

# INSTITUTE OF ACTUARIES OF INDIA

## EXAMINATIONS

11<sup>th</sup> November 2014

**Subject ST6 – Finance and Investment B**

**Time allowed: Three Hours (10.15\* – 13.30 Hrs)**

**Total Marks: 100**

### INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
2. *\* You have 15 minutes at the start of the examination in which you are required to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You have then three hours to complete the paper.*
3. *You must not start writing your answers in the answer sheet unless instructed to do so by the supervisor.*
4. *The answers are not expected to be any country or jurisdiction specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.*
5. *Attempt all questions, beginning your answer to each question on a separate sheet.*
6. *Mark allocations are shown in brackets.*
7. *Please check if you have received complete Question paper and no page is missing. If so, kindly get a new set of Question paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

**Q.1)** Silverstone Assurance is required to hold the capital which will protect the policyholders in the event of a 50% equity market fall. The company can reduce this capital requirement by purchasing put options on the NSE Nifty index. The current spot value of the NSE Nifty is 8000.

- i) Explain how put options can be used to reduce the capital requirement and what risks are associated with holding put options compared to cash deposits as capital.

The company currently has the following put options

Strike	5500
Number of option	20000
Time to expiry	6

Current economic indicators are as follows:

Dividend yield on NSE Nifty index – 5%

Interest rates

Years:	Rates
4.0	5.50%
5.0	6.00%
6.0	6.50%
7.0	7.00%
8.0	7.50%
9.0	8.00%
10.0	8.50%

Implied volatility

Implied Expiry	Strike as a Percentage of the Forward												
	40	50	60	70	80	90	100	110	120	130	140	150	160
4yr	27%	25%	23%	21%	20%	19%	17%	16%	15%	15%	14%	14%	14%
5yr	27%	25%	23%	22%	20%	19%	18%	17%	16%	16%	15%	15%	15%
6yr	27%	26%	25%	24%	23%	22%	21%	20%	19%	18%	17%	16%	15%
7yr	29%	28%	27%	26%	25%	24%	23%	22%	21%	20%	19%	18%	17%
8yr	31%	30%	29%	28%	27%	26%	25%	24%	23%	22%	21%	20%	19%
9yr	33%	32%	31%	30%	29%	28%	27%	26%	25%	24%	23%	22%	21%
10yr	35%	34%	33%	32%	31%	30%	29%	28%	27%	26%	25%	24%	23%

\*Please note that implied volatilities are at strike as percentage of forward **not at spot** (3)

- ii) What is the current market price of the put option?

The ALCO (Asset-Liability Committee) presents a paper noting that equity markets are up and implied volatilities are low compared to past. This presents an opportunity to extend the hedge either by increasing the duration of the option or by increasing the strike price of the option. The options to consider are of duration 9 years at strike 5500 or duration 6 years at strike 6000. (4)

- iii) Calculate the price of the above 2 options. (8)

- iv) Estimate the reduction in capital requirement due to holding the above options at 50% fall in equity values. State clearly any assumptions used including dividend yields. (8)

- v) Recommend which option is better value for money for the purpose of capital reduction. Mention other costs and risk ALCO should be aware of before making purchase decisions. (3)

[26]

- Q. 2)** After an occupational accident, Mumbai court awarded regular annual compensation to be paid to the victim after their retirement in 15 years time. The compensation, which is determined now, should increase in line with RPI, subject to a maximum of 6% per annum until the retirement age. While the rate of inflation measured by RPI may be negative, regular annual compensation cannot be less than the amount determined now.
- i) Write down an expression for the annual compensation at the retirement date. (Let  $A$  be the payment at date of court decision). Explain how this can be viewed as combination of options and describe these options. (6)

- ii) If the RPI follows lognormal process

$$d(\log q(t)) = \mu(t, q(t))dt + \sigma dz$$

with a continuous rate of return of  $r$  and interest rate  $\delta$ , derive a formula to estimate the present value of expected annual compensation commencing after 15 years. (6)

[12]

- Q. 3)** i) Describe the method of Monte Carlo (MC) simulation applied to a bond (swap) yield curve developing under the influence of a stochastic process. (3)
- ii) Discuss the advantages and disadvantages of using this approach to value interest rate options, compared with other approaches such as binomial/trinomial trees. (6)
- iii) Describe three ways that the MC approach can be refined, from its raw form, to reduce the number of iterations without affecting the accuracy of the end result. (6)

[15]

- Q. 4)** Party A wants to take a floating rate loan and Party B wants to take a fixed rate loan. They face borrowing rates as below:-

	<b>Fixed</b>	<b>Floating</b>
<b>Party A</b>	$F_A$ %	$\text{LIBOR} + S_A$ %
<b>Party B</b>	$F_B$ %	$\text{LIBOR} + S_B$ %

Party A agrees to pay a bank a floating rate of  $(\text{LIBOR} - S'_A)$  % in exchange of fixed rate of  $(F_A + F'_A)$  %, and Party B agrees to pay the same bank a fixed rate of  $(F_B + F'_B)$  %, in exchange for a floating rate of  $(\text{LIBOR} - S'_B)$  %. Prove that

$$0 < S_A + F'_A + S'_A < S_A + F_B - F_A + F'_B + S'_B < S_A + F_B - S_B + F_A$$

must be true for both A and B to enter into a swap with the bank in which A effectively takes out a floating rate loan and B a fixed rate loan. (6)

- Q. 5)** Let  $S_0$  be the initial price of a share of stock that pays no dividends. Let  $r$  be one plus the risk free rate, let  $\mu$  be one plus the mean stock return, and let  $\sigma$  be the standard deviation of the stock return, all per period. What is the price today of a claim maturing 10 periods from now that pays the average stock price over the next 10 periods? (7)

- Q. 6)** Let  $\{W(t); t \geq 0\}$  be a Brownian motion. Verify whether  $\frac{W(t)^2}{2}$  is a martingale or not. (5)

- Q. 7)** Let the short risk-free interest rate in the market be 25% per period. Consider a stock, trading in the same market, with the current price being \$100. The stock price either doubles or falls by half in each period. The actual probability of an increase is 60% and that of a decrease is 40%. Consider European and American options with a strike price of \$75 and maturity two periods from now. Assume discrete compounding of the interest rate process.

Answer the following questions based on this information.

- i) What are the risk neutral probabilities? (2)
- ii) What is the European put values at each node? (3)
- iii) What is the American put values at each node? (3)
- iv) What is the price of a European call option with strike \$75 maturing two periods from now? Use put-call parity. (2)

[10]

- Q. 8)** Happy Burgers has an exclusive contract to supply burger buns to the organizers of the annual burger eating contest. The contract states that the contest organizers will take delivery of 10,000 burger buns in one year at the market price. It will cost Happy Burgers 1,000 to provide 10,000 burger buns and today's market price is 0.12 for one burger bun. The continuously compounded risk-free interest rate is 6%.

Happy Burgers has decided to hedge as follows (both options are one-year, European): Buy 10,000 0.12-strike put options for 84.30 and sell 10,000 0.14-strike call options for 74.80.

Happy Burgers believe the market price in one year will be somewhere between 0.10 and 0.15 per burger bun. Calculate an interval that represents the range of possible profit one year from now for Happy Burgers.

[6]

- Q. 9)** i) Describe briefly the Vasicek one-factor model of interest rates and its key statistical properties. (3)
- ii) Let  $r_t$  denote the short rate of interest applicable at time  $t$ . According to a particular parameterization of this model, the instantaneous forward rate applicable at time  $t > 0$  implied by the market prices at time 0 is found to be:

$$f(t) = r_0 e^{-\alpha t} + r_\infty (1 - e^{-\alpha t}) + k(1 - e^{-\alpha t})e^{-\alpha t}$$

where  $\alpha$  is a positive constant.

- a) Show that, if the graph of  $f(\tau)$  has a stationary point, then

$$e^{-\alpha t} = \frac{r_0 - r_\infty + k}{2k}$$

- b) Hence describe the shape of the graph of  $f(t)$  in each of the following cases

1.  $r_0 = 0.04, r_\infty = 0.06, k = 0.1$
2.  $r_0 = 0.06, r_\infty = 0.04, k = 0.02$

- c) State a possible explanation for a yield curve with the shape of the graph in (ii) (b) 1. (6)

- iii) Describe briefly the main advantages and limitations of the Vasicek model. (4)

[13]

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