

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

10th November 2014

Subject CT8 – Financial Economics

Time allowed: Three Hours (10.30 – 13.30 Hrs.)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

- 1. Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
- 2. Mark allocations are shown in brackets.*
- 3. Attempt all questions, beginning your answer to each question on a separate sheet.*
- 4. Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

- Q. 1)** The price of a stock is currently Rs. 500. Over each of the next two 3-month periods the stock price is expected to go up by 6% or down by 5%. The risk free interest is 5% per annum with continuous compounding.
- i) What is the value of a six-month European call option with a strike price of Rs. 510? (3)
 - ii) What is the value of a six-month European put option with a strike price of Rs. 510? Verify that the European call and put prices satisfy put-call parity theorem. (3)
 - iii) What is the value of a six-month American put option with a strike price of Rs. 510? (3)
 - iv) Suppose in the real world the expected return is 9%. What is the expected payoff of the European call option after 3 months? Explain with reasons whether you would be able to calculate the no-arbitrage value of such an option or not. (2)
- [11]**
- Q. 2)** Suppose the stock price S follows geometric Brownian motion with expected return μ and volatility σ :
- $$dS = \mu S dt + \sigma S dz$$
- where dz is a Wiener process
- The risk free rate of interest (with continuous compounding) is r .
- i) Determine the process followed by the variable S^k (where k is a positive integer). Find the expected return and variance of S^k . (5)
 - ii) For the same measure corresponding to dz , show that the stock price discounted at risk-free rate is a martingale only for a specific value of μ . (3)
 - iii) Can you make any similar claim about the process followed by S^k in part (i)? What conditions are needed? (1)
- [9]**
- Q. 3)** Assume that the market portfolio of an economy constitutes two portfolios of growth stocks and value stocks. The market capitalization of growth stocks portfolio constitutes 70% of the total market portfolio and the market capitalization of value stocks portfolio constitutes 30% of the total market portfolio. The expected return on the portfolio of value stocks is 13% and on the portfolio of growth stocks is 17%. The standard deviation of the rate of return on the portfolio of value stocks is 12% and on the portfolio of growth stock is 25%. The correlation coefficient between the returns of the two portfolios is 0.5. The risk-free rate of interest is 6%.
- i) What is the expected return and volatility of the market portfolio? (2)
 - ii) What is the beta of each portfolio? (2)
 - iii) What is the residual variance of each portfolio? (2)
 - iv) Explain whether CAPM holds in this economy. (2)
- [8]**

- Q. 4)** i) Define martingale in continuous time period (1)
- Let $f: [0, \infty] \rightarrow \mathbb{R}$ be a real-valued deterministic function, and $\{B_t\}, t \geq 0$ be a Brownian motion. Then:
- ii) State the distribution of $\int_u^t f(s) dB_s, t > u > 0$ (2)
- iii) Show that $M_{0,t} = e^{\int_0^t f(s) dB_s - \frac{1}{2} \int_0^t f^2(s) ds}$ is a martingale (3)
- [6]
- Q. 5)** i) For a non-dividend-paying stock, define and find the relationship between the delta of a European call and the delta of a European put (1)
- ii) For a non-dividend-paying stock, define and find the relationship between the gamma of a European call and the gamma of a European put (1)
- iii) For a non-dividend-paying stock, define and find the relationship between the theta of a European call and the theta of a European put (1)
- iv) Calculate the delta of an at-the-money 6 month European call option and delta of an at-the-money put option on a non dividend paying stock with risk free rate of interest of 6% per annum and stock price volatility of 25% per annum. What is the gamma of both the options? Without computing Theta from the first principle, determine the value of the Theta for both the options using the relationship between delta, theta and gamma for a derivative. Interpret the results obtained. (7)
- [10]
- Q. 6)** i) Suppose that the risk-neutral default probability for a one-year zero-coupon bond is 1% and the real-world default probability for the bond is 0.30%. The risk-free yield curve is flat at 6% and the excess return of the market over the risk-free rate is 4%. What is the bond's beta? Assume no recovery in the event of default. (4)
- ii) Suppose that a one-year coupon bond pays coupon semiannually at the rate of 7% per annum (with semiannual compounding), the risk free zero curve is flat at 6% (with semiannual compounding) and expected recovery rate from default is 30%. The bond yield is 6.60% (with semiannual compounding). What is the probability of default, assuming defaults can happen only at bond maturity dates? (4)
- [8]
- Q. 7)** Consider a process $X_n = \alpha X_{n-1} + e_n$ where e_n is a white noise process with zero mean and variance σ_e^2 .
- i) Identify the process. (1)
- ii) Show that the process has a mean and variance given by $E[X_n] = \alpha^n X_0$ and $Var[X_n] = \sigma_e^2(1 - \alpha^{2n})/(1 - \alpha^2)$. (5)
- iii) Define an Ornstein-Uhlenbeck process and determine the mean and variance of the process. Show that the first two moments coincide with the process for certain values of α and σ_e . (4)
- iv) What can you say about the properties of such a process and why? (1)
- [11]

- Q. 8)** An investor can construct a portfolio using only two assets A and B. The statistical properties of the two assets are shown below:

	<i>A</i>	<i>B</i>
Expected return	12%	8%
Variance of return	30%	15%

Correlation coefficient between the returns of assets A and B is 0.5

Assuming that the investor cannot borrow to make the investment:

- i) Determine the portfolio composition which will give the highest expected return. (1)
 - ii) Calculate the portfolio composition which will give the investor the minimum variance. (3)
 - iii) Explain and sketch how the investor would choose a utility maximizing portfolio. (3)
- [7]**
- Q. 9)** Consider a portfolio, P, which consists of N assets held in equal proportions.
- i) Mathematically prove that the variance of the returns on the portfolio tends to zero when the portfolio is well diversified and the individual asset returns are uncorrelated. Interpret the result. (3)
 - ii) State why in real world it is a difficult to achieve condition specified in (i). (1)
 - iii) Prove mathematically that the variance of the returns on the portfolio gets closer to the average covariance between the pairs of assets in that portfolio when the portfolio is well diversified and individual asset returns are positively correlated. Interpret the result. (3)
- [7]**
- Q. 10)**
- i) State the principal theme of behavioural finance. State the assumption of expected utility theory that is challenged by this theme. (2)
 - ii) In a fast developing country, house prices have been rising rapidly for several years fuelled by loans of up to 90% of the value of the house and black money. The government has recently introduced rules that restrict the role of black money by increasing registration rates. Also, the amount of borrowing has been restricted to 50% of the value of the house. Following these changes, house prices have started to fall and are expected to do so for the next couple of years.

Identify and explain the themes of behavioural finance exhibited by the following people:

- a) An investor who has been successful in the past buying residential property to profit from capital appreciation, who has decided to continue to buy residential property over the next couple of years.

- b) An economist who has written a recent article commenting that house prices were always expected to fall. The economist uses historic data to prove this point. All news articles written prior to the government intervention only indicated that prices would continue to rise. (5)
[7]
- Q. 11)** i) State eight desirable characteristics of a term-structure model. (4)
- ii) Explain the impact of a movement in the short rate on the volatility term in the Vasicek model and the Cox-Ingersoll-Ross model. (3)
- iii) Compare & contrast Vasicek and Hull & White models of interest rates. (5)
[12]
- Q. 12)** i) State two key properties of Value at Risk (VaR). (2)
- ii) VaR is frequently calculated assuming a normal distribution of returns. State an advantage and a disadvantage of this approach. (2)
[4]
