INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

05th November 2007

Subject CT8 – Financial Economics

Time allowed: Three Hours (14.30 – 17.30 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1) *Do not write your name anywhere on the answer sheet/s. You have only to write your Candidate Number on each answer sheet/s.*

2) *Mark allocations are shown in brackets.*

3) *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*

4) *Fasten your answer sheet/s together in numerical order of questions. This, you may complete immediately after expiry of the examination time.*

5) *In addition to this paper you should have available Graph paper, Actuarial Tables and an electronic calculator.*

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Please return your answer sheet/s and this question paper to the supervisor separately.

Q. 1)

i) Outline the central role that the inflation model s within the Wilkie Model (3)

ii) The Wilkie Model proposes an AR(1) process for the continuously-compounded rate of inflation, I(t) that can be written as:

I(t)=a + b(t-1)+e(t)

Where e(t)~N(0, σ^2) and a, b are constants, with –1<b<1.

(a) Derive an expression for the long-term average inflation rate in terms of a and b (1.5)

(b) Explain an economic justification for using an AR(1) process to model inflation (2)

(c) Explain why a model of the above form would not be suitable for share prices (4.5)

[11]

Q. 2) The following expression for the price of a discount security is obtained by fitting the Cox-Ingersoll-Ross short-rate model to current market prices:

P(t,T)=exp[A(t,T) – B(t,T)*r_t]

Where A(t,T)=2 ln [[10*exp{0.225(T-t)}]/[9 exp{0.25(T-t)}+1]]

And

B(t,T)=40[exp{0.25(T-t)}-1]/[9 exp{0.25(T-t)}+1]

(i) Show that P(t,T) is a solution of :

$$\frac{\partial P}{\partial t} + 0.5\mathrm{r}\sigma^2 \frac{\partial^2 P}{\partial r_t^{\,2}} - rP + a(\mu - r_t)\frac{\partial P}{\partial r_t} = 0$$ (9)

(ii) Following from the above, deduce the values of a, μ, and σ (3)

(iii) Derive a formula for the spot rate R(0,T) and deduce the limiting value of the spot rate as T---> infinity (3)

[15]

Q. 3) A certain share pays a dividend every quarter which is 1% of the share price immediately before the ex-dividend date. Immediately before one such ex-dividend date, the share price is Rs 100. The risk-free interest rate is 5% per annum.

i) Deduce whether it might be advantageous to exercise either of following American put options immediately:

(a) expiry date in one month, exercise price=Rs 150 (4)

(b) expiry date in three months, exercise price=Rs 140 (1)

ii) An American put option should be exercise immediately if the underlying share price falls below a critical value S_c. No dividends are paid on this share.

Explain with reasons, how the value of S_c depends on :

(a) the risk-free interest rate (1)

(b) the exercise price of the option (1)

(c) the volatility of the share price (1)

[8]

Q. 4) An institutional fund divides its assets between an equity index fund and a bond index fund. The mean and standard deviation of the annual return from each fund and the correlation between returns is given below:

	Mean(μ)	Std Dev(σ)	ρ EQUITY	ρ BOND
EQUITY FUND	0.10	0.20	1	0.6
BOND FUND	0.07	0.10		1

The fund can borrow or lend at the risk-free annual rate of 0.05.

(i) What is the optimal split between the equity index and bond index funds? (7)

(ii) Deduce the gradient of the transformation line passing through this optimal portfolio. (2)

[9]

Q. 5) Consider a multiple state, one-period pricing model in which there are two assets X and Y that provide payoffs in each of the next-period states as follows.

State	Probability of state occurance	X	Y
Good	42%	1.65	2.20
Bad	58%	1.20	1.10
Market price		1.35	1.50

(i) Find the state prices for both the good and bad states. (5)

(ii) Find the expected return on each asset. (2)

[7]

Q. 6) The time decay (theta) of a European call option on a share which pays no dividend is given by:

$$\theta_c = \frac{\partial c}{\partial t} = \frac{-\sigma S \exp(-\frac{1}{2}z^2)}{2\sqrt{\pi(T-t)}} - Er\exp(-r(T-t))\Phi(y)$$

Where

z=$\left[\ln(S/E)+(r+\frac{1}{2}\sigma^2)\tau\right]/\sigma\sqrt{T-t}$ and y=z-$\sigma\sqrt{(T-t)}$

i) Use put-call parity to derive the theta of a European put option on the same stock, with the same expiry date and exercise price. (3)

ii) Derive the limiting values of theta, for both puts and calls, for deep in-the-money and deep out-of-the-money options. (2)

iii) The following parameters apply to a European put option:

S=Rs 100, E=Rs 125, sigma=0.20, T-t=1, r=0.05.

Estimate by how much the premium of the option would change in one day if the share price stayed the same. (3)

[8]

Q. 7)

i) Defining all the symbols you use, write down for the Arbitrage Pricing Theory:

(a) the assumed relationship between the return on a risky asset and the value of N economic factors denoted by F_i (where i =1, 2, …N) (2)

(b) the formula for the expected risk premium on the asset, stating the statistical requirements for this formula to hold. (2)

ii) Use the formula you gave in (i) (b) above to show that the Capital Asset Pricing Model can be thought of as a special case of the Arbitrage pricing Theory (2)

iii) The annual returns on assets A and B can be fully explained by the following equations involving two economic factors F_1 and F_2, where $E[F_1] = E[F_2] = 0$

$R_A = 0.13 + 3F_1 + 2F_2$
$R_B = 0.07 + F_1 + 2F_2$

(a) derive the two portfolios which exactly follow the movements in each of the economic factors. (2)

(b) if the annual risk-free return is 0.05, deduce the expected return on each of these portfolios. (2)

(c) hence verify that the formula you stated in (i)(b) above gives the correct expected risk premiums on assets A and B. (2)

[12]

Q. 8) List the three main types of multi-factor asset return models and describe the differences in their approach. **[5]**

Q. 9)

i) State the assumptions underlying the Black-Scholes option pricing formula and discuss how realistic they are. (6)

ii) An investment bank has written a number, N, of European call options on a non-dividend paying stock with strike price Rs 140, current stock price Rs 120, time to expiry of 6 months and an assumed continuously-compounded interest rate of 5% p.a.

The bank is delta-hedging the option position assuming the Black-Scholes framework holds. It has 200,000 shares of the stock and is short Rs 190,00,000 in cash.

(a) By using the hedging position and the Black-Scholes formula for the value of the option, derive two equations satisfied by N and σ, the bank's assumed volatility. (3)

(b) Estimate σ by interpolation. (5)

(c) Deduce the value of N. (1)

[15]

Q. 10)

i) Use a binomial tree to illustrate and determine the value a European put option using the following parameters:

Time to maturity:	3 months
Time interval for tree:	1 month
Risk free rate:	10% p.a., compounded continuously
Current stock price:	Rs 100
Option exercise price:	Rs 110
Dividend:	Rs 2, immediately after month two
Probability of upward move	50%

An upward or downward movement always results in a change of 10% (up or down).

(hint: the share price drops by Rs 2 after the dividend is paid at t=2) (6)

ii) Describe in words (no formulae) what the delta of a derivative is. (1)

iii) Estimate the delta of the put option for the second time step using the tree in your answer to (i). (3)

[10]

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