Actuarial Society of India

EXAMINATIONS

23rd November 2005

Subject ST6 – Finance and Investment B

Time allowed: Three Hours (2.15* - 5.30 pm)

INSTRUCTIONS TO THE CANDIDATE

1. ***You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only but notes may be made. You then have three hours to complete the paper.***
2. ***You must not start writing your answers until instructed to do so by the supervisor.***
3. ***The answers are not expected to be any country or jurisdiction specific. However, if examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.***
4. ***Mark allocations are shown in brackets.***
5. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
6. ***Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.***

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION
Hand in BOTH your answer script and this question paper to the supervisor.

Q.1)

(a) The current price of gold is $450 per ounce. Assuming that the continuously compounded risk free interest rate is 5% per annum and the storage cost is $1.5 per annum per ounce payable quarterly in advance, calculate the futures price of gold for delivery in 9 months.

(3)

(b) State for each of the factors below, whether an increase in the factor leads to an increase or decrease in the forward/futures price :

- Risk-free rate of interest
- Dividend yield on asset
- Storage costs of asset
- Convenience yield of asset **(4 x ½=2)**

Total [5]

Q.2) Ram Lagan wants to save for his daughter's marriage which will require a lump sum of Rs. 25 lacs in 5 years time. He has just bought Rs. 15 lacs of high growth shares. The share price can be modelled using Brownian motion with the parameters á = 0.1 and ó = 0.6. How likely that Ram Lagan will be able to meet the marriage expenses. **[5]**

Q.3)

(a) State the "Tower Law" **(2)**

Let B_t, $t \geq 0$ denote standard Brownian motion.

(b) Without assuming anything about the martingale properties of Brownian motion, show that $E[B_n^2 / F_{n-1}] = B_{n-1}^2 + 1$ for any positive integer n **(3)**

(c) Hence derive an expression for $E\{E[B_3^2/ F_2]/ F_1\}$ **(1)**

(d) Evaluate the expression in (c) using the Tower Law **(3)**

(e) Use the result in (b) to find a function $f(B_t)$ that is a martingale. **(1)**

Total [10]

Q.4) An equity is currently priced at Rs. 80. Over each of the next two 3-month periods its price is expected to go up by 2.5% or down by 5%. No dividends are expected to be paid during this period. The risk-free interest rate is 4% per annum, continuously compounded.

(a) Calculate, using the risk-neutral valuation, the value of a 6-month at-the-money European put. **(3)**

(b)

(i) Verify your answer to (a), using instead only a no-arbitrage argument **(1)**

(ii) Suppose that a trader followed the strategy you have used in (b)(i). Write down the constituents of the portfolio he would have immediately before and immediately after reaching the upper 3-month node, and comment on your answers. **(4)**

(c) Calculate the value of a 6-month at-the-money American put option **(2)**

Total [10]

Q.5) The continuously compounded risk-free rate is 5% per period.
The risk-neutral probability is q = 0.43588.
The share price either goes up to 80 or down to 50 at time 2 (the exercise date).
The call option has a strike price of Rs. 45.

Suppose that S_1, the share price at time 1, is equal to Rs. 60. Find the corresponding value of $\hat{S}_1$, the discounted stock-price process, and show that the martingale property $E_Q\left[\hat{S}_1 / F_1\right] = \hat{S}_1$ is satisfied.

[5]

Q.6) The Black-Scholes formula for the value of a European Call option on a dividend paying share is:

$$c = S_t e^{-q(T-t)}\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2)$$

Where:

$$d_1 = \frac{\ln(\frac{S_t}{K}) + (r - q + s^2/2)(T-t)}{s\sqrt{T-t}}, \quad d_2 = d_1 - s\sqrt{T-t}$$

and Ô(z) is the cumulative distribution function of the standard normal distribution. The variable c is the European call price, S_t is the stock price at time t, K is the exercise price, r is the continuously compounded risk-free rate, ó is the stock price volatility, q is the continuously compounded dividend yield, and (T-t) is the time to maturity of the option.

(a) Derive expressions for $\frac{\partial d_1}{\partial S_t}$, $\frac{\partial d_2}{\partial S_t}$, $\frac{\partial d_1}{\partial t}$ and $\frac{\partial d_2}{\partial t}$. **(4)**

(b) Prove that $S_t e^{-q(T-t)} f(d_1) = Ke^{-r(T-t)} f(d_2)$, where $f(z)$ is the density function of the standard normal distribution. **(3)**

(c) Use your answers in (a) and (b) to derive expressions for Delta (Ä), Gamma (Ã) and theta (È) of call option.

(4)

(d) If q = 0, show that $\Theta + rS_t\Delta + \frac{1}{2}s^2 S_t^2\Gamma = rc$. **(3)**

Total [14]

Q.7)

(a) The market price of a European call is Rs 150 and its Black-Scholes price is Rs 175. The Black-Scholes price of a European put with the same strike price and time to maturity is Rs 50.

What should be the market price of this option be (European put)? Explain the reasons for your answer. **(3)**

(b) What is implied volatility? How can it be calculated? Explain with help of an example. **(3)**

(c) What is the difference between volatility smile and term structure of volatility? **(3)**

(d) The closing price of a share over the last ten trading days were as follows:

20.00 20.10 19.90 20.00 20.50 20.25 20.90 20.90 20.90 20.77

Estimate the volatility parameter ó, expressing your answer as percentage per annum compounded continuously. Assume that there are 252 trading days in the year. **(3)**

Total [12]

Q.8)

(a) Consider a four month European put option on NSE Nifty. The current value of Nifty is 2745, the strike price is 2700, the dividend yield is 3% per annum, the risk-free interest rate is 8% per annum, and the volatility of the index is 25% per annum. Calculate the vega (kappa) of the option. Interpret the calculated value of kappa. **(3)**

(b) A company has a Rs 54 million portfolio with a beta (with respect to NSE Nifty) of 1.2. It would like to use futures contract on the NSE Nifty to hedge its risk. The index is currently standing at 2700, and each contact is for delivery of 200 times the index. What is the hedge that minimizes risk? What should the company do if it wants to reduce the beta of the portfolio to 0.6? **(3)**

(c) Jet Airways expects to purchase four million gallons of jet fuel in one month and decides to use heating oil futures for hedging. The standard deviation of monthly change in the spot price of jet fuel is Re 1 per gallon. The standard deviation of monthly change in the futures price of heating oil is Rs 1.50 per gallon. The correlation coefficient between the change in future price of heating oil and change in spot price of jet fuel is 0.9. Each heating oil futures contract is for the delivery of 40,000 gallons of heating oil. What strategy should Jet Airways follow? **(3)**

Total [9]

Q.9)

(a) Define “Swaption” **(1)**

(b) Company Z holds a 1 year European Swaption on a 3 year interest rate swap under which it will pay a fixed rate of 6% pa in return for receiving LIBOR.Under what circumstances will the company choose to exercise the swaption in 1 year’s time? **(2)**

(c) The following data are available on the annual effective spot rates as on June 30, 2005:

Maturing Date	Spot Rate (%) pa
30 June 2006	6.25%
30 June 2007	6.50%
30 June 2008	6.75%

Consider an European swaption on a 2 year "pay floating and receive 7.5% pa fixed" interest rate swap with a strike date of 30 June 2006. The underlying swap involves annual payments based on a principal of Rs.100 million.
Calculate the value of the swaption as on 30 June, 2005 assuming that the annual volatility of the swap rate is 25%. State other assumptions, if any . **(5)**

Total [8]

Q.10)

(a) Write down the stochastic differential equation for the short rate of interest under the Cox - Ingersoll-Ross model under the risk neutral measure Q, defining the symbols you use. **(2)**

(b) A bond analyst has developed the following model for bond prices:

- The short rate of interest r_t obeys the stochastic differential equation $dr_t = s\, dW_t$ where Wt is the standard Brownian motion under the real world probability measure P.
- The price of a Zero coupon bond maturing at a fixed time T is related to the short rate via the equation $B(t,T) = \exp(a - br_t)$ where a and b are constants.

(i) Show that for this model, the stochastic differential equation for the bond price under P is :

$$dB(t,T) = B(t,T)[\frac{1}{2}b^2 s^2 dt - bs\, dW_t] \quad \textbf{(4)}$$

(ii) Define the terms "Market Risk Premium" and "Market Price of Risk" for a bond pricing model. Calculate the market price of risk for the above bond model. **(3)**

(iii) State two drawbacks inherent in above model for r_t **(2)**

Total [11]

Q.11)

(a) Define "Market Risk". Based on the G30 report on derivatives, list the six components of market risk that should be considered across the term structure. **(3)**

(b) Consider an investment bank with the following portfolio:

- 12,500 shares of Remo Ltd at a current market price of Rs. 15 per share
- 25,000 European call options on the shares of Remo Limited at a strike price of Rs. 15 per share and a remaining term of 3 months.

Calculate the investment bank's value at risk(VAR) for the above portfolio using the following information:

- VAR is measured using probability analysis based upon a common confidence interval (eg: two standard deviations) and time horizon (eg: a one day exposure)
- The share price follows the lognormal model

$dSt = St[mdt + sdZt]$ with annualized parameter values $m = 0.06$ and $s = 0.2$

- The share is not expected to pay any dividends before the expiry date
- The option price is consistent with the Black-Scholes model

- The risk free force of interest is r = 0.05
- There are 240 trading days in the year

State other assumptions, if any **(8)**

Total [11]

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