INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

22nd October 2009

Subject CT4 – Models

Time allowed: Three Hours (10.00 – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1) *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*

2) *Mark allocations are shown in brackets.*

3) *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*

4) *In addition to this paper you will be provided with graph paper, if required.*

AT THE END OF THE EXAMINATION
Please return your answer book and this question paper to the supervisor separately.

Q 1) **a)** Explain the terms "under-graduation" and "over-graduation". (3)

b) State four advantages and four disadvantages of using the graphical method of graduation. (4)

[7]

Q 2) Given below is the distribution of the number of medical claims per policy during a one-year period for a block of 3000 insurance policies:

No of claims per policy	Number of policies
0	1000
1	1200
2	600
3	200
4+	0

(i) Fit the Poisson model to the number of claims per policy using the method of maximum likelihood. (2)

(ii) Construct the large-sample 90% confidence interval for the mean of the underlying Poisson model that is symmetric around the mean and determine the lower end-point of the confidence interval. (2)

(iii) Outline the differences between the two-state model and the Poisson model when used to estimate transition rates. (3)

[7]

Q 3) Consider a Markov chain with finite state space N. Suppose for all *i* and *j*

$$\lim_{n\to\infty} p^{(n)}_{ij} = \pi_j$$

which is not a function of *i*.

Then prove that

(i) $0 \le \pi_j \le 1$ and $\sum_{i=1}^{N} \pi_i = 1$ (3)

(ii) $\pi_k = \sum_{j=1}^{N} \pi_j p_{jk}$ (2)

(iii) **If** $X_m = (\pi_1, \pi_2, \ldots, \pi_N)$ **then** $X_n = X_m = (\pi_1, \pi_2, \ldots, \pi_N)$ **for all** $n \ge m$. (2)

[7]

Q 4) Consider a one-server system which follows continuous two state Markov Chain with two possible states: 0 (idle) and 1 (busy).

Assuming that the arrival process of the customers is a Poisson process with mean rate λ and the service time of the server have mean rate μ.

Let P_o (t) be the probability that the server is idle at time t and P_1 (t) be the probability that the server is busy at time t.

Determine the steady state probabilities P_o(t) and P_1 (t)

[8]

Q 5) A Cox proportional hazards model was used to study the mortality experience amongst group of lives classified according to smoker or non smoker and male or female.

(i) How you would use the Cox model in this study, assuming that there is only single covariate? Your answer should include a description of the relevant Cox model and you should define any notation you use. (4)

(ii) The following information are given:

a) There are two covariates; z_1=1 for smoker and z_1=0 for non smoker and z_2=1 for male z_2=0 for female

b) The parameter estimates are $\hat{\beta}_1 = 0.05 \, and \, \hat{\beta}_2 = 0.15$

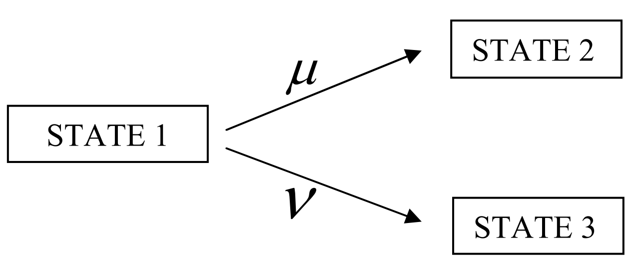
c) The covariance parameter matrix for $\hat{\beta}_1$ *and* $\hat{\beta}_2$ is

$$\begin{pmatrix} 0.0002 & 0.0001 \\ 0.0001 & 0.0003 \end{pmatrix}$$

Determine the upper limit of the 95% confidence interval for the relative risk of a female non-smoker compared to male smoker. (5)

[9]

Q 6) The multiple state model, with constant transition intensities μ and ν, is given below



a) Derive expressions for:

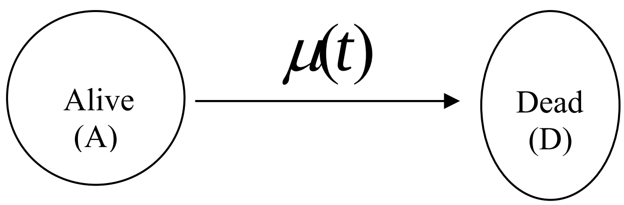
(i) ${}_tp_x^{11}$ (Prob : State1 -> State1) (4)

(ii) ${}_tp_x^{12}$ (Prob : State1 -> State2) (4)

b) State all the main assumptions underlying the model. (2)

[10]

Q 7) Drive all the four integrated form of Kolmogorov’s backward differential equations to the survival model given below:



[12]

Q 8) Describe the process of fitting a time homogenous simple stochastic Markov model to a set of observations. Also discuss as to how the simulation can be used to assess the goodness of fit.

[13]

Q 9) **a)** Describe what is meant by homogeneity in a mortality investigation and explain the problems which lack of homogeneity may cause for an insurer in setting premium rates. (2)

b) Suggest six factors in respect of which life office mortality statistics are often subdivided. (2)

c) The mortality of a large group of assured lives is to be investigated over a period of 3 years from 1st January 2006 to 31st December 2008. Sufficient information is available to allow deaths to be grouped in any one of the following basis:

- Age next birthday on 1st January prior to date of death.
- Age last birthday on the policy anniversary nearest to the date of death.
- Age nearest birthday at entry plus curtate duration at death.

In each case, state the rate interval and describe how you have arrived at the rate interval corresponding to the given age-definition.

Assuming that the deaths defined are divided by an appropriate exposed to risk, state the age to which the crude mortality rate applies and explain how you have arrived at that age.

State any assumptions you have made. (9)

[13]

Q 10) The population of elderly people in the old care home is observed during the period 1st January 2006 to 31st December 2008. The duration of residence (measured to the nearest number of months) is recorded for those who die during the period, for those who have gone back to their relatives during the period and for those who are still continuing on 31st December 2008.

The recorded data measured in months are

6^+ 6 6 6 7 9^+ 10^+ 10

11^+ 13 16 17+ 20 23^+

where + indicates those who have gone back to their relatives during the period or who were still in residence on 31st December 2008.

(i) Explain the type(s) of censoring present in these data. (3)

(ii) Calculate the product-limit (Kaplan-Meier) estimate of the survival function, *S(t)*, where *t* is the duration of residence in the old care home. (7)

(iii) State the assumptions underlying the estimate in (ii), and explain how each of these assumptions would apply to these data. (4)

[14]

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