Institute of Actuaries of India

Subject ST5 – Finance and Investment A

October 2014 Examination

INDICATIVE SOLUTION

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1:

i. Diversification is a strategy wherein your risk is lowered when one spreads capital across multiple assets and/or multiple asset classes. Instead of investing in one stock money is spread across say ten stocks. One is less likely to suffer large losses as one is not exposed to a single stock not doing well. And so, just by diversifying portfolio one can reduce risk.

[1]

ii. Diversification fails when assets happen to be too positively correlated to one another. At times of financial crisis, the correlations between various assets and asset classes around the world may go up as was witnessed during the 2008 financial crisis. In such times benefits of diversification may not be much. Portfolio diversification is supposed to help you in times of crisis. It is supposed to help you because the losses you'll make on some assets are offset by gains in others. But this is not what happen during crisis.

[2]

iii. Diversification works just fine when markets are in a normal state. In a normal state, diversification can reduce your risk without affecting your returns.

[1]

iv. Buying safe haven assets like gold
Buying Put options
Hedging using futures during times of crisis
Hedging using a combination of long and short assets
Investing in uncorrelated assets
Increasing the cash position of the portfolio by liquidating

[3]
[7 Marks]

Solution 2:

i. Risk:

Some phenomena which can be quantified and measured and hence can be priced.
Or we don't know what is going to happen next, but we do know what the distribution looks like.

Uncertainty:

Phenomenon which are hard to measure. There may be vague awareness of the situation but one is not sure about the frequency and severity of each factor
Or we don't know what is going to happen next, and we do not know what the possible distribution looks like.

[3]

ii.

	Default Probability		
	If uncorrelated	If correlated	Risk Multiple
Alpha pool	0.00003%	5%	1,60,000
Beta	0.0030%	5%	1,684
Gamma	0.11281%	5%	44
Theta	2.14344%	5%	2
Epsilon	20.3627%	5%	0.2

Correlated scenario will be possible when markets are healthy and stable. Uncorrelated scenario is similar to 2007 like situation which demonstrated the behavior of CDOs when the crises happen and everyone assumed and expected that default of one security will not affect the outcome of another but in reality all were linked. Alpha pool which is considered to be least risk is 1.6 lakh time riskier if we assume that securities are uncorrelated and situation is reverse for Epsilon pool

[12]

[15 Marks]

Solution 3:

The investor can effectively sell his entire portfolio of equities by selling the appropriate equity index futures depending on the beta of the portfolio with respect to the index allowing appropriately for the contract size of the futures used. The balance between the available equity index futures will depend on the constitution of the investor's equity portfolio.

At the same time he will buy futures (of the bonds of required duration) to the same total value. (Assuming that required bond futures are available)

Once this has been done, the investor effectively has no exposure to the equity market and full exposure to long-dated bonds.

Over the term of the futures contracts (*e.g.*3 months), the investor can gradually sell the shares in the cash market and buy bonds.

Simultaneously, the investor will unwind the futures contracts at the same rate, *i.e.*by buying the equity index futures and selling the bond futures.

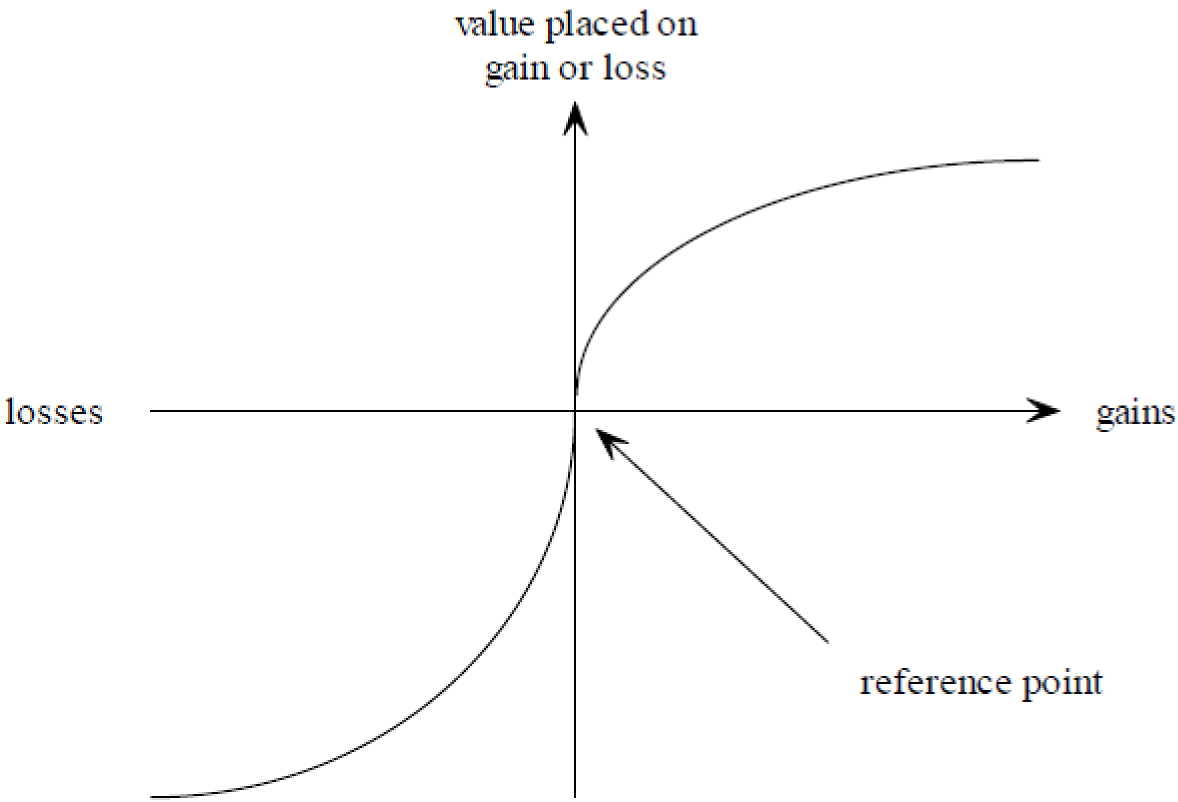
Such a strategy will help the fund manager to effect the desired transaction with much lesser impact cost.

[4 Marks]

Solution 4:

i. ***Prospect theory***

- Prospect theory is a theory of how people make decisions when faced with risk and uncertainty.
- It attempts to explain why people may make asymmetric choices when faced with similar possible gains and losses.
- It replaces the conventional theory of utility associated with a unit of gain or loss based on total wealth with a concept of value defined in terms of gains and losses relative to a reference point.
- In particular, individuals are assumed to be risk-averse when facing gains and risk seeking when facing losses defined in terms of some reference point.
- This generates utility curves that are concave above the reference point, convex below the reference point and with a point of inflexion at the chosen reference point.
- Prospect theory therefore suggests that the way in which alternatives are presented or framed can be very important.



[6]

ii.

- Straddle is an option based strategy which involves taking a position in both the call and put option of the same strike and date opf expiry – usually at the money
- A straddle behaves like a call/putoption after one significant move in the underlying
- VIX on the other hand helps the trader to continue to be long/short volatility even after significant moves in the underlying
- Straddle can be done on stocks whose options are listed
- VIX is usually a future based strategy on the index
- Straddles are too much dependent on the strike price. They are path dependent while Vix futures are not which probably makes it more suited for volatility trade if that is available

[6]
[12 Marks]

Solution 5:

i. The first step is to find the implied volatility of the option. The cost of a call option on NIFTY with strike 8000 is 175. The volatility has to be found using trial and error and works out to 16%.

Assume that impact of term structure of volatility can be ignored the same implied volatility can be used to find the price of the put option to be bought

	Call
S	8000
X	8000
R	9%
T	0.08
Q	1%
d1	0.17
d2	0.12
Price	175.00
Sigma (By trial and error)	16%

When the portfolio falls by 5% over 6 months, the expected fall in the index value needs to be calculated.

The return from portfolio over 6 months when it falls by 5% is -5%+1% = -4% i.e. annualized return of -8%.

From CAPM:

Rp-Rf = β*(Rm-Rf)

-8% - 9% = 1.5*(Rm-9%)

Rm = -2.33%

If Rm is -2.33%, the expected fall in the index over 6 months is ½* (-2.33%-1%) = -1.67%

So the expected index value at the end of 6 months is 7867 which becomes the strike for the protective put. The price of the put option from Black Scholes formula is

	Put
S	8,000
X	7,867
Sigma	16%
r	9%
T	0.5
q	1%
d1	0.56
d2	0.45
Price	172

Portfolio is 3, 12,500 times the NIFTY. So the fund manager needs to purchase 3, 12,500X1.5 = 4, 68,750 put options with strike 7867 expiring in 6 months time. The cost of this insurance will be about 8.06 Crores or 3.2% of the portfolio.

[10]

ii. The delta of the put option is given by –exp (-qT) N (-d1) = -0.287. So the overall delta is 4, 68,750 X -0.29 = -1, 34,531.

The delta of a 9 month future is exp {(r-q) T} = 1.062. (Note that T here is 0.75 being a 9 month future).

So number of future contracts to be shorted is 1, 26,696. {There may be round off errors depending on the number of decimal places used. The right approach is what matters}

[5]

[15 Marks]

Solution 6:

i. Merton model looks at stocks as a call option on the value of the assets of the company where the strike price is equal to the amount of debt D.

Suppose, for example, that the company has a zero-coupon bond outstanding, and that the bond matures at time *T*. We wish to determine the value of this bond.

Let:

- V_t= the value of the company's assets at time t
- E_t= the value of equity at time t
- D = debt (principal and interest) due to be paid at time T
- σ_V = firm volatility

If $V_T<D$, it is rational for the company to default in its debt at *T*. The value of the equity is then zero.

If $V_T>D$, the company should make the debt repayment and the value of the equity at this time is V_T-D.

The value of the firm's equity at time *T*, E_T, is therefore: max $(V_T- D, 0)$

This payoff is similar to that of a call option with strike D and current value V_T

The Black-Scholes formula can then be used in the usual way to give the current value of the equity, E_0, since:

$$E_0 = V_0\Phi(d_1) - De^{-rT}\Phi(d_2)$$

where:

$$d_1 = \frac{\ln(V_0/D) + \left(r + \sigma_V^2/2\right)T}{\sigma_V\sqrt{T}}$$

and:

$$d_2 = d_1 - \sigma_V\sqrt{T}$$

[4]

ii. The firm value of Target based on the EV/EBITDA multiple of 6 is 1800 while its debt itself is more than that. This would mean that value of equity is next to nothing. The company is probably in distress due to high debt and in such situations the market value to firm value drops will be a small portion of firm value. The payoffs start resembling an option more and more and hence this model is probably more suited to value the stock

[3]

iii. The face value of debt is 1000+1500 = 2500

The duration of ZCB is 4 years

The YTM of the ZCB is 10.95%

We may assume that the debenture YTM will be in line with the ZCB YTM

The duration of the debenture is 4.15.

The weighted average duration of the debt is 1000/2500*4+1500/2500*4.15 = 4.09 years

We need to know the firm's volatility. Since details about the equity volatility and debt volatility and correlation between them for Target is not known (being unlisted) we may assume the corresponding values of Acquirer as a proxy for Target's firm volatility.

Firm variance of Acquirer is 0.23^2*0.1^2+0.77^2*0.2^2+2*0.3*0.23*0.77*0.2*0.1= 0.0263. Volatility = 0.162. {D/E = 0.3 →D/(D+E) = 0.23; E/(D+E) = 0.77}

Risk free rate may be taken as 8.584% - using interpolation. (Using 8.58% as risk free rate is also acceptable)

Assume no dividends are going to paid

So all the required parameters are now available

V_0	1800
D	2500
T	4.087
R	8.584%
σ_V	16.2%
Equity Value using Black Scholes formula	**252**

[12]

iv. The value of equity is given by

$$E_0 = V_0\Phi(d_1) - De^{-rT}\Phi(d_2)$$

$\Phi(d_2)$ in the above formula represents a survival probability – *ie* the probability that the company does not default on its debt. Thus, **the risk-neutral probability that the company will default is $\Phi(-d_2)$.**

Using this, the probability of default can be estimated which is useful in calculating the theoretical price of a CDS. Here it is 53%.

[3]

v. The notion of "debt" for an insurance company is not well defined. An insurer's policyholder liabilities are essentially indistinguishable from other debt (if any) from the perspective of the equity holder. Due to the complexity of the policyholder liabilities, a single expiration date for all of an insurer's "debt" cannot be readily approximated. So the use of this model in the valuation of a General Insurer may not be appropriate.

[3]

[25 Marks]

Solution 7:

i. The first step is to calculate the Variance-Covariance Matrix which is

	A	B	C
A	0.0025	0.0060	0.0036
B	0.0060	0.0225	0.0072
C	0.0036	0.0072	0.0144

The portfolio variance is

$$x_A^2\sigma_A^2 + x_B^2\sigma_B^2 + x_C^2\sigma_C^2 + 2x_Ax_B\sigma_{AB} + 2x_Ax_C\sigma_{AC} + 2x_Bx_C\sigma_{BC}$$

Where x_A, x_B and x_C are the weights of A, B and C being 0.33, 0.27 and 0.40 respectively.

Portfolio variance is 0.008 and portfolio standard deviation is 0.088.

Portfolio VaR = 1.645x0.088x15 = 2.17

VaR – A = 1.645x0.05x5 = 0.41

VaR – B = 1.645x0.15x4 = 0.99

VaR – C = 1.645x0.12x6 = 1.18

Sum of individual VaR = 2.58

Benefit of diversification is 0.41

[4+4]

ii. Arriving at beta of each component with respect to the portfolio maybe done using first principles. For example the beta of a position 'a' with respect to the portfolio 'p' is given by

$$\beta_a = \frac{Cov(R_a, R_p)}{\sigma_p^2}$$

$$Cov(R_a, R_p) = Cov\{R_a, (x_aR_a + x_bR_b + x_cR_c)\}$$

$$= x_a\sigma_a^2 + x_b\sigma_{ab} + x_c\sigma_{ac}$$

Based on the above find Beta of A, B and C with respect to portfolio

Beta – A	Beta – B	Beta – C
0.50	1.40	1.15

You may cross check your calculations as follows:

$W_A \beta_A + W_B \beta_B + W_C \beta_C = 1$

(Rounding of errors maybe ignored)

Contribution of A to total VaR	2.17x0.33x0.5=	0.36
Contribution of B to total VaR	2.17x0.27x1.4 =	0.81
Contribution of C to total VaR	2.17x0.4x1.15 =	1.00
Total		**2.17**

[8]

iii. From CAPM we know that when portfolio is optimized the ratio of excess return of an asset to its Beta needs to be equal. If we calculate the ratio for the three positions it is as follows

	E_i/β_i
A	0.100
B	0.121
C	0.105

Where E is the excess return and i = A, B and C respectively

From the table it is clear that there is scope for optimization. Say an increased allocation to B is needed to maximize risk adjusted return.

The portfolio can be optimized by finding the combination of x_A, x_B and $x_C$which maximizes the Sharpe ratio given by E_p/σ_psubject to following constraints

$x_A + x_B + x_C = 1$

x_A, x_B and $x_C \geq 0$ (if short positions are not allowed)

Solution of this optimization problem is not expected as you are asked to just outline the approach! **[6]**

[22 Marks]

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