

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

05th November 2014

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (10.30 – 13.30 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

- 1. Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
- 2. Mark allocations are shown in brackets.*
- 3. Attempt all questions, beginning your answer to each question on a separate sheet.*
- 4. Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

- Q. 1)** For each of the following calculate the equivalent effective annual rate of interest:
- i)** An effective rate of interest of 12% paid every 2 years (1)
 - ii)** An effective rate of discount of 6% per annum (1)
 - iii)** A force of interest 1% per month (1)
 - iv)** A nominal rate of discount of 8% per annum convertible quarterly (1)
 - v)** A nominal rate of interest of 12% per annum convertible every 2 years (1)
- [5]**
- Q. 2)**
- i)** An investor is considering the purchase of an annuity, payable annually in arrears for 15 years. The first payment is ₹8000. Using a rate of interest of 9% per annum effective, calculate the duration of the annuity when:
 - a)** The payments remain level over the term. (2)
 - b)** The payments increase at a rate of 9% per annum compound. (3)
 - ii)** Compare and explain the results of (i) (a) and (i)(b) above (1)
- [6]**
- Q. 3)** A one-year forward contract on a stock is entered on 1st October 2013 when the stock price is at ₹91 and the risk-free force of interest is at 9% per annum. The stock is expected to pay an annual dividend of ₹3.80 with the next dividend due after six months.
- On 1st February 2014, the price of the stock is ₹109 and the risk-free force of interest is 8.5% per annum. The dividend expectation is unchanged.
- Calculate the value of the contract to the holder of the long forward position on 1st February 2014. **[6]**
- Q. 4)** The expected annual effective rate of return from an investment is 9% per annum and the standard deviation of annual effective returns is 10%. The annual effective returns are independent and $(1+i_t)$ is log normally distributed, where i_t is the return in the year t .
- Deriving any necessary formulae:
- i)** Calculate the expected value of an investment of ₹12, 00,000 after 12 years. (5)
 - ii)** Calculate the probability that the accumulation of the investment will be less than 90% of the expected value. (3)
- [8]**

- Q. 5)** The force of interest $\delta(t) = 0.04$ for $0 < t \leq 8$
 $= 0.005t$ for $8 < t \leq 20$
 $= 0.003t + 0.0002t^2$ for $20 < t$
- i) Calculate the accumulated value of ₹ 1000 at $t=22$, invested at $t=2$ (4)
- ii) Calculate the constant rate of discount per annum, convertible monthly, which would lead to the same accumulation in (i) above. (2)
- iii) Calculate the effective rate of interest per unit time from time $t=19$ to $t=20$ (2)
- iv) A continuous payment stream is paid at the rate of $e^{-0.03t}$ per unit time between time $t=0$ and $t=4$. Calculate the present value of the payment stream. (3)
- [11]

- Q. 6)** The Government issued zero coupon bonds on 1.4.2008 to be redeemed at a price of ₹ 100/- after five years for an issue price of ₹ 91/-. The bonds were traded in the market and the market prices at various different dates are shown in the table below.

Date	Market Price (₹)
1.4.2008	91
1.4.2009	93
1.4.2010	86
1.4.2011	93
1.4.2012	97
1.4.2013	100

- i) Explain why the price of the bonds may have fallen between 1st April 2009 and 1st April 2010. (1)

Two fund managers bought these bonds at various dates. Fund Manager X bought 1000 bonds on 1st April 2008 and 10,000 bonds on 1st April 2010. Fund Manager Y bought 1000 bonds each year on 1st April from 2008 to 2012 both inclusive. Both the fund managers held the bonds until maturity.

- ii) a) Calculate the effective money weighted rate of return per annum of the Fund Manager X during the period 1st April 2008 to 1st April 2013. (2)
- b) Calculate the effective time weighted rate of return per annum of the Fund Manager X during the period 1st April 2008 to 1st April 2013. (2)
- iii) a) Show that the effective money weighted rate of return of the Fund Manager Y is less than that earned by the Fund Manager X during the period 1st April 2008 to 1st April 2013. (3)
- b) Calculate the effective time weighted rate of return per annum of the Fund Manager Y during the period 1st April 2008 to 1st April 2013. (3)
- [11]

- Q. 7)** An investor borrowed ₹ 720,000/- at an effective rate of 6% per annum. The investor used the money to purchase an annuity of ₹84,000/- per annum payable half-yearly in arrears for a period of 25years. The investor can earn an interest at an effective rate of 4% per annum on the money invested from the annuity payments once the loan is repaid.
- i)** Determine the discounted payback period for this investment (4)
 - ii)** Determine the profit the investor will have made at the end of the term of the annuity. (6)
- [10]**
- Q. 8)** A loan is repayable by annual instalments in arrears for a period of 20 years. The initial instalment is ₹ 25,000, with each subsequent instalment decreasing by ₹ 1,000. The effective rate of interest over the period of the loan is 6% per annum.
- i)** Calculate the amount of the original loan. (3)
 - ii)** Calculate the capital repayment in the 10th instalment. (3)
- After the 10th instalment is paid, the borrower and the lender agree to a restructuring of the debt. The ₹1,000 reductions per year will no longer continue. Instead, future instalments will remain at the level of the 10th instalment and the remaining term of the debt will be shortened. The final payment will then be a reduced amount which will clear the debt.
- iii) a)** Calculate the remaining term of the revised loan. (3)
 - b)** Calculate the amount of the final reduced payment. (3)
 - c)** Calculate the total interest paid during the term of the loan. (2)
- [14]**
- Q. 9)**
- i)** Describe the characteristics of Eurobonds. (2)
 - ii)** Describe the characteristics of Preference Shares. (2)
 - iii)** An investor using the information from a benchmark bond spot yield curve calculates the present value of a corporate Eurobond with a term to redemption of exactly five years. The investor values each payment that is due from the Eurobond at a rate of interest equal to $b = a + 0.015 + 0.0008t$ where:
 - 't' is the time in years when the payment is due
 - 'a' is the annual t-year effective spot rate of the interest from the benchmark bond spot yield curve and $a = 0.03t$ for $t \leq 5$

The Eurobond pays annual coupons at 11% per annum of the nominal amount of the bond and is redeemed at par.

Calculate the present value of the Eurobond. (5)
- [9]**

- Q. 10)** A financial institution on 1st April 2013 issued an index-linked bond of term ten years which is redeemable at par. Half-yearly coupons in arrears are payable on 1st April and 1st October each year. The annual nominal coupon is 3% per annum. Interest and capital payments are indexed by reference to the value of an index with a time lag of seven months.

Values of the indices are given below:

<i>Date</i>	<i>Index</i>
September 2012	105
March 2013	107
September 2013	108
March 2014	110

The index is assumed to increase continuously at the rate of 4% per annum effective from its value in March 2014.

An investor, paying tax at the rate of 10% per annum on coupons only, purchased the stock on 1st April 2014, just after the coupon payment had been made.

Assuming the investor holds the bond until maturity; calculate the price per ₹100 nominal payable by the investor to obtain a real net yield of 5% per annum convertible half yearly.

[8]

- Q. 11)** The Central bank in a country has just issued three bonds with terms to redemption of exactly one, two and three years respectively. Each bond is redeemed at par and pays coupons of 9% per annum in arrears. The annual effective gross redemption yields from the one, two and three year bonds are 8%, 7% and 7% respectively.

i) Calculate the one-year, two-year and three-year spot rates of interest at the date of issue.

(8)

ii) Calculate all possible forward rates of interest from the above spot rates of interest.

(4)

[12]
