INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

29th October 2015

Subject CT4 – Models

Time allowed: Three Hours (10.30 – 13.30 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception***

2. ***Mark allocations are shown in brackets.***

3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***

4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) **i)** List the advantages and disadvantages of using models in actuarial work. (4)

ii) It has been decided to develop a Smart City in a semi urban area in the state of Gujarat. A model is to be developed to recommend the number and capacity of colleges and hospitals required in this new City. The proposed modelling approach is as follows:

- The current age distribution of the population in the area is multiplied by the projected population of the new smart city to produce an estimate of initial population distribution.
- Current national fertility, morbidity and mortality rates by age are used to estimate births, illness and deaths.
- The births and deaths are applied to the initial population distribution to generate a projected distribution of the smart city's population by age for each future year and hence the number of school-age children and number of patients required to be admitted to the hospital.

Describe points to be considered to assess the suitability of the proposed model. (4)

iii) Describe the ways in which the design of a model used to project over only a short time frame may differ from one used to project over long term. (4)

[12]

Q. 2) Out of 1,00,000 lives aged exactly 70 at the start of the year, 95,650 survived till the end of the year.

Calculate ${}_{0.5}p_{70}$ and ${}_{0.5}p_{70.5}$ assuming

i) Uniform distribution of deaths (1)

ii) Balducci assumption (1)

iii) Constant force of mortality (1)

[3]

Q. 3) **i)** Explain what is meant by a time-homogeneous Markov chain. (2)

ii) Distinguish between the conditions under which a Markov chain:

a) has at least one stationary distribution. (1)

b) has a unique stationary distribution. (1)

c) converges to a unique stationary distribution. (1)

[5]

Q. 4) A company is analysing the lapse experience of the policies by age at lapse. The age at lapse is defined as age last birthday when the policy was taken plus curate duration at date of lapse. The period of investigation is from 1st January 2011 to 31st December 2013.

The data of lapses classified by age at lapse (as defined above) is available. The number of in-force policies classified by age *x* last birthday at the 1st January of each year is also available ($P_{x,t}$) where t = 0,1,2,3 for 1st January 2011, 1st January 2012, 1st January 2013 and 1st January 2014 respectively.

i) Explain what is meant by rate interval and give the rate interval for the investigation (1)

ii) Give the age x for which the estimate of lapse rate would be applicable, stating all assumptions made (2)

iii) Derive an expression for the corresponding exposed to risk population required for estimation of the lapse rate (2)

iv) Explain how the given data of $P_{x,t}$ can be used to derive the required exposed to risk, giving details of any assumptions made (2)

[7]

Q. 5) The total number of claims received by an insurance company under individual products is described by a Poisson process with rate λ. and the total number of claims received by same insurance company under a group scheme is described by a Poisson process with rate μ.

i) Prove that the sum of events arising from either of these processes is also a Poisson process with rate $(\lambda + \mu)$. (3)

ii) Explain what is meant by a Markov jump chain. (1)

iii) Describe the circumstances in which the outcome of the Markov jump chain differs from the standard Markov chain with the same transition matrix. (2)

[6]

Q. 6) A company providing annuity to pensioners observed 20 pensioners aged 80 years for a period of 6 months. The pensioners can leave either by death or by transferring the account to another annuity provider. The Nelson-Aalen estimate for the deaths based on the above observation is given below.

t (months)	***ʌ (t)***
1	*0.05*
2	*0.175*
3	*0.425*
4	*0.758*
5	*0.958*

i) Calculate the number of pensioners dying in each month (3)

ii) Calculate the number of pensioners who transferred the account to another provider (3)

[6]

Q. 7) For the possibility of pricing of CI product only for Cancer, an Actuary analysed a four state Markov model for investigating mortality by different type of Cancer.

State 1: Alive, State 2: Death from Blood Cancer, State 3: Death from Brain Cancer, State 4 Death from Stomach Cancer

If μ^{ij}_{x+t} is the transition intensity from i state to j.

i) Draw the transition diagram (2)

ii) Show from first principles that

$$\frac{d}{dt}(\ {}_tP_{x\ 12}\ ) = \mu_{x+t12}\ {}_tP_{x11}$$ (5)

[7]

Q. 8) A company is conducting the mortality investigation of its term assurance policyholders aged exactly 50. The number of deaths between age 50 to 51 are assumed to have a Poisson distribution, the force of mortality being μ.

i) Explain why the company may be using a Poisson model (1)

ii) Why would the Poisson model not be an exact model (1)

iii) The investigation period was from 1st April 2013 to 31st March 2014. The table below gives the data collected for 10 lives.

Ref No	***Date of birth***	***Date of taking policy***	***Status as on 31st March 2014***	***Date of death/lapse (wherever applicable)***
A	*1st April 1963*	*10th Jan 2011*	*In-force*	-
B	*1st October 1963*	*1st May 2013*	*In-force*	-
C	*1st November 1962*	*25th March 2012*	*Died*	*1st December 2013*
D	*1st May 1962*	*20th July 2010*	*Died*	*1st July 2013*
E	*1st July 1964*	*1st June 2000*	*Lapsed*	*1st June 2013*
F	*1st March 1963*	*1st May 2000*	*In-force*	
G	*1st January 1963*	*1st April 2009*	*Lapsed*	*1st April 2013*
H	*1st July 1962*	*1st Aug 2013*	*In-force*	
I	*1st Feb 1963*	*1st Feb 2013*	*In-force*	
J	*1st May 1963*	*15th Dec 2008*	*Died*	*1st March 2014*

Derive the maximum likelihood estimator of the constant force of mortality under the Poisson model. (6)

iv) Comment on the advantage of using the Poisson model over the Binomial model for this analysis (2)

[10]

Q. 9) Customers come in a very small pizza joint at an average rate of 0.5 per minute. Each customer spent time in ordering pizza for a random duration which is exponentially distributed with mean 3 minutes, independently of the duration of the other customers. Two employees are assigned to handle those customers. If a customer arrives when both employees are busy, the customer has to wait unless there are already two customers in waiting, in which case the new customer (the fifth one) is sent to another joint. When a customer leaves, one of the customers in waiting is immediately engaged to the newly free employee.

i) Identify the five states which are required if this system is to be modelled as a Markov jump process. (2)

ii) Draw the transition diagram for this system. (2)

iii) Write down the transition matrix of the Markov jump chain associated with the process and calculate the stationary distribution. (5)

[9]

Q. 10) A life insurance company has used the Cox model for analyzing the persistency experience of its policies by the following parameters

- Premium frequency (Annual, non-annual)
- Distribution channel (Online, Agency, Bancassurance)
- Method of premium payment (Direct Debit/Cheque)

using the following co-variates

F - Value 0 if frequency is annual and 1 if non-annual

D - Value 0, 1 and 2 for Online, Agency and Bancassurance respectively

M – Value 0 for Direct debit and 1 for Cheque

The associated parameters are β_F, β_D and β_M respectively.

i) Why is the Cox model also called a proportional hazard model (1)

ii) Give the equation for the Cox model used in the analysis, defining any additional terms used (2)

iii) State the characteristics of the policy to which the baseline hazard applies (1)

The results showed that

- The hazard rate of lapse for Annual policy sold through Agency and paid by direct debit is 25% lower than hazard rate of non-annual policy sold through Agency and paid by cheque
- Premiums paid through direct debit, annual policies of Agency channel had same hazard rate of lapse as non-annual policies of Online channel
- The hazard rate of lapse for Annual policies of Online channel where premiums were paid through cheque was $3/4^{th}$ the hazard rate of lapse for annual policies of Bancassurance channel where premiums were paid through direct debit

iv) Calculate the estimated values of the parameters β_F, β_D and β_M (6)

[10]

Q. 11) A General Insurance company is designing a product for covering the warranties of electronic items and is subject to a maximum of 3 claims over the year of coverage.

The probability of a specific item breaking down has been estimated to follow an Exponential distribution with the following annualised frequencies λ

0.1 If the item has not suffered any previous breakdown.

0.2 If the item has broken down once previously.

0.25 If the item has broken down on two or more occasions.

It can be assumed that the repair is made immediately, and that it is always possible to repair the item. The warranties are sold only with new products so at time t=0 product is new and never broken down before. Pi(t) is the probability that the electronic item has suffered i breakdowns by time t.

i) Draw a transition diagram for the process defined by the number of breakdowns occurring up to time t. (1)

ii) Write down the Kolmogorov equations obeyed by P'0 (t), P'1(t) and P'2 (t). (2)

iii) **a)** Derive an expression for P0 (t) and (1)

b) Demonstrate that P1(t) = exp(-0.1*t) – exp (-0.2*t) (2)

iv) Derive an expression for P2 (t). (3)

v) Calculate the expected number of claims under the policy. (3)

[12]

Q. 12) A pension scheme which provides different types of annuity benefits to its members has analysed the mortality experience of its pensioners and graduated the crude mortality rates by reference to a standard table for annuitants, by using the formula

$q_x^o = q_x^s$

The summary of the comparison of the observed deaths with the graduated rates is given below

Age-group	Pensioners	Actual deaths	Exp deaths	Stddevn
50-54	37,259	248	227.28	1.3786
55-59	28,057	392	367.55	1.2839
60-64	25,654	680	672.13	0.3074
65-69	20,475	987	997.13	-0.3290
70-74	16,219	1380	1360.77	0.5445
75-79	11,843	1625	1584.59	1.0906
80-84	7,532	1564	1487.57	2.2121
85-89	3,294	925	891.36	1.3195
90+	450	130	155.48	-2.5254

i) Explain the reasons for the company performing a graduation (2)

ii) List the different methods used for graduation (1)

iii) Using the chi-squared test, check the appropriateness of the graduated rates for representing the mortality of the pensioners (4)

iv) Suggest two other tests that may help in deciding the appropriateness of graduation and perform those tests (4)

v) Comment on the impact of using the graduated rates in calculating the provisions for the annuity benefits payable under the scheme (2)

[13]

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