INSTITUTE OF ACTUARIES OF INDIA

CT8 – Financial Economics

OCTOBER 2009 EXAMINATION

INDICATIVE SOLUTION

Solution 1

(a) A state price security is a contract that agrees to pay one unit of currency if a particular state occurs at a particular time in the future and zero in all other states.

State price securities can be regarded as the stochastic equivalents of zero-coupon bonds, as they provide a return (of unity) at a specified future point in time, but only if a specified state of affairs exists at that time.

Equivalently, you could think of a state price security as a form of derivative that pays one unit if a particular state occurs at a particular time in the future and zero in all other states.

[Max 2 for state price security]

The price of a state price security is known as the state price. It can found using risk neutral valuation within a binomial tree.

(b)

u = 1.25
d = 0.75

The risk neutral probability q = $\frac{e^{.015} - 0.75}{1.25 - 0.75}$ = 0.53023

State Prices

S1 = 1 x $(0.53023)^2$ x exp(-0.015x2) = 0.2728

S2 = 1 x 2 x (0.53023) x (1 - 0.53023) x exp(-0.015x2) = 0.48345

S3 = 1 x $(1 - 0.53023)^2$ x exp(-0.015x2) = 0.2142

(c)

The stock price at the end of two months

S1 = 312.5
S2 = 187.5
S3 = 112.5

Real world probabilities

S1 0.2728 / 0.7622 = 0.358
S2 0.48345 / 1.0089 = 0.479
S3 0.2142 / 1.3142 = 0.163

Expected stock price after two months = 312.5*0.358 + 187.5*0.479 + 112.5*0.163
= 220

Monthly rate of return = $(220 / 200)^{0.5}$ = 4.88%

(d)

The option payoffs will be

S1	91.95
S2	117.8
S3	0

Value of call option = 91.95*0.358*0.7622 + 117.8*0.479*1.0089 + 0*0.163*1.3142
= 82.03

(e)

We know that the replicating portfolio for a one period model is

$$V_0 = \phi S_0 + \psi$$

Where

$$\phi = \frac{c_u - c_d}{S_0(u-d)}$$

$$\psi = e^{-r}\left(\frac{c_d u - c_u d}{u-d}\right)$$

ϕ is the amount of shares held and ψ is the amount of cash held in the replicating portfolio.

c_u and c_d are the payoffs of the option in case of increase or decrease in the price of the share at the end of the first period.

u and d are increase or decrease in the share price for each period

r is the risk free rate of interest for the period

At the end of first month we have two nodes. The replicating portfolios at these nodes are

Upside node at the end of the first month

Φ = (91.95 – 117.8) / (312.5 – 187.5) = -0.207

Ψ = exp(-0.015) * (187.5*1.25 – 312.5*0.75) / (1.25 – 0.75) = 154.24

$V_{1,1}$ = -0.207 * 250 + 154.24 = 102.54

Downside node at the end of first period

Φ = (117.8– 0) / (187.5 – 112.5) = 1.57

Ψ = exp(-0.015) * (112.5*1.25 – 187.5*0.75) / (1.25 – 0.75) = -174.07

$V_{1,2}$ = 1.57 * 150 - 174.04 = 61.53

At time zero

Φ = (102.54– 61.53) / (250 – 150) = 0.41

Ψ = exp(-0.015) * (61.53*1.25 – 102.54*0.75) / (1.25 – 0.75) = 0

V_0 = 0.41 * 200 + 0 = 82.03

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Solution 2

(a) The put–call parity relationship is given by:

$$c_t + Ke^{-r(T-t)} = p_t + S_t e^{-q(T-t)}$$

where c_t = price of call option at time *t*; p_t is the price of put option; K = strike price; S_t is the price of the underlying and T − t is the time to expiry.

Therefore
41 + 430 * exp(-0.03 * 2) = p_t + 350

P_t = 95.96
The assumptions made for this calculation are:
No dividends are assumed to be payable during the life of this option.
The markets in which the share and options are traded, there is no arbitrage.

(b)
Intrinsic value of put option = K – S = 430 – 350 = 80
Time value of the put option = Price of put option – Intrinsic value = 95.96 – 80 = 15.16

(c)
The factors that might cause the time value to increase with no change in intrinsic value are:
An increase in market expectations of volatility of the share price.
Decrease in interest rates
Increase in dividend payments

(d)
The holder makes a loss when he doesn't exercise the put option due to the market price of the share being higher than the strike price. The maximum loss the holder can make in such scenario is the price of the option i.e. Rs.95.96

The writer makes a loss if the market price of the share is lower than the strike price. The maximum loss the writer will make is when the share price falls to zero. The loss made by writer in this scenario = 430 – 95.96 = 334.04.

[11]

Solution 3

(a)

- The model should be arbitrage-free.
- Interest rates should be positive.
- Interest rates should be mean-reverting.
- Bonds and derivative contracts should be easy to price.
- It should produce realistic interest rate dynamics.
- It should fit historical interest rate data adequately.
- It should be easy to calibrate to current market data.
- It should be flexible enough to cope with a range of derivatives.

(b)

(i)

$B(t) = \exp [- \int f(u)\, du]$

Where $f(u) = e^{-\alpha u} R + (1 - e^{-\alpha u})L$

$\int f(u)\, du = Lt + (R – L) \int e^{-\alpha u}\, du$

$= Lt + (L – R) \dfrac{e^{-\alpha t} - 1}{\alpha}$

Or

$B(t) = \exp[-Lt + (R – L) \dfrac{e^{-\alpha t} - 1}{\alpha}]$

(ii)

Spot Yield $R(t) = - \dfrac{LnB(t)}{t}$

Therefore

$R(t) = - [-Lt + (L – R) \dfrac{e^{-\alpha t} - 1}{\alpha}] / t$

$= L + \dfrac{(R - L)}{t} * \dfrac{e^{-\alpha t} - 1}{\alpha}$

$\lim_{t \to \infty} \dfrac{e^{-\alpha t} - 1}{\alpha t} = 0$

Therefore

$\lim_{t \to \infty} R(t) = L$

[9]

Solution 4

(a) Structural models

Structural models are explicit models for a corporate entity issuing both equity and debt.
They aim to link default events explicitly to the fortunes of the issuing corporate entity.
The models are simple and cannot be realistically used to price credit risk.
However, studying them does give an insight into the nature of default and the interaction between bondholders and shareholders.
An example of a structural model is the Merton model.

Reduced – form models

Reduced-form models are statistical models, which use observed market statistics rather than specific data relating to the issuing corporate entity.
The market statistics most commonly used are the credit ratings issued by credit rating agencies such as Standard and Poor's and Moody's.
The credit rating agencies will have used detailed data specific to the issuing corporate entity when setting their rating.
They will also regularly review the data to ensure that the rating remains appropriate and will re-rate the bond either up or down as necessary.
Reduced-form models use market statistics along with data on the default-free market to model the movement of the credit rating of the bonds issued by a corporate entity over time.
The output of such models is a distribution of the time to default.

Intensity-based models
An intensity-based model is a particular type of continuous-time reduced form model.
It typically models the "jumps" between different states (usually credit ratings) using transition intensities.

(b)
The Jarrow, Lando and Turnbull approach assumes that the transition intensities between states are deterministic.
Although the JLT model allows the values of the transition intensities between any two states to vary over time, it assumes that the functions involved are known with certainty at the outset.

However, in real life, the economic conditions can change unpredictably. If, for example, a recession struck, we would expect the transition intensities corresponding to jumps to a higher-numbered state (i.e. a state closer to the default State n) to increase significantly, as companies struggled to remain profitable.
By using a stochastic approach, the transition intensities can be allowed to vary with the company fortunes and other economic factors.

[8]

Solution 5

a) The portfolio may outperform because it is a higher risk portfolio than the market. This does not contradict the EMH. The market rewards investors for taking higher risk and it is expected that on an average, a higher risk portfolio will result in higher return.

b) (i) The premise is that if markets over-react or under-react and this takes a long time to correct itself, traders could take advantage of the slow correction and efficiency would not hold.

(ii) Examples of over-reaction:
- Past performance – past winners tend to be future losers and vice-versa. Market tends to over-react to past performance.
- Certain accounting ratios seem to have predictive powers, e.g. companies with high earnings to price, cashflow to price and book value to market value, i.e. generally poor past performers, tend to have higher future returns. Again, an example of markets apparently overreacting to past performance.
- Firms coming to market (through IPO or subsequent offers) in the US have had poor subsequent performance.

Examples of under-reaction:
- Stocks continue to respond to earning announcements up to a year after their announcement. This is an under-reaction that corrects itself slowly.

- Abnormal excess returns for both the parent and subsidiary firms following a de-merger. Another example of market being slow to recognise the benefits of an event.
- Abnormal negative returns following mergers.

c) The extra assumptions for CAPM are:
- All investors have the same one-period horizon.
- All investors can borrow or lend unlimited amounts at the same risk-free rate
- The markets for risky assets are perfect, i.e. information is freely and instantly available to all investors and no investor believes that they can affect the prices by their own actions.
- Investors have the same estimates of the expected returns, standard deviations and covariances of securities over the one-period horizon.
- All investors measure in the same currency.

d) APT is based on the factor model of returns and the "no arbitrage" assumption. CAPM is based on the investors' portfolio demand and equilibrium arguments.

[12]

Solution 6

a)

We know,

$$\beta = \frac{Cov(R_i, R_m)}{V_M}$$

For LoRisk,

$$\beta = \frac{0.5 \times 24\% \times 30\%}{30\%^2} = 0.4$$

For MedRisk,

$$\beta = \frac{1.0 \times 24\% \times 30\%}{30\%^2} = 0.8$$

For the market portfolio, the beta is the weighted average of the betas of the constituents and is equal to 1. Therefore, for HiRisk,

$$\beta = \frac{1 - 20/95 \times 0.4 - 30/95 \times 0.8}{45/95} = 1.4$$

b)

As per CAPM,

$R_i = R_f + \beta(R_M - R_f)$

We have to solve for R_M and R_f.

For LoRisk and MedRisk,

$9\% = R_f + 0.4(R_M - R_f)$

$13\% = R_f + 0.8(R_M - R_f)$

Solving these equations, we get

$R_f = 5\%$

$R_M = 15\%$

c)

As per CAPM, expected return on HiRisk is given by

$R_{HiRisk} = 5\% + 1.4(15\% - 5\%) = 19\%$

The expected return on HiRisk is lower than that indicated by CAPM. Therefore, the security is over priced.

(Credit should be given if the numerical answer to this part is incorrect because of an error in the earlier parts)

d) The opportunity can be exploited by creating a portfolio of LoRisk and MedRisk which has the same beta as HiRisk and then by going long on that portfolio and short on MedRisk. This would give a portfolio of zero beta and a positive expected return.

Let w be the weight of LoRisk in the abovementioned portfolio:
Then, $w \times 0.4 + (1-w) \times 0.8 = 1.4$
Or $w = -1.5$.

For the portfolio, we will have to go short on LoRisk and long on MedRisk with the weights being -1.5 and +2.5.

(Credit should be given if the numerical answer to this part is incorrect because of an error in the earlier parts)

[12]

Solution 7

a)

Let the proportion invested in A be x_a.
Let the proportion invested in B be x_b
Therefore the investment in C is $1 - x_a - x_b$

Variance of the portfolio is:

$$V = x_a^2\sigma_a^2 + x_b^2\sigma_b^2 + 2\rho x_a x_b \sigma_a \sigma_b \ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

Expected return of the portfolio is:

$$E = x_a E_a + x_b E_b + (1 - x_a - x_b)E_c \ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

The Lagrangian function is:

$$W = V - \lambda(E - E_p)$$
$$= x_a^2\sigma_a^2 + x_b^2\sigma_b^2 + 2\rho x_a x_b \sigma_a \sigma_b - \lambda(x_a E_a + x_b E_b + (1 - x_a - x_b)E_c - E_p)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

where,
E_p = target return of the portfolio

Now, we have:

$$\frac{\delta W}{\delta x_a} = 2x_a\sigma_a^2 + 2\rho x_b\sigma_a\sigma_b - \lambda E_a + \lambda E_c = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

$$\Rightarrow 2x_a\sigma_a^2 + 2\rho x_b\sigma_a\sigma_b = \lambda(E_a - E_c)$$

$$\frac{\delta W}{\delta x_b} = 2x_b\sigma_b^2 + 2\rho x_a\sigma_a\sigma_b - \lambda E_b + \lambda E_c = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

$$\Rightarrow 2x_b\sigma_b^2 + 2\rho x_a\sigma_a\sigma_b = \lambda(E_b - E_c)$$

$$\frac{\delta W}{\delta \lambda} = -(x_aE_a + x_bE_b + (1 - x_a - x_b)E_c - E_p) = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

$$\Rightarrow x_aE_a + x_bE_b + (1 - x_a - x_b)E_c = E_p$$

Dividing (4) by (5), we have

$$\frac{2x_a\sigma_a^2 + 2\rho x_b\sigma_a\sigma_b}{2x_b\sigma_b^2 + 2\rho x_a\sigma_a\sigma_b} = \frac{E_a - E_c}{E_b - E_c}$$

or,

$$\frac{0.18x_a + 0.12x_b}{0.32x_b + 0.12x_a} = \frac{5}{10}$$

$$\Rightarrow \frac{0.18\alpha + 0.12}{0.32 + 0.12\alpha} = \frac{1}{2}$$

where, $\alpha = x_a / x_b$

$$\Rightarrow 0.36\alpha - 0.12\alpha = 0.32 - 0.24$$

$$\Rightarrow \alpha = \frac{0.08}{0.24} = \frac{1}{3}$$

For any portfolio with a target return Ep, the minimum variance will be achieved when investments in A and B are in the ratio 1/3.

b)

For a target return of 10% with minimum variance, we use the equation 6 from part (a) and the result that the ratio of investment between A and B should be 1/3;

$$0.1x_a + 3x_a \times 0.15 + (1 - x_a - 3x_a)0.05 = 0.1$$

$$\Rightarrow x_a = 1/7$$

Therefore, x_b= 3/7 and x_c = 3/7

(Please give credit for the approach and if numerical answer is incorrect because of error in previous part)

c)

For the portfolio,

Expected return = 10%.

$$Variance = x_a^2\sigma_a^2 + x_b^2\sigma_b^2 + 2\rho x_a x_b \sigma_a \sigma_b$$
$$= 0.09 \times 1/49 + 0.16 \times 9/49 + 0.12 \times 3/49$$
$$= .0386$$

Therefore, σ = 19.64%

Assuming that the returns are normally distributed, at 95% CL

VaR = 100×(10%-1.65×19.64%)=22.4

(Please give credit for the approach and if numerical answer is incorrect because of error in previous part)

[13]

Solution 8

a)

The three categories are:

i) Macroeconomic factor models – these use observable economic time series as factors, e.g. annual rates of inflation, economic growth, short-term interest rates, etc.

ii) Fundamental factor models – these are closely related to the macroeconomic models but instead of macroeconomic factors, use company specific fundamental factors, e.g. gearing ratio, P/E ration etc. They may use macroeconomic factors also.

iii) Statistical factor models – these do not rely on specifying factors independently of the historical returns data. Instead, they use a technique called Principal Components Analysis to determine a set of indices that can explain as much as possible of the observed variance.

b) The parameters for the models can be determined using regression of security returns with the market returns.

c)

(i) For any security:

$E_i = \alpha_i + \beta_i E_M$

Therefore,

$E_A = 3\% + 1.2\times10\% = 15\%$

$E_B = 2\% + 0.8\times10\% = 10\%$

$E_A = 5\% + 0\times10\% = 5\%$

(ii)

$V_i = \beta_i^2 V_M + V_{\varepsilon i}$

Therefore,

$V_A = 1.44\times0.04+0.01 = 0.0676$

$V_B = 0.64\times0.04+0.0225 = 0.0481$

Therefore, the standard deviations are: 26% and 21.93% respectively

(iii) Covariance of A and B = $\beta_A \beta_B V_M = 1.2 \times 0.8 \times .04 = 0.0384$

(iv) While the beta for security C is 0, it is not risk free as it still has a variance of returns given by the variance of the residual errors. It is just not correlated to the factor.

[10]

Solution 9

(i)

The change in bank balance in one year is given by:
dBB = 100×12+ 63.24555×sqrt(12) ×dW
dBB = Change in bank balance
Drift p.a. = Drift p.m. × 12
SD p.a. = SD p.m. ×sqrt(12) = 63.24555×sqrt(12)
dBB = N(1200, 219.089)

Starting bank balance = 200
Closing bank balance = N(1400,219.089)

For bank balance to be greater than 0, the z-value is 1400/219.089=6.39

Probabaility = 1 (from tables)

(ii) If x be the starting bank balance, the bank balance at the end of the year is N (1200+x, 219.089).

At 99% confidence level, the bank balance would be:
1200+x-219.089×2.33 (from tables)

For year end bank balance to be greater than 0 with 99% probability;
1200+x-510.4775>0
or, x > - 689.52 (Overdraft)

[5]

Total [100] Marks

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