# Institute of Actuaries of India 

## October 2009 EXAMINATION

# Subject ST6 - Finance and Investment B Specialist Technical 

1. 

(a) Value of the bond $=\sum_{t=1}^{\infty} \frac{1}{(1+y)^{t}}=\frac{1}{y}$

Duration of the bond $=D=\frac{\sum_{t=0}^{\infty} t \frac{1}{(1+y)^{t}}}{\frac{1}{y}}$
$D=y \frac{1}{1+y}+y \frac{2}{(1+y)^{2}}+y \frac{3}{(1+y)^{3}}+\ldots+y \frac{\infty}{(1+y)^{\infty}}$
$D(1+y)=y+y \frac{2}{(1+y)^{1}}+y \frac{3}{(1+y)^{2}}+\frac{4}{(1+y)^{3}} \cdots$
$D(1+y)-D=y+y \frac{1}{(1+y)^{1}}+y \frac{1}{(1+y)^{2}}+\frac{1}{(1+y)^{3}} \cdots=y \frac{1+y}{y}=1+y$
$D=\frac{1+y}{y}$
(b)
i. The duration of the perpetuity is: $1 \cdot 10 / 0 \cdot 10=11$ years

Call w the weight of the zero-coupon bond. Then:

$$
(w \times 6)+[(1-w) \times 11]=8 \Rightarrow w=3 / 5=0.60
$$

Therefore, the portfolio weights would be as follows: $60 \%$ invested in the zero and $40 \%$ in the perpetuity.
ii. Next year, the zero-coupon bond will have duration of 5 years and the perpetuity will still have 11 -year duration. To obtain the target duration of seven years, which is now the duration of the obligation, we again solve for w:

$$
(w \times 5)+[(1-w) \times 11]=7 \Rightarrow w=2 / 3=0.7059
$$

So, the proportion of the portfolio invested in the zero increases to $66 \frac{2}{3} \%$ and the proportion invested in the perpetuity falls to $33 \frac{1}{3} \%$.
2.
a. $\quad X=\operatorname{Max}\left(1.10,0.95 S_{T}{ }^{*}\right)$
$X=1.10+0.95 \operatorname{Max}\left(S_{T} *-\frac{1.10}{0.95}, 0\right)$
The replicating portfolio thus consists of 1.10 zero coupon risk-free bonds with redemption value Re 1 and maturity T, and 0.95 call options with time to maturity T and strike price $1.10 / 0.95=22 / 19$. The price of the claim at time 0 is therefore given by
$\Pi(0 ; X)=1.10 e^{-0.06 \times 2}+0.95 C(0,1,22 / 19,2,35 \%, 3 \%, 6 \%)$

Where $C(t, s, K, T, \sigma, \delta, r)$ denotes the price at time $t$ of a European call option with expiry date T and the strike price K , when the value of the underlying at time t is s and the underlying has a dividend yield $\delta$ and volatility $\sigma$, and the risk-free interest rate is r .

$$
\begin{aligned}
& C(0,1,22 / 19,2,35 \%, 3 \%, 6 \%)=e^{-0.03 \times 2} \Phi\left(d_{1}\right)-\frac{22}{19} e^{-0.06 \times 2} \Phi\left(d_{2}\right) \\
& \begin{aligned}
d_{1}=\frac{\ln \left(\frac{1}{22 / 19}\right)+\left(0.06-0.03+\frac{0.35^{2}}{2}\right) 2}{0.35 \sqrt{2}}=\frac{-0.1466+0.1825}{0.4950}=0.0725
\end{aligned} \\
& \begin{array}{r}
d_{2}=d_{1}-\sigma \sqrt{T}=0.0725-0.4950=-0.4225
\end{array} \\
& \begin{aligned}
C(0,1,22 / 19,2,35 \%, 3 \%, 6 \%) & =e^{-0.03 \times 2} \Phi(0.0725)-\frac{22}{19} e^{-0.06 \times 2} \Phi(-0.4225)
\end{aligned} \\
& \begin{array}{r}
\quad=0.9418 \times 0.5289-(22 / 19) \times \mathrm{e}^{-0.06 \times 2} \times 0.3363 \\
\\
\begin{aligned}
& =0.4981-0.3454=0.1527
\end{aligned} \\
\begin{aligned}
(0 ; X)=1.10 e^{-0.06 \times 2}+0.95 \times 0.1527=0.9756+0.1451=1.1207
\end{aligned}
\end{array}
\end{aligned}
$$

(b)

The value of the stock is likely to go down by the level of the dividend paid out by the company and hence value of the call is likely to be lower as expected payoff is lower. Hence, ignoring the dividend (i.e. as if the stock is not paying dividend) the value of the call should be higher compared to a dividend paying stock.

Now, the dividend would have an impact only in the formula for $C(t, s, K, T, \sigma, \delta, r)$. Approximately the impact may be computed as follows
$=0.95 *[\mathrm{~N}(\mathrm{~d} 1)-\exp (-0.03 * 2) * \mathrm{~N}(\mathrm{~d} 1)]=0.95 * 0.5289 * 0.5824=0.0292$
Approximately, the value of the option is likely to go up by 0.0292 to 1.1499 .
(c)

It is exactly the same contract as part a except that it is limited from the above by 1.50 which is similar to selling a call option at 1.50 . Therefore, it may be rewritten as

$$
\begin{aligned}
Y= & \operatorname{Min}\left\{\operatorname{Max}\left(1.10,0.95 S_{T} *\right), 1.50\right\} \\
& =1.10+0.95 \operatorname{Max}\left(S_{T} *-\frac{1.10}{0.95}, 0\right)-\operatorname{Max}\left(0.95 S_{T} *-1.50,0\right)
\end{aligned}
$$

The replicating portfolio thus consists of 1.10 zero coupon risk-free bonds with redemption value Re 1 and maturity T, 0.95 call options with time to maturity T and strike price $1.10 / 0.95=22 / 19$, and a short position of 0.95 call options with time to maturity T and strike price $1.50 / 0.95=30 / 19$ The price of the claim at time 0 is therefore given by
$\Pi(0 ; X)=1.10 e^{-0.06 \times 2}+0.95 C(0,1,22 / 19,2,35 \%, 3 \%, 6 \%)-0.95 C(0,1,30 / 19,2,35 \%, 3 \%, 6 \%)$
$C(0,1,30 / 19,2,35 \%, 3 \%, 6 \%)=e^{-0.03 \times 2} \Phi\left(d_{1}\right)-\frac{30}{19} e^{-0.06 \times 2} \Phi\left(d_{2}\right)$
$d_{1}=\frac{\ln \left(\frac{1}{30 / 19}\right)+\left(0.06-0.03+\frac{0.35^{2}}{2}\right) 2}{0.35 \sqrt{2}}=\frac{-0.4568+0.1825}{0.4950}=-0.5541$
$d_{2}=d_{1}-\sigma \sqrt{T}=-0.5541-0.4950=-1.0491$
$C(0,1,30 / 19,2,35 \%, 3 \%, 6 \%)=e^{-0.03 \times 2} \Phi(-0.5541)-\frac{30}{19} e^{-0.06 \times 2} \Phi(-1.0491)$

$$
\begin{aligned}
& =0.9418 \times 0.2898-(30 / 19) \times \mathrm{e}^{-0.06 \times 2} \times 0.1471 \\
& =0.2729-0.2060=0.0669
\end{aligned}
$$

$\Pi(0 ; Y)=1.10 e^{-0.06 x 2}+0.95 \times 0.1527-0.95 \times 0.0699=0.9756+0.1451-0.0636=1.0571$

The modified contract $(\mathrm{Y})$ is cheaper than the original contract by $(1.1207-1.0571) / 1.0571$ or $5.67 \%$.

## 3.

(a) The payoff of a forward start call option at time $T_{2}$ is given by the stochastic variable X:

$$
X=\operatorname{Max}\left(S_{T_{2}}-S_{T_{1}}, 0\right)
$$

(b) The price of a forward start call option at time 0 is given by:

$$
\begin{aligned}
& \Pi(0, X)=e^{-r T_{2}} E_{Q}\left[\operatorname{Max}\left(S_{T_{2}}-S_{T_{1}}, 0\right)\right] \\
& =e^{-r T_{2}} E_{Q}\left[E_{Q}\left\{\operatorname{Max}\left(S_{T_{2}}-S_{T_{1}}, 0\right)\right\} \mid F_{T_{1}}\right] \\
& S_{T_{2}}=S_{T_{1}} \exp \left[\left(r-\frac{\sigma^{2}}{2}\right)\left(T_{2}-T_{1}\right)+\sigma\left(\bar{W}_{T_{2}}-\bar{W}_{T_{1}}\right)\right] \\
& \Pi(0, X)=e^{-r T_{2}} E_{Q}\left[\left.E_{Q}\left\{S_{T_{1}} \operatorname{Max}\left(e^{\left(r-\frac{\sigma^{2}}{2}\right)\left(T_{2}-T_{1}\right)+\sigma\left(\bar{W}_{T_{2}}-\bar{W}_{T_{1}}\right)}-1,0\right)\right\} \right\rvert\, F_{T_{1}}\right] \\
& =e^{-r T_{2}} E_{Q}\left[\left.\frac{S_{T_{1}}}{e^{-r\left(T_{2}-T_{1}\right)}} x e^{-r\left(T_{2}-T_{1}\right)} E_{Q}\left\{\operatorname{Max}\left(e^{\left(r-\frac{\sigma^{2}}{2}\right)\left(T_{2}-T_{1}\right)+\sigma\left(\bar{W}_{T_{2}}-\bar{W}_{T_{1}}\right)}-1,0\right)\right\} \right\rvert\, F_{T_{1}}\right]_{=} \\
& E_{Q}\left[\frac{S_{T_{1}}}{e^{T_{1}}} C\left(T_{1}, 1,1, T_{2}, \sigma, r\right)\right] \\
& =S_{0} C\left(T_{1}, 1,1, T_{2}, \sigma, r\right)
\end{aligned}
$$

Where $C(t, s, K, T, \sigma, r)$ denotes the price at time t of a European call option with expiry date T and the strike price K , when the value of the underlying at time t is s and the volatility $\sigma$, and the riskfree interest rate is $r$.
4.
(a) Since risk-free rate of interest is zero the call option price is given by:
$C=E_{Q}\left[\operatorname{Max}\left(S_{t}-K, 0\right)\right]$
$S_{t}-K=S_{0}+\sigma S_{0} W_{t}-K$
$S_{t}-K_{\text {follows normal distribution with mean }} S_{0}-K_{\text {and variance }} \sigma^{2} S_{0}{ }^{2} t$
Assume $S_{t}-K=x ; S_{0}-K=\mu ;$ and $\sigma S_{0} \sqrt{t}=s$
$C=E_{Q}\left[\operatorname{Max}\left(S_{t}-K, 0\right)\right]=\int_{0}^{\infty} x \frac{1}{\sqrt{2 \pi} s} e^{-\frac{1}{2}\left(\frac{x-\mu}{s}\right)^{2}} d x$
$\frac{x-\mu}{s}=z$
$d x=s d z$
$C=\int_{-\mu / s}^{\infty} s z \frac{1}{\sqrt{2 \pi}} e^{-\frac{1}{2} z^{2}} d z+\mu \int_{-\mu / s}^{\infty} \frac{1}{\sqrt{2 \pi}} e^{-\frac{1}{2} z^{2}} d z$
$C=\mu \Phi\left(\frac{\mu}{s}\right)+s \phi\left(\frac{\mu}{s}\right)$
$C=\left(S_{0}-K\right) \Phi\left(\frac{S_{0}-K}{\sigma S_{0} \sqrt{t}}\right)+\sigma S_{0} \sqrt{t} \phi\left(\frac{S_{0}-K}{\sigma S_{0} \sqrt{t}}\right)$
(b) $\Delta_{C}=\frac{\delta C}{\delta S}=\Phi\left(\frac{S-K}{\sigma S \sqrt{t}}\right)+(S-K) \phi\left(\frac{S-K}{\sigma S \sqrt{t}}\right)\left(0+\frac{K}{\delta^{2} \sqrt{t}}\right)+\phi\left(\frac{S-K}{\sigma S \sqrt{t}}\right) \sigma \sqrt{t}-\sigma \sqrt{t} \phi\left(\frac{S-K}{\sigma S \sqrt{t}}\right)\left(0+\frac{K}{\delta^{2} \sqrt{t}}\right)\left(\frac{S-K}{\sigma S \sqrt{t}}\right)$
$\Delta_{C}=\frac{\delta C}{\delta S}=\Phi\left(\frac{S-K}{\sigma S \sqrt{t}}\right)+\phi\left(\frac{S-K}{\sigma S \sqrt{t}}\right) \sigma \sqrt{t}$
Substitute $S=S_{0}$ to obtain today's value of $\Delta$.

$$
\Delta_{C}=\frac{\delta C}{\delta S}=\Phi\left(\frac{S_{0}-K}{\sigma S_{0} \sqrt{t}}\right)+\phi\left(\frac{S_{0}-K}{\sigma S_{0} \sqrt{t}}\right) \sigma \sqrt{t}
$$

If you have sold the options $\Delta_{C}$ tells you how many you have to buy of the underlying in order for your portfolio to become delta neutral i.e. insensitive to small changes in the stock price.

5(a)
(i)

$p=\frac{1-0.80}{1.20-0.80}=0.5$
Value of a European call option = Rs. 110
(ii)

To obtain the replicating portfolio at A one has to solve the following set of equations:
$1200 \phi+\psi=220$
$800 \phi+\psi=0$
$\phi=\frac{22}{40} ; \psi=-440$
To obtain the replicating portfolio at B one has to solve the following set of equations:
$1440 \phi+\psi=440$
$960 \phi+\psi=0$
$\phi=\frac{11}{12} ; \psi=-880$
To obtain the replicating portfolio at C one has to solve the following set of equations:
$960 \phi+\psi=0$
$640 \phi+\psi=0$
$\phi=0 ; \psi=0$
That the portfolio is self financing is seen from the following equation
$-440+\frac{22}{40} \times 1200=-880+\frac{11}{12} \times 1200=220$
$-440+\frac{22}{40} \times 800=0+0 \times 800=0$
5(b)
(i) $\mathrm{u}=1.50 ; \mathrm{d}=0.50$

The arbitrage bounds for the interest rate $r$ are
$0.5 \leq(1+r) \leq 1.5$
$0 \leq r \leq 50 \%$
Where $r$ is the rate per annum with annual compounding.
(ii) Both the price of stock and the price of the option have to satisfy the risk-neutral principle. Thus, we have
$100=\frac{1}{1+r}[150 q+50(1-q)]$
$22=\frac{1}{1+r}[42 q+0(1-q)]$
$\frac{100}{22}=\frac{100 q+50}{42 q}$
$2000 q=1100$
$q=0.55(0.5)$
$\mathrm{r}=5 \%(0.5)$
5(c)
The model is free from arbitrage if an only if there exists a martingale measure. Thus one needs to prove that there exist $q_{1}, q_{2}$, and $q_{3}$ all strictly between 0 and 1 , and such that
$500=\frac{1}{1.10}\left(750 q_{1}+500 q_{2}+250 q_{3}\right)$
$q_{1}+q_{2}+q_{3}=1$
$q_{2}=2\left(0.6-q_{1}\right), \quad q_{3}=q_{1}-0.2$
From this we see that all values of $q_{1}$ such that $0.2<q_{1}<0.6$ will result in a martingale measure, and therefore under the above martingale measure the stock price is free from arbitrage.

6(a)
(i)
(a) The share price at time 7 will be :

$$
\frac{\operatorname{Max}[1200-900,0]}{20}=\frac{300}{20}=\text { Rs. } 15
$$

(b) Here the share price at time 7 will be zero because the value of the outstanding debt exceeds the total value of the company.
(ii) The Merton model values shares as call options on the company's assets with the strike price equal to the face value of the company's debt

The value of equity at time 7 (in Rs. Million) will be:
$E_{7}=\operatorname{Max}\left(V_{7}-900,0\right)$

Where $V_{7}$ is the total value of the company (in Rs. Million) at time 7.
The model assumes that the company will default completely on payment of the debt if the total value of its assets is less than the promised debt payment at that time.
There are 20 million shares outstanding, so the share price (in Rs.) at time 7 will be:
$S_{7}=\frac{\operatorname{Max}\left[V_{7}-900,0\right]}{20}$
An appropriate option pricing formula to value this call option at time 0 is then
$S_{0}=\frac{V_{0} \Phi\left(d_{1}\right)-900 e^{-0.05 \times 7} \Phi\left(d_{2}\right)}{20}$
Where $d_{1}=\frac{\ln \left(\frac{V_{0}}{900}\right)+7\left(0.05+\frac{\sigma_{V}{ }^{2}}{2}\right)}{\sigma_{V} \sqrt{7}}, d_{2}=d_{1}-\sigma_{V} \sqrt{7}$
Where $\sigma_{V}$ is the volatility per annum (with continuous compounding) of the company assets.
7. $d r(t)=[\theta-\alpha r(t)] d t+\sigma d \bar{W}(t)$

Put all the terms containing $r(t)$ on the left hand side

$$
d r(t)+\alpha r(t) d t=\theta d t+\sigma d \bar{W}(t)
$$

Multiplying both side of the equation by $e^{\alpha t}$

$$
\begin{aligned}
& e^{\alpha t} d r(t)+\alpha e^{\alpha t} r(t) d t=\theta e^{\alpha t} d t+\sigma e^{\alpha t} d \bar{W}(t) \\
& d\left(e^{\alpha t} r(t)\right) d t=\theta e^{\alpha t} d t+\sigma e^{\alpha t} d \bar{W}(t)
\end{aligned}
$$

Renaming the variable $t$ as $s$, and integrating over the range $0 \leq s \leq t$

$$
\left[e^{\alpha s} r(s)\right]_{0}^{t}=\int_{0}^{t} \theta e^{\alpha s} d s+\int_{0}^{t} \sigma e^{\alpha s} d \bar{W}^{-}(s)
$$

$$
e^{\alpha t} r(t)-r(0)=\frac{\theta}{\alpha}\left(e^{\alpha t}-1\right)+\int_{0}^{t} \sigma e^{\alpha s} d \bar{W}(s)
$$

Dividing both side by $e^{\alpha t}$ and putting r(0)=r
$r(t)=e^{-\alpha t} r_{0}+\frac{\theta}{\alpha}\left(1-e^{-\alpha t}\right)+\int_{0}^{t} \sigma e^{-\alpha(t-s)} d \bar{W}(s)$
(b)

Under the risk-neutral measure, $W(t)$ is standard Brownian motion. Thus, $d W(s)$ is normally distributed with mean 0 and variance ds. Thus, the Ito integral in (a) is also normally distributed. (1 mark) The mean of the Ito integral in (a) has mean zero.
The variance of the Ito integral in (a) is:

$$
\begin{aligned}
\operatorname{Var}\left[\int_{0}^{t} \sigma e^{-\alpha(t-s)} d \bar{W}\right. & (s)]=\int_{0}^{t} \sigma^{2} e^{-2 \alpha(t-s)} d s \\
& =\left[\frac{\sigma^{2}}{2 \alpha} e^{-2 \alpha(t-s)}\right]_{0}^{t} \\
& =\frac{\sigma^{2}}{2 \alpha}\left(1-e^{-2 \alpha t}\right)
\end{aligned}
$$

Thus, the conditional distribution of $\mathrm{r}(\mathrm{t})$ given $\mathrm{r}(0)$ is normally distributed with
Mean: $e^{-\alpha t} r_{0}+\frac{\theta}{\alpha}\left(1-e^{-\alpha t}\right)$, and
Variance: $\frac{\sigma^{2}}{2 \alpha}\left(1-e^{-2 \alpha t}\right)$
(c)

The table below shows the conditional mean and standard deviation of $r(t)$ together with the upper and lower limits of the confidence interval, which are calculated as mean $\pm 1.94313$ standard deviation:

|  | $\mathrm{t}=0$ | $\mathrm{t}=4$ | $\mathrm{t}=8$ | $\mathrm{t}=20$ | $\mathrm{t}=\infty$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Mean | $6.00 \%$ | $6.76 \%$ | $7.23 \%$ | $7.82 \%$ | $8.00 \%$ |
| Standard <br> Deviation | $0.00 \%$ | $3.21 \%$ | $3.77 \%$ | $4.07 \%$ | $4.08 \%$ |
| Upper limit | $6.00 \%$ | $12.99 \%$ | $14.56 \%$ | $15.72 \%$ | $15.93 \%$ |
| Lower limit | $6.00 \%$ | $0.53 \%$ | $-0.09 \%$ | $-0.08 \%$ | $0.07 \%$ |

The following comments can be on the basis of the results of the table:
a. According to the model, the short rate can take negative values, which is not realistic.
b. However, the probability of negative rates occurring is quite low.
c. Negative rates are more likely to occur for medium terms than for long or short.
d. The yield curve slopes upwards.
e. The standard deviation increases with the term but has a limiting value of $4.08 \%$.
8.
a. A bank that is asset sensitive loses net interest income as interest rates fall, in general. To protect against falling interest rates, a bank should buy an interest rate floor as a hedge. If rates do fall, the floor counterparty must pay the bank and the payment will at least partially offset the lost net interest income.
b. A reverse collar would similarly provide a hedge because it involves the simultaneous purchase of a floor and sale of a cap. The bank receives cash if rates fall below the floor rate, but will have to pay cash if rates rise.
c. The benefit of a reverse collar over a floor, or collar over a cap, is that it costs less in terms of the upfront premium. Of course, the buyer of a collar or reverse collar gives up some or all of any beneficial rate move that is retained with simple purchase of a floor or cap..
9.

## Payoff: Butterfly Spread Using Calls

| Position | $\mathrm{S}_{\mathrm{T}}<$ | $\mathrm{K}_{1} \leq \mathrm{S}_{\mathrm{T}} \leq \mathrm{K}_{2}$ | $\mathrm{~K}_{2}<\mathrm{S}_{\mathrm{T}} \leq \mathrm{K}_{3}$ | $\mathrm{~K}_{3}<\mathrm{S}_{\mathrm{T}}$ |
| :--- | :---: | :---: | :---: | :---: |
| Long call $\left(\mathrm{K}_{1}\right)$ | 0 | $\mathrm{~S}_{\mathrm{T}}-\mathrm{K}_{1}$ | $\mathrm{~S}_{\mathrm{T}}-\mathrm{K}_{1}$ | $\mathrm{~S}_{\mathrm{T}}-\mathrm{K}_{1}$ |
| Short 2 calls $\left(\mathrm{K}_{2}\right)$ | 0 | 0 | $-2\left(\mathrm{~S}_{\mathrm{T}}-\mathrm{K}_{2}\right)$ | $-2\left(\mathrm{~S}_{\mathrm{T}}-\mathrm{K}_{2}\right)$ |
| Long call $\left(\mathrm{K}_{3}\right)$ | 0 | 0 | 0 | $\mathrm{~S}_{\mathrm{T}}-\mathrm{K}_{3}$ |
| Total | 0 | $\mathrm{~S}_{\mathrm{T}}-\mathrm{K}_{1}$ | $2 \mathrm{~K}_{2}-\mathrm{K}_{1}-\mathrm{S}_{\mathrm{T}}$ | $\left(\mathrm{K}_{2}-\mathrm{K}_{1}\right)-\left(\mathrm{K}_{3}-\mathrm{K}_{2}\right)=0$ |

Payoff: Butterfly Spread Using Puts

| Position | $\mathrm{S}_{\mathrm{T}}<\mathrm{K}_{1}$ | $\mathrm{~K}_{1} \leq \mathrm{S}_{\mathrm{T}} \leq$ | $\mathrm{K}_{2}<\mathrm{S}_{\mathrm{T}} \leq \mathrm{K}_{3}$ | $\mathrm{~K}_{3}<\mathrm{S}_{\mathrm{T}}$ |
| :--- | :---: | :---: | :---: | :---: |
| Long put $\left(\mathrm{K}_{1}\right)$ | $\mathrm{K}_{1}-\mathrm{S}_{\mathrm{T}}$ | 0 | 0 | 0 |
| Short 2 puts $\left(\mathrm{K}_{2}\right)$ | $-2\left(\mathrm{~K}_{2}-\mathrm{S}_{\mathrm{T}}\right)$ | $-2\left(\mathrm{~K}_{2}-\mathrm{S}_{\mathrm{T}}\right)$ | 0 | 0 |
| Long put ( $\left.\mathrm{K}_{3}\right)$ | $\mathrm{K}_{3}-\mathrm{S}_{\mathrm{T}}$ | $\mathrm{K}_{3}-\mathrm{S}_{\mathrm{T}}$ | $\mathrm{K}_{3}-\mathrm{S}_{\mathrm{T}}$ | 0 |
| Total | 0 | $\mathrm{~S}_{\mathrm{T}}+\mathrm{K}_{3}-2 \mathrm{~K}_{2}$ | $\mathrm{~K}_{3}-\mathrm{S}_{\mathrm{T}}$ | 0 |



The butterfly spread would generate maximum profit at $\mathrm{S}_{\mathrm{T}}=\mathrm{K}_{2}$.
b.

Payoff: Butterfly Spread Using Calls/Puts

| Position | S | $\mathrm{K}_{1} \leq \mathrm{S}_{\mathrm{T}} \leq \mathrm{K}_{2}$ | $\mathrm{~K}_{2}<\mathrm{S}_{\mathrm{T}} \leq \mathrm{K}_{3}$ | $\mathrm{~K}_{3}<\mathrm{S}$ |
| :--- | :---: | :---: | :---: | :---: |
| Butterfly Spread Using Calls | 0 | $\mathrm{~S}_{\mathrm{T}}-\mathrm{K}_{1}$ | $2 \mathrm{~K}_{2}-\mathrm{K}_{1}-\mathrm{S}_{\mathrm{T}}$ | 0 |
| Butterfly Spread Using Puts | 0 | $\mathrm{~S}_{\mathrm{T}}+\mathrm{K}_{3}-2 \mathrm{~K}_{2}=\mathrm{S}_{\mathrm{T}}$ | $\mathrm{K}_{3}-\mathrm{S}_{\mathrm{T}}=2 \mathrm{~K}_{2}-\mathrm{K}_{1}$ | 0 |
|  | $-\mathrm{K}_{1}$ | $-\mathrm{S}_{\mathrm{T}}$ |  |  |

Cost of butterfly spread using calls $=2 \mathrm{C}_{2}-\mathrm{C}_{1}-\mathrm{C}_{3}$

Cost of butterfly spread using puts $=2 \mathrm{P}_{2}-\mathrm{P}_{1}-\mathrm{P}_{3}$
The payoff of butterfly spread using calls is the same as that of butterfly spread using puts. If two portfolios have the same payoff, they must have the same cost to establish to avoid any arbitrage opportunities.
Thus, $2 \mathrm{C}_{2}-\mathrm{C}_{1}-\mathrm{C}_{3}=2 \mathrm{P}_{2}-\mathrm{P}_{1}-\mathrm{P}_{3}$
(c)

Arbitrage opportunities will exist if:
$2 C_{4350, \text { Bid }}-C_{4300, \text { Ask }}-C_{4400, A s k}-2 P_{4350, \text { Ask }}+P_{4300, \text { Bid }}+P_{4400, \text { Bid }}>0$; and/or
$2 P_{4350, B i d}-P_{4300, A s k}-P_{4400, A s k}-2 C_{4350, A s k}+P C_{4300, \text { Bid }}+C_{4400, \text { Bid }}>0$
$2 C_{4350, B i d}-C_{4300, A s k}-C_{4400, A s k}-2 P_{4350, A s k}+P_{4300, B i d}+P_{4400, B i d}=2 x 63.60-90-68-2 \times 90.50+65+119=-4.80<0$
$2 P_{4350, B i d}-P_{430 Q, A s k}-P_{4400, A s k}-2 C_{435,, A s k}+C_{4300, B i d}+C_{4400, B i d}=2 \times 82-68-121.30-2 \times 68+89+44.40=-27.90<0$

Thus, the arbitrage opportunities were not existing on June 192009 at 2.50 PM.
[10]
10.
a. A straddle is appropriate when an investor is expecting a large movement in a stock price but does not know in which direction the move will be.
b. Payoff and Profit: Straddle

| Position | $\mathrm{S}_{\mathrm{T}} \leq 60$ | $\mathrm{~S}_{\mathrm{T}}>60$ |
| :--- | :---: | :---: |
| Long call | 0 | $\mathrm{~S}_{\mathrm{T}}-60$ |
| Long put | $60-\mathrm{S}_{\mathrm{T}}$ | 0 |
| Total payoff of | $60-\mathrm{S}_{\mathrm{T}}$ | $\mathrm{S}_{\mathrm{T}}-60$ |
| Straddle |  |  |
| Total Profit of | $60-\mathrm{S}_{\mathrm{T}}-6-4=50-$ | $\mathrm{S}_{\mathrm{T}}-60-6-4=\mathrm{S}_{\mathrm{T}}$ |
| Straddle | $\mathrm{S}_{\mathrm{T}}$ | -70 |

The straddle will generate profit if stock price on maturity is less than Rs. 50 or more than Rs. 70. If the stock price on maturity lies between Rs. 50 and Rs. 70, the straddle will lead to a loss. $\mathrm{P}\left(50<\mathrm{S}_{\mathrm{T}}<70\right)=\mathrm{P}\left(\ln 50<\ln \mathrm{S}_{\mathrm{T}}<\ln 70\right)$

$$
\begin{aligned}
& =P\left[\frac{\ln 50-\left[\ln 60+\left(0.18-\frac{0.38^{2}}{2}\right) 0.25\right]}{0.38 \sqrt{0.25}}<\frac{\ln S_{T}-\left[\ln 60+\left(0.18-\frac{0.38^{2}}{2}\right) 0.25\right]}{0.38 \sqrt{0.25}}<\frac{\ln 70-\left[\ln 60+\left(0.18-\frac{0.38^{2}}{2}\right) 0.25\right]}{0.38 \sqrt{0.25}}\right] \\
& \quad=P\left[\frac{\ln 50-4.1213}{0.19}<\frac{\ln S_{T}-4.1213}{0.19}<\frac{\ln 70-4.1213}{0.19}\right]
\end{aligned}
$$

$=\mathrm{P}(-1.1014<\mathrm{Z}<0.6695)$
$=\Phi(0.6695)-\Phi(-1.1014)$
$=0.7484-0.1354$
$=0.6131$

11(a) Implied volatility has increased. If not, the call price would have fallen as a result of the decrease in stock price.

11(b) Implied volatility has increased. If not, the put price would have fallen as a result of the decreased time to maturity.

11(c) A call option with a high exercise price has a lower delta. This call option is less in the money. Both $\mathrm{d}_{1}$ and $\Phi\left(d_{1}\right)$ are lower when K is higher.

11(d) If a poor harvest today indicates a worse than average harvest in future years, then the futures prices will rise in response to today's harvest, although presumably the two-year price will change by less than the one-year price. The same reasoning holds if wheat is stored across the harvest. Next year's price is determined by the available supply at harvest time, which is the actual harvest plus the stored wheat. A smaller harvest today means less stored wheat for next year which can lead to higher prices.
Suppose first that wheat is never stored across a harvest, and second that the quality of a harvest is not related to the quality of past harvests. Under these circumstances, there is no link between the current price of wheat and the expected future price of wheat. The quantity of wheat stored will fall to zero before the next harvest, and thus the quantity of wheat and the price in one year will depend solely on the quantity of next year's harvest, which has nothing to do with this year's harvest.
11(e) Because long positions equal short positions, futures trading must entail a "canceling out" of bets on the asset. Moreover, no cash is exchanged at the inception of futures trading. Thus, there should be minimal impact on the spot market for the asset, and futures trading should not be expected to reduce capital available for other uses.

The bank may use any of the following methods (or combination) to reduce the risk of default:

1) Credit line limit
2) Collaterals
3) Design of derivative contract
4) Credit Derivatives

Credit Line Limit
The overall limit and frequency of monitoring has to take into account the volatility of underlying variables and the overall exposures. The main problem with this limit is that it prevents to write profitable business even if the customer is currently in profit and may impact the relationship and future business.

Collateral
It needs to be enforceable. There has to be agreement to the valuation of the contract and also of the collateral. It may be based on gross or net exposure or mark to market. It may be unilateral or may be bilateral. There is also cost associated with the administration of collateral. It may also give rise to interest cost if the collateral is cash along with potential problems to non-bank customers if mark to market goes against them.

## Contract Design

It may design the contract where the bank is in the money at the beginning of the contract though bank does not pay counter party upfront and pays up only at the expiry adjusting for the interest. The simplest example is buying of a call option by the bank or an in the money collar etc. The problem with this approach is that the protection is limited. There may also be pay-offs linked to the credit rating of the counterparty wherein the contract is closed out if there is a downgrade.

Credit Derivatives
The bank may hedge the credit risk exposure by using credit derivatives. There are various problems associated with the credit derivatives like

1) the credit risk hedged and the credit risk of the counter-party for credit-derivative may be highly correlated
2) The hedging may not be perfect in type, nature or time
3) There may be legal uncertainty regarding enforceability of the contract
4) The payoff of the derivative may not match the uncertain amount that is recoverable under default
(Any of the above three are acceptable)
