INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

02 November 2007

Subject CT6 – Statistical Methods

Time allowed: Three Hours (10.00 – 13.00 pm)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.***
4. ***Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.***
5. ***In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.***

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Please return your answer scripts and this question paper to the supervisor separately.

Q. 1) The total amounts claimed each year from a portfolio of general insurance policies over n years were $X_1, X_2, X_3, \ldots\ldots X_n$. The annual claims have a normal distribution with mean θ and variance σ_1^2, where θ is unknown. The prior distribution of θ is assumed to be normal with mean μ and variance σ_2^2.

i) Derive the posterior distribution of θ. (5)

ii) If $\sigma_1^2 = 1$, $\sigma_2^2 = 4$, $\mu = 2$, and the claims observed in the last three years are 2.5, 3.5 and 4, calculate the posterior probability that θ is more than μ. (3)

[8]

Q. 2) A no claims discount system has four levels, 0%, 25%, 50% and 75%. The rules for moving between levels are as follows:

- If no claims is made in a year, the policyholder moves to the next higher level, or remains at the 75% level (if already at 75% level);
- If one claim is made in a year, the policyholder moves down one level, or remains at the 0% level;
- If two or more claims are made in a year, the policyholder moves straight down to, or remains at, the 0% level.

Policyholders at different levels are found to experience different rates of claiming. The number of claims made per year follows a Poisson distribution with parameter λ as follows:

Level	0%	25%	50%	75%
λ	0.29	0.22	0.18	0.10

(i) Derive the transition matrix. (5)

(ii) Calculate the proportions at each different level when the system reaches a steady state. (5)

[10]

Q. 3) A generalized linear model has independent Binomial responses $Z_1, Z_2, \ldots, Z_k$ with

$E(Z_i) = n\mu$; $\mathrm{Var}(Z_i) = n\mu(1-\mu)$ for $i = 1,2,\ldots,k$, $0 < \mu < 1$.

i) Show that $Y_i = Z_i / n$ belongs to an exponential family. (2)

ii) Identify the natural parameter and canonical link function, and derive the variance function. (4)

[6]

Q. 4) **i)** State the assumptions underlying the basic chain ladder method (2)

ii) You are given the following data in respect of claims originating from recent years from the portfolio of a general insurance company, split by year of payment (in Rs. 000). The table below shows the total amounts paid in each development year for each accident year (in Rs. 000).

		Settlement delay in years (Development Year)				
		0	1	2	3	4
Year of accident (Origin Year)	2004	125	104	65	32	9
	2005	120	98	57	27	
	2006	148	124	75		
	2007	143	110			
	2008	138				

Use the inflation adjusted chain ladder method to estimate the total outstanding claim Payments. Assume that inflation for each of the last four years has been 6% per annum. Claims inflation is expected to be 2% pa for 2008-09 and 4% per annum thereafter. (9)

[11]

Q. 5) A general insurance company has a portfolio of m independent fire insurance policies. Under each policy, there can be either one claim or no claim with respective probabilities of occurrence θ and (1– θ), respectively; where $0 < \theta < 1$. The prior distribution of θ follows the following density:

$$f(\theta) \propto \{\theta(1-\theta)\}^{\beta-1}, \ 0<\theta<1, \quad \text{for some constant } \beta>0.$$

Let the total number of claims for the past n years, $x_1, x_2, \ldots, x_n$, be available to you.

(i) Derive the posterior distribution of θ given $x_1, x_2, \ldots, x_n$. (3)

(ii) Derive the Bayesian estimate of θ under quadratic loss, and show that it takes the form of a credibility estimate:

$$Z\overline{X} + (1-Z)\mu,$$

where $\overline{X}$ is the maximum likelihood estimate of θ and μ is a quantity obtained from the prior distribution of θ. (6)

(iii) By considering the prior variance, comment on the effect on Z of increasing β from 5 to 15 and relate this effect to the quality of the prior information about θ in each case. (4)

[13]

Q. 6) **(i)** Define the term "(weakly) stationary". (1)

(ii) Show that the process $\{X_n\}$ satisfying the equation $X_n = Z_n + \beta Z_{n-1}$, where $\{Z_n\}$ is a white noise process with mean μ and variance σ^2, is weakly stationary. (3)

(iii) Is the process $\{X_n\}$ satisfying the equation $X_n = 1.5X_{n-1} - 0.5X_{n-2} + Z_n$, where $\{Z_n\}$ is a white noise process with mean 0 and variance σ^2, weakly stationary? (2)

(iv) Determine the autocorrelation at lag 1 of a suitably differenced version of $\{X_n\}$ in (iii) above. (3)

[9]

Q. 7) **(i)** Explain two disadvantages of using truly random, as opposed to pseudo random numbers. (2)

(ii) Explain the main advantage of the Polar method compared with the Box-Muller method for pairs of uncorrelated pseudo-random values from a standard normal distribution. (1)

(iii) State briefly the method you would use to generate pseudo-random numbers on a computer for each of the following statistical distributions:

a) the continuous uniform distribution on the range (0,1],
b) the chi square distribution with 6 degree of freedom,
c) the distribution of X given X>10, where X has the exponential distribution with mean 5,
d) the Burr distribution. (5)

[8]

Q. 8) Two taxi drivers, A and B, are waiting at the bus stand for two regular customers, C and D, who will arrive by different buses. The amount which a taxi driver can earn by driving customer C to his home is Rs. 50, while the corresponding amount for customer D is Rs. 100. The bus carrying customer C is scheduled to arrive at 6:00 pm, but it may be delayed by X minutes where X has the exponential distribution with mean 30. The bus carrying customer D is scheduled to arrive at 6:15 pm, but it may be delayed by Y minutes where Y has the exponential distribution with mean 15. The two taxi drivers will have to serve the two customers, and there is no other customer or taxi driver. Driver A has the choice of serving the customer who arrives first, or waiting for the customer arriving second. Driver B does not have this choice, but he insists that driver A makes his choice before 6:00 pm.

(i) Show that the decision problem of driver A is a statistical game, and clearly identify Nature's choice in this case. (2)

(ii) Describe the loss matrix for driver A. (2)

(iii) Calculate the probabilities of the events corresponding to Nature's choices. (2)

(iv) What will be the Bayes solution for the decision problem of driver A, with respect to the usual squared error loss function? (3)

[9]

Q. 9) An insurer insures a series of Indian satellites for unsuccessful placement in orbit. The events of unsuccessful placement for different launches are independent, and each event has probability 0.06. The claim amount is Rs. 1 crore. The insurer collects a premium c before every single launch. The insurer starts with a surplus of Rs. 0.5 crores.

(i) Calculate the premium amount which would result in a 100% expected profit for the insurer at every launch. (1)

(ii) If ~~and~~ *c* is equal to the premium calculated in part (i), calculate the probability of ruin for the insurer at the first launch. (2)

(iii) If ~~and~~ *c* is equal to the premium calculated in part (i), calculate the probability of ruin for the insurer at the first event of unsuccessful placement of satellite. (4)

[7]

Q. 10) Briefly discuss the premium rating process for the general insurance business **[4]**

Q. 11) The number of claims N arising from a portfolio of general insurance polices has the Poisson distribution with mean 10. The individual claim sizes, represented by $X_1, X_2, \ldots, X_N$, are independent and identically distributed. The first three moments of the claim size random variable (X) are $E(X) = 40$, $E(X^2) = 2{,}500$ and $E(X^3) = 80{,}000$.

(i) Derive the moment generating function of the total claim amount, in terms of the moment generating function of the claim size distribution. (2)

(ii) Calculate the mean of the aggregate annual claims. (2)

(iii) Calculate the variance of the aggregate annual claims. (2)

(iv) Calculate the third moment of the aggregate annual claims (3)

[9]

Q. 12)

i) Briefly explain the concept of Proportional Reinsurance arrangement (3)

ii) The aggregate annual claims for a risk have a compound Poisson distribution with Poisson parameter λ. Before reinsurance, individual claim amounts have a Pareto distribution with parameters 4 and 300. The insurer effects proportional reinsurance with retained proportion 0.8. The variance of the insurer's aggregate retained claims is 288,000. Calculate the value of λ. (3)

[6]

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