number of risks per band, total anticipated premiums)
4. The kind of treaty (proportional or non-proportional)
5. Kind of pricing model (experience rating / exposure ration / mix – credibility based model)
6. Type of loss model (attritional loss model only with large loss loadings or a separate large loss model
7. Kind of business – short tailed or long tailed
8. Ceding commissions (sliding scale, profit commission, fixed rate etc.)

[3]

[18 Marks]

Solution 3:

i) Using a simple average over all periods provides the following assumptions
2 =288; 3 =181; 4 =99; 5 = 65; 6 = 61; 7 = 36; 8 = 15; 9 = 8;

Note that 2014Q1 ACPC values appears low. Have chosen to include it (assuming that such low quarters will occur periodically)
Adding in a reasonable figure for 10+ (tail factor) say 20 using quarter on quarter reductions from delay 6 onwards as a guide. (Anything else reasonable should gain a mark)

Total = 890+288+181+99+65+61+36+15+8+20 = 1,663 [5]

ii) **a)** Currency changes will affect cost of OD claims since parts represent some (but certainly not all) of the cost of repair. A proportion of this change could be factored in– the proportion of costs impacted by this change would be needed. For example if 30% of costs relate to parts then the impact would be a reduction in assumed future ACPC factors for all accident periods of 30% * 20% * 0.5 = 3%.

b)
For 2014Q1 it appears that the most obvious thing to do is to omit the data from analysis. Benign weather is more likely to impact claim frequency rather than claim severity. If 2014Q1 is omitted the adjusted averages for the first five quarters would then be 300, 195, 107 and 74

c)
2nd last and 3rd last diagonals appear lower but last diagonal is higher. Thus the delayed payments would largely appear to have caught up. Hence the total ACPC would be expected to be correct (but some adjustment by development quarter may be necessary).

[6]

iii)
- Claim numbers by accident period – to assess whether 2014Q1 should be excluded and to help assess whether backlog has been cleared.
- Case estimates to assist with estimation (especially for older accident years), including historical view of case estimates to understand adequacy.

- The currency exchange rates with the currencies from the major parts markets should be examined in greater detail by quarter over the last two years, say. A breakdown of the cost of claim, by labour expense, parts expense and other costs should be made.

- Examination of large OD claim experience over time to determine whether to exclude 2014Q1 accident quarter or perhaps to model large and small claims separately.

- More detailed review to determine whether backlog of payments has been cleared (to check the backlogs in numbers and amounts of claim.)

- Relevant split of claims costs by subtype (collision, theft, other) to potentially analyse separately. [6]

[17 Marks]

Solution 4:

i) The change in business strategy would affect the following:

- Business overview: This section covers general operations, plans and projections. It would require amendment to reflect the increased operations in Karnataka, as well as the change in distribution channels. Some projections around each of these items may be required. The FCR needs to comment on any material risks arising out of these plans.
- Pricing and premium adequacy: Given the change in strategy, the actuary will most likely need to investigate the pricing of the new business in Karnataka, including any impact on pricing or profitability from the use of brokers. The FCR will need to include comments on the pricing processes and underwriting.
- Asset and liability management – risks arising from having exposure to an aggregation of liability exposure through a natural catastrophe (e.g. a cyclone) as well as assets whose value might deteriorate due to the catastrophe, as well as becoming difficult to liquidate in these circumstances.
- Capital management and capital adequacy: This section is intended to include comment on the insurer's capacity to meet capital targets in the next three years. The impact of the move into Karnataka and therefore potential growth in business will need to be considered, as this may influence the level of capital held, or the need for capital in future.

- Reinsurance arrangements: The actuary should investigate the reinsurance arrangements in place for the new business, and also understand any impact on the maximum event retention. Any increase in concentration risk, or change in exposure due to the differing region (and potentially different exposure to catastrophes or similar) will need to be considered. This may also affect the capital required above.
- Risk management: The FCR primarily comments on the risk management strategy, so the actuary will need to comment on the extent to which the risk management strategy covers the new circumstances and strategy. [8]

ii) One component of the FCR is an assessment of risks and while reliance may be placed on the work of others, there are some gaps in the current situation:

- The RMS may not take into account the new risks (such as to cyclones and adequacy of reinsurance protection and concentration of distribution in a single broker), a fact which should be highlighted in the FCR. The actuary may therefore be unable to offer an opinion, and should outline the implications for the FCR.
- The FCR should clearly note any reliance which has been placed on the RMS document. There may be a key person risk if the Head of Risk Management has not

been adequately replaced. This also highlights the risk of concentrating the risk management function in one person.

- The actuary is also required to comment on the extent of implementation of the RMS. The fact that no one is able to answer questions on the document is not a good sign [4]

iii) The following approach should generally be adopted:

- The difference should be discussed with the AA to get an understanding of material differences in order to try to resolve the issue.
- The EPR actuary may also further investigate his own projection basis or that adopted by the AA to determine other causes of the difference.
- If the difference was not resolved, the EPR actuary would need to make a decision regarding whether the difference was material, and therefore whether it needed to be noted in the EPR report. A 5% differential would not be considered significant in some cases, but circumstances vary, so this is not a rule that can be applied without further consideration of circumstances.
- If the EPR actuary believes the difference to be material and is of the opinion that provisions have been understated then he/she will need to comment on the same in the peer review report. The peer review report may then be placed before the Audit Committee of the Board, depending on DHV's practices. [6]

[18 Marks]

Solution 5:

i) Comments on analysis:

Chain Ladder Factors on Cumulative Claim Numbers

- No obvious trend in the chain ladder factors over accident years. Adopted averages appear reasonable

Finalisation Rates

- Finalisation rates are accelerating over accident years
- Averages over all accident years are inappropriate as they do not allow appropriately for the changing rate of finalisation
- Projected finalisation rates need to allow for changing experience over time and will require different projected finalisation rates for different accident years

Average cost Per Claim Incurred

- Significant upward trend in average cost per claim incurred over accident years
- In the absence of other information, suggests significant superimposed inflation
- Acceleration of finalisation rates suggest that claims are being finalised faster, which could indicate that payments are being brought forward
- Adopted averages are inappropriate given the underlying trend in payments
- The adopted pattern is also inappropriate as it assumes no payments after development year 6
- The adopted payment pattern needs to take into account both the acceleration of payments over accident year, and any underlying superimposed inflation

Average cost per claim Finalised

- Superimposed inflation is also evident in the average cost per claim finalised
- Adoption of overall averages is inappropriate due to the upward trend over

accident years
- The adopted pattern is also inappropriate as it assumes no payments after development year 6

[7]

ii) Additional information from and questions for the company
- Earned premium by year to help assess the volume of business written
- Changes to premium rates over this period
- Case estimates by accident year and development year
- Information on large claims
- Subdivision of claims between personal injury and property damage
- What is the nature of business underwritten
- Understanding of changes to claims management
- Changes to case estimation standards
- What has impacted the change in finalisation rates

[6]

iii) Additional analysis:
- As finalisation rates have been changing so significantly, analysis in operational time is crucial.
- Both PPCI and PPCF analysis should be performed in operational time
- Claims experience should be projected beyond development year 6
- Comparison of case estimates and projected case estimate method should be performed
- Perform B-F analysis or at least compare loss ratios for each accident year
- Large claims should be analysed separately and an explicit allowance should be made
- Depending upon the information gained about the composition of the portfolio, analysis could be performed on separate segments of the portfolio.

[6]
[19 Marks]

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