INSTITUTE OF ACTUARIES OF INDIA

CT5 – General Insurance, Life and Health Contingencies

OCTOBER 2009 EXAMINATION

INDICATIVE SOLUTION

Question 1:

Profit = Present value of premiums – Present value of benefits

Let P be the annual premium for the sum assured, S.

Let K be a random variable representing the curtate future life time of the policy.

Profit = $P * \ddot{a}_{K+1} - S * v^{(K+1)}$
$= P * [(1- v^{K+1}) / d] - S * v^{(K+1)}$

$= P/d - [P/d + S] * v^{(K+1)}$

Variance of Profit = Var(P/d) + Var [P/d +S] $*v^{(K+1)]}$

= 0 + [P/d +S]^2 * Var ($v^{(K+1)}$)

= [P/d +S]^2 * [$^2A_x - A_x^2$]

[4]

Question 2:

(i)
Death Strain at Risk in a policy year is defined as the benefit payable on death during the year in excess of the amount required to set up the reserve and to pay out the survival benefit, if any, at the end of the year.

Expected Death Strain is the expected amount of the death strain. This is the amount that the life insurance company expects to pay in addition to the year-end reserve and survival benefit payments for the policy.

Actual Death Strain is calculated in respect of the policies where the death has occurred during the year. It is the actual amount of the benefit paid on death less the survival benefits payable at year end and the reserve required to be set up at the end of the year in respect of such policies had death not happened for such polices.

Mortality profit during the year is defined as the excess of the expected death strain over the actual death strain for the year.

(ii)

(a) Reserve Calculations:

Immediate Annuity Policies:

Reserve required at year end for each surviving policy = 10,000* a51
= 10,000*(19.291-1) = 182,910

Pure Endowment Policies:

Let P be the annual premium

P * ä35:25 = 100000*(D60/D35)
P * 16.027 = 100000 * (882.85/ 2507.40)

P = 2196.90

Reserve for each survivor at the end of year 11;

11V = 100000 * (D60/ D46) - 2196.90 * ä46:14
= 100,000 * 882.85 / 1611.07 – 2196.90 * 10.818
= 31032.92

Total reserve as at 31 Dec 2008 = 995* 182910 +997*31032.92= 212,935,271

(b) Mortality Profit Calculations:

Annuity payment due at the end of the year for each policy = 10,000

Death strain at Risk for each inforce policy at the beginning of the year
= 0 – (182,910 + 10,000) = - 192,910

Actual death strain = 5 * (-192910) = - 964550

Expected Death stain = q50 * 1000 * death strain at risk per policy
= 0.000527 * 1000* (- 192910) = - 101663.57

Mortality profit per immediate annuity policy = (- 101663.57) – (- 964550.00) = 862886.43

Pure Endowment Policies:

Death strain at risk for each inforce policy = 0 – 31032.92= - 31032.92

Expected death stain = 1000* q45* (-31032.92)
= 1000* 0.001465* (-31032.92)
= - 45463.23

Actual death strain = no of deaths times death strain at risk per inforce policy at beginning
= 3 * (-31032.92) = - 93098.76

Mortality Profit = EDS – ADS = (- 45463.23) – (- 93098.76) = 47635.53

Total Mortality profit = 862886.43 + 47635.53 = 910521.96 = 910,522

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Question 3:

The net premium reserve is calculated as the present value of the future benefits less the present value of the future net premiums on the valuation date. No explicit allowance is made for either expenses or the actual premiums payable from the policyholders (the office premium).

A. Increase in interest rate assumption reduced the net premium reserve.
B. No impact of change in expense levels
C. Increase in mortality rate assumption increases the net premium reserve.

[3]

Question 4:
(i)

Cost Head	Overhead or Direct	Initial/ Renewal/ Termination
(a) Salary of the Chief Executive Officer	Overhead - as increase in business does not increase this cost	To be apportioned appropriately to all three - CEO is responsible for all activities of the Company
(b) Incentive to sales managers for procuring new business	Direct - as increase in new premium income increases this cost	Initial Expense – as this cost incurs on policy issue
(c) Claims Investigation Costs	Direct – as higher the number of claims, higher the cost	Termination expense – as this cost incurs on claims cases only.

(ii)

There is no reserve required at the end of 3rd to 5th year as the profit (nest cash flow) is positive in subsequent years.

Reserve at the end of year 2 = 50/ 1.05 = 47.62

Reserve at the end of year 1 = (47.62*.99+100)/1.05 = 140.14

Revised profit vector after allowing for reserves is

(- 638.73, 0, 0, 600, 400)

[10]

Question 5:

$$\text{Variance of } \bar{a}_{\overline{T_x}|} = \frac{{}^2\bar{A}_x - (\bar{A}_x)^2}{\delta^2}$$

$$\bar{A}_x = \int_0^\infty e^{-\delta t}\, {}_tp_x \mu_{x+t} dt = \int_0^\infty e^{-\delta t} e^{-\mu t} \mu dt = \mu \int_0^\infty e^{-t(\delta+\mu)} dt = -\frac{\mu}{\mu+\delta}\left(e^{-t}\Big|_0^\infty\right) = -\frac{\mu}{\mu+\delta}(0-1)$$

= (μ/ μ + δ) = 0.01/ (0.01+ 0.08) = 1/9

$$\text{Similarly, } {}^2\bar{A}_x = \int_0^\infty e^{-2\delta t}\, {}_tp_x \mu_{x+t} dt = \int_0^\infty e^{-\delta t} e^{-\mu t} \mu dt = \frac{\mu}{\mu+2\delta}$$

= 0.01/ (0.01 + 2 * 0.08) = 1/17

So, the variance of the present value of benefit is

[1/17 – (1/9)^2] / 0.08^2 = 7.26

[6]

Question 6:

The prospective reserve formula for non profit endowment assurance, ${}_tV_{x+t:n-t}$, is as under:

${}_tV_{x+t:n-t} = 1 - [\ddot{a}_{x+t:n-t} / \ddot{a}_{x+t:n-t}]$

500,000 * ${}_tV_{x+t:n-t}$ = 1 – [8.314/ 13.927] = 201515 **[2]**

Question 7:

1)

a. The main advantage is the simplicity and comparison compared to the use of a set of age specific rates.
The main disadvantage is the loss of information as a result of summarizing the set of age specific rates, and any distortions that may be introduced by the choice of weights for the averaging process.

b. Its main advantage is that knowledge of $E^{c}_{x,t}$ and $m_{x,t}$ separately for each age is not needed.

Its main disadvantage is that the differences in age structure between populations will be confounded with differences between mortality rates using this index to make comparisons of mortality levels between populations.

c. Its main advantage is that it only reflects differences in $m_{x,t}$ and not differences in age structures between populations.
Its main disadvantage is that it requires age specific mortality rates $m_{x,t}$ for its calculation.

d. The advantages of the indirectly standardized rate are the:
   i. ease of availability of data to calculate it
   ii. And the fact that is a good approximation to the directly standardized rate and so removes almost all the effect of differing age structures between populations when any comparisons are made.

[9]

Question 8:

a. Let P be the annual premium.

Multiple decrement table:

Age	Death Rate (DR)	Withdrawal rate (WR)	Number of policies at start of year	Number of Deaths	Number of withdrawals	Number of policies at end of year
45	0.001465	0.05	1	0.001465	0.0499268	0.94860825
46	0.001622	0.05	0.9486083	0.00153864	0.0473574	0.89971217
47	0.001802	0.05	0.8997122	0.00162128	0.0449127	0.85317822

Number of Deaths = DR * Number of policies at start of year
Number of withdrawals = Number of policies at start of year*(1- DR)*WR

Unit cashflows:

Policy Year	Premium	Allocation Charge	Fund Value (Start of Year)	Interest on Fund	Fund management Charge	Fund Value (End of Year)
1	P	0.1P	0.9P	0.09P	0.0099P	0.9801P
2	P	0.1P	1.8801P	0.188P	0.0207P	2.0474P
3	P	0.1P	2.9474P	0.2947P	0.0324P	3.2097P

Non unit cashflows:

Policy Year	Unallocated Premium	Expenses	Interest on non unit CF	Fund Managment Charge	End of year cashflows
1	0.1P	500+0.075P	0.00125P-25	0.0099P	0.03615P-525
2	0.1P	100+0.02P	0.004P-5	0.0207P	0.1047P-105
3	0.1P	100+0.02P	0.004P-5	0.0324P	0.1164P-105

Policy Year	Probability In force	Discount factor
1	1	0.869565
2	0.948608	0.756144
3	0.899712	0.657516

Expected Present Value of profit = 0.175394P – 595.95

Therefore: 0.12P = 0.175394P – 595.95
P = Rs 10,758.38

b.

- The profit margin will be same as calculated earlier.
- Reserves do not reduce profit but only defer the release of profits

So if the interest earned on reserves is equal to the risk discount rate than the there is no effect of deferring the profit as the opportunity cost on the money held as reserves is zero.

c. The surrender / maturity value will now become the maximum of fund value or sum of premiums paid.

Calculating the loss on surrenders:

Policy Year	Fund Value at the end of Year	Minimum Surrender Value payable	Loss on surrenders/Maturity
1	10544.29	11834.22	1289.93
2	22026.71	23668.44	1641.73
3	34531.17	35502.65	971.48

Non Unit Cashflows:

Policy Year	Previous Year end cashflows	Loss on surrenders	Loss on Maturity	Revised year end cashflows
1	-136.08	64.40	0	-200.48
2	1021.40	77.75	0	943.65
3	1147.28	43.63	828.65	274.80

Taking into account the discount factor as well as the in-force probability, the expected present value of profit = Rs 665.1

Value of new business = 6.18%

d. Two approaches that could be adopted to increase the value of new business are:
1) Reduce the first year commission payable to the sales force.
2) Claw-back the first year commission on lapses. This will however depend on the financial advisor still being active.

[25]

Question 9:

Let us split the benefits in three parts:

a. X = A guarantee benefit of Rs 500,000 payable per annum for 10 years and then for the life time of Amit.
b. Y = Rs 300,000 per annum payable to Puja for lifetime from the age of 70.

c. Z = Rs 300,000 per annum payable from tenth policy anniversary if both Amit and Puja are alive.

Premium = X + Y – Z

$$X = 500{,}000\left(\ddot{a}_{\overline{10}|} + \frac{D^m_{80}}{D^m_{70}}\,\ddot{a}^m_{80}\right)$$

Y = 300,000 * $\frac{D^f_{70}}{D^f_{60}}$ $\ddot{a}^f_{70}$)

Z = 300,000 * $\frac{l^m_{80}}{l^m_{70}}$ * $\frac{D^f_{70}}{D^f_{60}}$ $\ddot{a}_{80^m:70^f}$

$\ddot{a}_{\overline{10|}}$ = 8.4353

$\frac{D^m_{80}}{D^m_{70}}$ = v^{10} * $\frac{6953.536}{9238.134}$ = 0.508497

$\ddot{a}^m_{80}$ = 7.506

$\frac{D^f_{70}}{D^f_{60}}$ = v^{10} * $\frac{9392.621}{9848.431}$ = 0.644297

$\ddot{a}^f_{70}$ = 12.934

$\frac{l^m_{80}}{l^m_{70}}$ = $\frac{6953.536}{9238.134}$ = 0.752699

$\ddot{a}_{80^m:70^f}$ = 6.876

Therefore:
X = 6,126,039
Y = 2,500,001
Z = 1,000,379

Premium = 6,126,039 + 2,500,001 – 1,000,379
= Rs 7,625,661

[10]

Question 10:

Expected present value:

$$= X \int_0^{25} {}_t p^{hh}_{35} * \sigma_{35+t} * {}_1 p^{\overline{ss}}_{35+t} * \left(\int_0^{24-t} e^{-\delta(t+u-1)} \, {}_u p^{\overline{ss}}_{36+t} \, du \right) dt$$

Where δ is the force of interest

${}_t p_{35}^{hh}$ - is the probability of a healthy life aged 35 being healthy at age 35+t

${}_1 p_{35+t}^{\overline{ss}}$ - is the probability that a life who is sick at age 35+t is sick continuously for one year thereafter

${}_u p_{36+t}^{\overline{ss}}$ - is the probability that a life who is sick at age 36+t is sick is still sick at age 36+t+u

[5]

Question 11:

The expected present value at the valuation date (age x) of benefits arising from future service in the year of age (x+t, x+t+1) is:

$$\frac{1}{2} * \frac{1}{80} * \frac{z_{x+t+1/2}}{s_{x-1}} * S * \frac{r_{x+t}}{l_x} * \frac{v^{x+t+1/2}}{v^x} \overline{a^r_{x+t+1/2}}$$

(In respect of retirement in the year of service itself - and if retirement is half through on average, the entitlement is only for half an year)

$$+ \frac{1}{80} * \frac{z_{x+t+1\frac{1}{2}}}{s_{x-1}} * S * \frac{r_{x+t+1}}{l_x} * \frac{v^{x+t+1\frac{1}{2}}}{v^x} \overline{a^r_{x+t+1\frac{1}{2}}}$$

(In respect of retirement in the year of age x+t+1 to x+t+2)

+ ....+

$$+ \frac{1}{80} * \frac{z_{NPA-1+1/2}}{s_{x-1}} * S * \frac{r_{NPA-1}}{l_x} * \frac{v^{NPA-1+1/2}}{v^x} \overline{a^r_{NPA-1+1/2}}$$

(In respect of retirement in the year of age NPA-1 to NPA)

$$+ \frac{1}{80} * \frac{z_{NPA}}{s_{x-1}} * S * \frac{r_{NPA}}{l_x} * \frac{v^{NPA}}{v^x} \overline{a^r_{NPA}}$$

(In respect of retirement at exact age NPA)

<u>Definitions:</u>

The probability of retiring between ages x+t and x+t+1 is:

$$\frac{r_{x+t}}{l_x}$$

The value of an annuity of 1 pa payable according to scheme rules, if we write on average at age x+t+1/2 is:

$$\overline{a^r_{x+t+1/2}}$$

A factor to discount from time x+t+1/2 to current time (age x) is:

$$\frac{v^{x+t+1/2}}{v^x}$$

Z_{x+t} is the 5 year average salary scale at age x+t

NPA – Normal retirement Age

Let's define the commutation function:

$${}^{z}C^{ra}_{x+t} = z_{x+t+1/2} * r_{x+t} * v^{x+t+1/2} * \overline{a^{r}_{x+t+1/2}}$$

$${}^{s}D_{x} = l_x * v^x * s_{x-1}$$

Therefore the whole expression can be written as:

$$\frac{S}{80}\ \frac{1}{{}^{s}D_x}\{\ \frac{1}{2}\,{}^{z}C^{ra}_{x+t} + {}^{z}C^{ra}_{x+t+1} + \ldots + {}^{z}C^{ra}_{NPA}\ \}$$

We define the following additional commutation function:

$$\overline{{}^{z}M}{}^{ra}_{x} = \sum_{t=0}^{t=NPA-x} {}^{z}C^{ra}_{x+t} - \frac{1}{2}\,{}^{z}C^{ra}_{x}$$

And the expected present value can be written as:

$$\frac{1}{80}\ S\ \frac{\overline{{}^{z}M^{ra}_{x+t}}}{{}^{s}D_x}$$

So this is the present value of the benefit in respect of the future year of service from x+t to x+t+1. We now need to consider all the future years of service.

Now we add over all possible years of future service i.e. (x, x+1), (x+1, x+2)… (NPA-1, NPA) to give the total expected present value of the age retirement entitlement arising from all future service:

$$\sum_{t=0}^{t=NPA-x-1} \frac{1}{80}\ S\ \frac{\overline{{}^{z}M^{ra}_{x+t}}}{{}^{s}D_x}$$

If we define a commutation function:

$$\overline{{}^{z}R^{ra}_{x}} = \sum_{t=0}^{t=NPA-x-1} \overline{{}^{z}M}{}^{ra}_{x+t}$$

Then the future liability can be expressed as follows:

EPV of Future service liability of age retirement $= \frac{1}{80}\ S\ \frac{{}^{Z}\overline{R^{ra}_{x}}}{{}^{s}D_x}$

[8]
[Total 100 Marks]

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