INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

26th October 2009

Subject CT6 – Statistical Methods

Time allowed: Three Hours (10.00 – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*
2. *Mark allocations are shown in brackets.*
3. *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*
4. *In addition to this paper you will be provided with graph paper, if required.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q 1) **(i)** Identify any four risks that a large pharmaceutical company is likely to be exposed to, and the insurance cover/product that would help the company to cover each of these risks? (4)

(ii) State the conditions for a risk to be insurable. (3)

[7]

Q 2) An insurer has two reinsurance treaties covering claims under product liability portfolio: and individual excess of loss treaty, which is applied first, and a quota share treaty, which is applied second. Under the excess of loss teaty, the reinsurer pays the portion of each individual claim in excess of Rs. 80,000. Under the quota share treaty, the insurer passes on 25% of the retained amount to the reinsurer.

Assume that the original claim amounts have a Pareto distribution with parameter $\alpha = 3$ and $\lambda = 20{,}000$.

(i) Calculate the percentage of claims that would involve payment by the excess of loss reinsurer. (1)

(ii) Determine the mean of the claim amount paid by the direct insurer, net of reinsurance. (5)

[6]

Q 3) An insurance company plans to install a custom-built software to handle night-time complaints, and wants to evaluate the associated risks. It estimates that the probability of the software failing on the i^{th} day after installation is e^{-10-i}, while the costs associated with software failure on a particular day has the exponential distribution with mean Rs. 1 million. Calculate the expected aggregate of all future losses.

[3]

Q 4) **(i)** The table below shows the total amount of the claim payments made in each development year for a portfolio of general insurance policies. The italicized numbers show the projected amounts calculated using the basic chain ladder method. Unfortunately, some of the entries (indicated by *) are illegible.

Claim payments (Rs.'000s)		Development year			
		0	1	2	3
Accident year	1997	32.7	19.7	*	4.7
	1998	41.6	*	12.9	*
	1999	*	37	*	*6.7*
	2000	40.2	*32.1*	*11.1*	*6.7*

Determine the total amount of claims paid during the 1999 calendar year. (6)

(ii) Use the Bornheutter Fergusson method (BF) to estimate the outstanding claims reserve for the data given in the table below. The total claims paid so far in respect of these policies are Rs. 2,500,000.

Claims incurred (Rs.'000s)		Earned premium (Rs. '000s)	Development year			
			0	1	2	3
Accident year	1996	1050	650	146	72	23
	1997	1135	685	152	84	
	1998	1180	704	144		
	1999	1260	722			

[Use the loss ratio in respect of AY 1996 as the initial estimate]. (8)

[14]

Q 5) The total amount of claims for each year from a portfolio of five insurance policies over t years were found to be $X_1, X_2, \ldots, X_t$. The insurer believes that the annual claims have a normal distribution with mean θ and variance σ^2, where θ is unknown. The prior distribution of θ is assumed to be normal with mean μ and variance τ^2.

(i) Derive the posterior distribution of θ. (4)

(ii) Using the posterior distribution found in (i), write down the Bayesian point estimate of θ under the quadratic (squared error) loss function. (1)

(iii) Show that the answer in (ii) can be expressed in the form of a credibility estimate, and derive the credibility factor. (2)

(iv) If one uses the all-or-nothing loss function, can the corresponding Bayesian estimate of θ be written as a credibility factor? Explain. (2)

[9]

Q 6) An insurer operates an NCD system with discounts 0%, 30% and 50%. Losses arrive for a policyholder according to a Poisson process. In the event of at least one claim during the year, the policyholder moves down one level or remains at the 0% discount level. In the event of a claim free year, the policyholder moves up one level or remains at the 50% discount level. The full premium for these insurance contracts is Rs. 300 per annum.

(i) Assuming that every policyholder has an n-year time horizon (where $n > 1$), show that a policyholder paying full premium at the 0% discount level will make a claim only if the loss amount is above Rs. 150, and policyholders paying premium at the 30% and 50% discount levels will make a claim only if the loss amount is above Rs. 210 and Rs. 60, respectively. (3)

(ii) The claim size distribution for this portfolio is known to be lognormal with parameters $\mu = 6.012$ and $\sigma^2 = 1.792$. Calculate the probability that in respect of a given loss, a policyholder with a two-year time horizon will choose to claim under the policy. (3)

(iii) On the assumption that losses for different policyholders arise according to independent Poisson processes with common incidence rate of 0.1 per year and the same two-year time horizon, obtain the underlying transition matrix for the policyholders of the company and estimate the proportion of policyholders in the various NCD categories in a stationary non-growth situation. (6)

[12]

Q7) **(i)** In the context of generalised linear models, explain the term 'linear predictor'. (1)

(ii) An insurance company is trying to model the number of claims in a year on its home insurance policies. The following data are available for each policyholder: area (classified into ten categories), class of property (classified into six categories) and age of property (expressed in months).

Write down an expression for the linear predictor for the following models, stating the number of parameters (excluding redundant parameters) contained in each:

(a) area + class,

(b) area + class + age + age*area. (4)

[5]

Q8) **(i)** Calculate the auto-covariance function $\{\gamma_k, k \geq 0\}$ and the auto-correlation function $\{\rho_k, k \geq 0\}$ for the m^{th} order moving average process:

$$X_t = \mu + \frac{1}{m+1} \times \left(e_t + e_{t-1} + e_{t-2} + \cdots + e_{t-m}\right),$$

where $\{e_k, k \geq 0\}$ is a sequence of uncorrelated, zero mean random variables with common variance σ_e^2. (4)

(ii) Explain whether or not the process is invertible in the case when $m = 2$. (2)

[6]

Q9) It is thought that the daily closing price of a stock $\{X_t : \ t = 0,1,2,\ldots\}$ may be modeled by the following relationship

$$X_t = 1.7X_{t-1} - 0.4X_{t-2} - 0.3X_{t-3} + e_t - 0.7e_{t-1} + 0.12e_{t-2}.$$

For this model:

(i) Write the equation in terms of the backward shift operator B in the form $\phi(B)(1-B)^d X_t = \theta(B)e_t$, where $\phi(B)$ and $\theta(B)$ are polynomials in B. (2)

(ii) Identify the values of p, d and q such that X_t is an ARIMA(p, d, q) process. (1)

(iii) Explain whether the process $\{X_t : \ t = 0,1,2,\ldots\}$ is stationary. (1)

(iv) For the value of d from part (ii), put $W_t = (1-B)^d X_t$. Explain how the model can be written in the equivalent form: $W_t = \sum_{i=0}^{\infty} \psi_i e_{t-i}$, and calculate ψ_i for $i = 0,1$ and 2. (2)

(v) If the above model has been obtained through fitting of an ARIMA model to a time series data set (X_t for t = 1 to 1000), indicate how the fitted model can be used for forecasting X_{1002}, using the Box-Jenkins approach. (3)

[9]

Q10) **(i)** Describe the three integers one has to choose in order to form a linear congruential generator (LCG) and set out a recursive relationship, with subsequent scaling, for generating a sequence of pseudo random numbers in the range –1 to 1. (3)

(ii) Given a random number U uniformly distributed over [0,1], give an expression in terms in terms of U for another random variable X whose density function is $f(x) = 5\exp(-5x)$. Justify the expression. (1)

(iii) It is required to generate a sequence of simulated observations from the density function

$$g(x) = \frac{k\exp(-5x)}{1+x}, \quad x > 0,$$

where k is a constant not involving x. Describe a procedure that applies the Acceptance Rejection method to obtain pseudo random samples from this distribution. Is it necessary to determine the constant k in order to use this method? (3)

[7]

Q11) A large diversified company wants to get into the insurance business and apply to the regulator agency for a license. They have identified three categories of products: Protection, Saving and Investment, and they would choose to focus on only one of these during their first few years of operation. The insurance company has to pay a fixed amount, Rs. 2 million, annually to the regulator, plus some other variable costs that depend on the chosen strategy. These variable costs and expected revenue per policy for the three possible strategies are as follows:

Category of product	Variable cost	Expected revenue per policy
Protection	Rs. 300,000	Rs. 50
Saving	Rs. 600,000	Rs. 100
Investment	Rs. 900,000	Rs. 150

The company is uncertain about the number of policies, and has prepared a profit forecast based on Cautious, Best estimate and Optimistic number of policies. The projected numbers under these scenarios are 10,000 (Cautious), 20,000 (Best estimate) and 30,000 (Optimistic).

(i) Determine the annual loss under each possible combination of category and scenario for forecast. (3)

(ii) Determine the minimax strategy regarding the choice of the category of product. (2)

(iii) Determine the Bayes strategy based on the following probabilities of the different scenarios:

P(Cautious) = 0.15, P(Best estimate) = 0.75, P(Optimistic) = 0.10. (2)

[7]

Q12) Let $S = \sum_{i=1}^{N} X_i$, where $N, X_1, X_2, \ldots$ are independent, N has the Poisson distribution with mean λ and $X_1, X_2, \ldots$ have the common distribution $P(x)$ with mean m_1, second moment m_2 and third moment m_3.

(i) Show that the distribution of S is positively skewed. (3)

(ii) Comment on the skewness of Z for finite λ and also as $\lambda \to \infty$. (2)

[5]

Q13) An insurer insures a large building. The probability of a claim on a given day is p, and the daily claims are independent. Premium of Rs. c is payable on a daily basis at the start of each day. The claim size is independent of the day of the claim, and follows an exponential distribution with mean $1/\lambda$. The insurer has an initial surplus of Rs. U_0.

(i) Derive an expression for the probability that the first claim results in the ruin of the insurer. (5)

(ii) If $p = 0.01$, $\lambda = 0.0125$ and $c = 10$, determine how large U_0 must be so that the probability that the first claim causes ruin is less than 1%. (2)

(iii) Derive and expression for the probability that the insurer is not in a state of ruin at the end of the first two days, evaluate it for $p = 0.01$, $\lambda = 0.0125$, $c = 10$, and U_0 chosen as in part (ii). (3)

[10]

****