

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

03rd November 2015

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (10.30 – 13.30 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

- 1. Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
- 2. Mark allocations are shown in brackets.*
- 3. Attempt all questions, beginning your answer to each question on a separate sheet.*
- 4. Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) i) The manufacturer of a certain toy sells to retailers on either of the following terms:

a) Cash payment : 30% below the recommended retail price

b) Six months credit: 25% below the recommended retail price.

Find the effective annual rate of discount offered by the manufacturer to the retailers who pay cash as opposed to those retailers who accept the credit terms. (1)

ii) Find $d^{(12)}$, $i^{(2)}$ and δ . (3)

[4]

Q. 2) Assuming that the force of interest per annum at time t is $\delta(t) = ae^{-bt}$

i) Show that the present value of 1 due at time t is

$$v(t) = [a/b(e^{-bt} - 1)] \quad (1)$$

ii) If the force of interest per annum falls by 50% over ten years from value 0.01 at time 0, find the present value of a series of four annual payments each of amount 1000, the first payment being made at time $t = 1$. (2)

iii) At what constant force of interest per annum does the series of payments have the same present value as that found in (ii) above. (1)

[4]

Q. 3) 9 months ago, John entered into a one year forward contract to purchase Company ABC's shares for INR 1,000. Just before the contract was executed, the company had paid a dividend and is not expected to pay any further dividends during period of the contract. The government bond yield at the beginning of the contract was 8.75% p.a., however the yield has reduced to 7.80% p.a. currently.

Calculate the increase in the price of shares to neutralize the effect of the change in the market condition of the forward contract. (5)

[5]

Q. 4) A company has set up a new project and will invest INR 10 crore per annum for the first two years of the project, the investment being made continuously during this period. The project will then start to receive payments at the end of each year, the first payment occurring at the end of year 3 of the project. The amount of payment at the end of year 3 will be 8 crores, reducing by 0.5 crores in each of the subsequent years until the annual amount is 3 crores, after which the annual reduction will be 1 crore. When the payments have reduced to zero, the company's involvement in the project will end.

Calculate the net present value of the project at a rate of interest of 10% per annum effective. (5)

[5]

- Q. 5)** A local bank offers a 5 - year one time deposit scheme to its customers. For an initial deposit of INR 2,400, a customer can expect an accumulation to INR 3,600. The accumulated value has a standard deviation of INR 50. The annual interest rates offered on this deposit conform to the varying interest rate model and follow a Gamma (α, λ) distribution..

Determine the parameters α, λ of the Gamma distribution for the interest rate.

[7]

- Q. 6)** The n-year spot rate of interest S_n is given by:

$$S_n = 0.09 - n^2/200 + n/500 \quad \text{for } n = 1, 2, 3$$

- i) An Investor purchases a 3 year bond that provides coupons payable annually in arrears. Find the par yield of the bond.

(5)

- ii) Also define and determine the coupon bias.

(2)

[7]

- Q. 7)** Mr. Patel moved into a rented flat on 1st January 2010. He planned to live there for two years. In order to pay his rent at the end of each month, he put a sum of money INR X into his bank account on 1st January 2010 and instructed his bank to withdraw from his bank account each rent payment when it was due. The rent was INR 360,000 per month and Mr. Patel calculated that if his bank account earned interest of 6% p.a. effective, he would have just sufficient money in his account to pay for his rent for the next two years.

- i) Find X

(2)

- ii) During the first year his rent remained constant and his Bank account earned an interest of 6% p.a. On 1st January 2011 his bank informed him that his account would now earn an interest rate of 4% p.a. convertible quarterly and his landlord informed him that from 1st April 2011 all the future rent would be increased to INR 460,000 per month, starting with the payment due at the end of April. Suppose Mr. Patel does not put any extra money into his account, show that the rent payment due at the end of October 2011 is the first payment for which there would not be sufficient money in his account.

(4)

- iii) How much extra money should he have put into his account on 1st April 2011 in order that there would be just sufficient money to cover all his remaining payments of rent?

(3)

[9]

- Q. 8)** Given below is the information concerning a pension fund:
(Figures in crores)

Date	Fund Value	Cash flow
1/1/2015	460	
31/3/2015	500	
1/4/2015		40
31/12/2015		-50
1/1/2016	550	
1/7/2016	600	
31/12/2016	X	

- i) If the time weighted rate of return earned on the fund during the period from 1/1/2015 to 31/12/2016 is 20% p.a. effective, calculate X, the value of the fund on 31/12/2016 (2)
- ii) Calculate the annual effective money weighted rate of return (2)
- iii) Calculate the annual effective linked rate of return using subintervals of one year (2)
- iv) State when the linked internal rate of return will be identical to the time weighted rate of return. (1)
- v) Which is a better measure of fund performance TWRR or MWRR? Why? (2)
- [9]**
- Q. 9)** A businessman has decided to purchase a leasehold property for INR 800,000 with a further payment of INR 50,000 for repairs in one year's time. The income associated with letting the property will be INR 100,000 per annum payable continuously for 20 years commencing in two years' time.
- i) Explain what is a discounted payback period (DPP) for a project (1)
- ii) a) Given that the venture will be financed by bank loans on the basis of an effective annual rate of 7% and that the loans may be repaid continuously, find the discounted payback period (DPP) for the project. (3)
- b) After the loan has been repaid the businessman will deposit all the available income in an account which will earn interest of 6% p.a. Find the accumulated amount in the account in 22 years' time. (2)
- iii) Assuming that the bank loan is repaid partially at the end of each complete year, interest is paid annually in arrear and the businessman earns interest on his rental income at an annual effective rate of 6%, find the DPP and the accumulated amount in the businessman's account after 22 years. (6)
- [12]**
- Q.10)** A bank has issued zero coupon bonds that will need repayment of INR 5m in 8 years and INR 10m in 11 years. It plans to use the money to give loan to its customers that would have lump sum repayments at 6 years and 15 years. The current effective interest rate is 8% per annum. The bank wishes that the portfolio together remains immunized against small changes in the rate of interest.
- i) Explain what is meant by immunization of portfolio. (2)
- ii) Determine the size of the issue of the two zero coupon bonds issued by bank. (6)
- iii) Demonstrate that the portfolio is immunized. (3)
- [11]**

Q. 11) A loan of INR 988,000 is granted on 10th July 2015. The loan is repayable by a level annuity payable monthly in arrears (on the 10th of each month) for 25 years and is calculated on the basis of interest rate of 7% p.a. effective.

Determine the following:-

- i)** The monthly repayment (2)
 - ii)** The loan outstanding immediately after the repayment on 10th March 2029 (2)
 - iii)** The capital to be repaid on 10th October 2026 (4)
 - iv)** The capital to be repaid and the amount of interest to be paid in monthly installments due between 10th April 2033 and 10th March 2034 (both dates inclusive). (4)
- [12]**

Q. 12) A Company is looking to raise INR 1000 million capital by issuing unsecured debentures in the market. The issue will be open to both the retail and the corporate investors with the following criteria:

- Debentures issued to the corporate investors have a nominal value of INR 100,000 and those issued to the retail investors have a nominal value of INR 1,000.
- Coupon rate for both the investors is 6% p.a. payable quarterly in arrears.
- The tenure of debentures for both the investors is proposed at 10 years.
- The debentures are redeemed at par for the retail investors and at 105% of nominal value for the corporate investors.

It is additionally known that

- The Income tax rate applicable for Corporate & retail investor is 20% p.a. & 15% p.a. respectively.
- The Capital gain tax is 10% p.a. for both the set of investors.
- Zero coupon Government bonds of the same tenure i.e. 10 years have a current market price of INR 500 for a nominal value INR 1,000.

Determine the maximum issue price of debentures under each category.

[15]
