

# **INSTITUTE OF ACTUARIES OF INDIA**

## **EXAMINATIONS**

**05<sup>th</sup> November 2015**

**Subject CT8 – Financial Economics**

**Time allowed: Three Hours (10.30 – 13.30 Hrs.)**

**Total Marks: 100**

### **INSTRUCTIONS TO THE CANDIDATES**

- 1. Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
- 2. Mark allocations are shown in brackets.*
- 3. Attempt all questions, beginning your answer to each question on a separate sheet.*
- 4. Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

**AT THE END OF THE EXAMINATION**

**Please return your answer book and this question paper to the supervisor separately.**

- Q. 1)** **Derive** the formula of a fair price of a future contract expiring at time T on a share priced  $S_t$  and declaring dividend of amount  $q$  once at time 't'. Define any terms you use and state any additional assumptions you make. [6]
- Q. 2)**
- i)** **Derive** the Black Scholes partial differential equation of fair price of a derivative  $f(S_t, t)$  contingent on stock price  $S_t$  following Geometric Brownian motion stating any other assumptions you use. (5)
  - ii)** An investment bank has announced that it is issuing a security that pays off a rupee amount equal to  $\ln S_T$  at time T. **Determine** the price of the security at time 't' using risk neutral valuation. (3)
  - iii)** Confirm that the price of this security satisfies the equation in (i) above (3)
  - iv) a)** What will be the PDE for a delta - neutral portfolio of the same stock? (2)
  - b)** What can you say about the relation between the portfolio greeks? (1)
  - c)** What can you say about the rate of return on the portfolio if it is both delta and gamma neutral? (1)
- [15]
- Q. 3)** A stock following Geometric Brownian Motion is currently priced Rs. 300, risk free interest rate is 6% per annum, and the volatility is 30% per annum.
- i)** An investor decides to buy a 2-month European put option assuming the stock is non-dividend paying and with a current strike price of Rs. 300. **Calculate** how much the investor should be prepared to pay. (3)
  - ii)** He later identifies that a dividend of Rs. 9 is expected in a month's time. **Calculate** the price the investor will be expected to pay. (3)
  - iii)** The investor attempts to find the price of the put option on the basis of initial assumption using a two-step binomial tree. **Derive** the price he will calculate, stating your assumptions. (5)
- [11]
- Q. 4)** Interest rates 'r' in a particular country are modelled using a Black-Karasinski Model given by:
- $$(\ln r(t)) = k(t)(\theta(t) - \ln r(t))dt - \sigma(t)dW(t)$$
- where W is the Weiner Process under risk neutral measure.
- i)** Compare and contrast the given model with the Vasicek Model. (5)
  - ii)** **State** the distribution that 'r' follows (1)
- [6]

**Q. 5) State giving brief explanation** for each of the following cases which theme/themes of behavioural finance may be applicable:

i) An investor trades in many stocks and has already booked profit on most of them but there is one stock on which he had to book a small loss. He has now decided to buy that stock again with a hope that he will make good for the loss already booked and earn some money over and above that. (1)

ii) An investor has bought a call option at a premium of Rs. 50 and will expire in 30 days. After just 5 days the premium has reduced to Rs. 5. A friend of the investor who is working in a broking firm advised the investor to sell the option at whatever price available but the investor has decided to hold the option to expiry. (2)

iii) Three persons A, B and C have a sum of Rs. 100,000 each that was invested in a bank's term deposit yielding 10%. The deposit is due to mature in the next week. All of them can invest the proceeds in another term deposit, shares, mutual funds, insurance policies, gold or a newly launched Pension Scheme. 'A' chooses to renew the term deposit. 'B' decides to appoint an advisor to help him decide and 'C' decides to invest in the pension scheme. (4)  
[7]

**Q. 6)** Consider a zero coupon bond that pays Rupee 1 at time T and has a yield to maturity with continuous compounding.

The process followed by  $y$  is  $dy = a(y_0 - y)dt + s.y.dw$  where  $a$ ,  $y_0$  and  $s$  are constants and  $dw$  is a Weiner process.

i) **Derive** the process followed by the bond price. (5)

ii) Show that the volatility of the zero coupon bond price declines to zero when it approaches maturity. (1)

iii) Assume that 'y' is some inverse function of the stock price S following geometric Brownian motion with expected return  $\mu$  and volatility  $\sigma$  such that  $y = \frac{e^{r(T-t)}}{S}$ . **Derive** the process followed by  $y$ . What can you say about the process if  $\mu = \sigma^2 - 1$ ? (4)  
[10]

**Q. 7)** An investor is trying to evaluate the probability of default of a zero coupon corporate bond having a principal amount P with yield to maturity  $y$  and time to maturity T. The yield on a risk free zero coupon bond is  $y_r$

i) Show that the default probability  $q$  is given by  $q = 1 - e^{-[y-y_r]T}$  (4)

ii) What is the expected value of the loss from default on the given bond over the term to maturity? (1)

iii) Suppose now the bond has a recovery rate defined as R, which is the proportion of the claimed amount received in the event of default. Determine the probability of default. (3)  
[8]

- Q. 8)** In a country, shares of four companies and a risk free asset are available for investment:

		<b>Probability of return</b>			
<b>Company</b>	Company Code	15%	30%	50%	5%
<b>Narrow plc</b>	N	-5%	2%	5%	10%
<b>Volatile plc</b>	V	-12%	-4%	7%	18%
<b>Logsystems plc</b>	L	Lognormal with parameters (-2.70, 1.17)			
<b>ProLog plc</b>	P	Lognormal with parameters (-3.09, 1.37)			

$$\rho_{NV} = 0.25, \rho_{LP} = -0.75,$$

- i) Fund manager 'A' creates a fund consisting of shares of only Narrow plc and Volatile plc. Determine the proportion of shares of Narrow plc in A's portfolio if he wants the portfolio to have minimum variance. Derive the results used for the calculations. (5)
- ii) Fund manager B creates a fund consisting of shares of only Logsystems plc and ProLog plc. Determine the mean and variance of returns on this fund if the proportion of ProLog shares is 60%. (4)

[9]

- Q. 9)** You are currently invested in Fund F. It has an expected return of 14% with a volatility of 20%. The risk-free rate is 5.5%. Your financial advisor suggests you add Stock B to your portfolio with a positive weight. Stock B has an expected return of 19%, a volatility of 60% and a correlation of 0 with Fund F.

- i) State giving reasons whether your advisor is right? (2)

You wish to earn returns similar to the stock B relative to the risk undertaken.

- ii) You follow your advisor's advice and make a substantial investment in Stock B so that now 60% is in Fund F and 40% is in Stock B. You tell your colleague about your investment and he says you made a mistake and should reduce your investment in Stock B. State giving reasons whether your colleague right? (3)
- iii) You decide to follow your colleague's advice and reduce your exposure to Stock B. Now Stock B represents only 15% of your risky portfolio with the rest invested in Fund F. Determine if this is the correct amount of Stock B to be held? (3)
- iv) An analyst wishes to model the returns using Arbitrage Pricing Model. He has arrived at an equation of expected return for Fund F as  $(7I_1 + 2I_2 - 2)/100$ . The equation for expected return on stock B is  $(10I_1 + 1I_2 + 4.5)/100$ . Determine the values of  $I_1$  and  $I_2$  for which the expected returns on fund F and stock B are same as that given earlier in the question. Hence determine the weight of B in the overall portfolio so that the expected returns on overall portfolio are 19% if the actual value of  $I_1$  and  $I_2$  in the economy turns out to be 1.7 and 4 respectively. (6)

[14]

- Q. 10)** i) Describe the nature of the investor's utility function in each of the following cases if the investor measures the risk as per the following criteria:
- a) Variance (1)
- b) Down-side semi variance (2)
- c) 97.5% TailVar (2)
- ii) What is an appropriate risk measure if a cubic equation describes an investor's utility function? (1)
- iii) There are two investors A & B with utility function  $U_A(w) = w + 1$  and  $U_B(w) = w^{0.5}$  respectively. There are two portfolios with returns given as below. State giving reasons which investor will choose which of the following portfolio. (8)

Return	Probability	
	Portfolio U	Portfolio V
6%	0.25	0
7%	0.25	0.75
8%	0.25	0
9%	0.25	0.25

[14]

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