Actuarial Society of India

Examinations

November 2005

ST3 – General Insurance

Indicative Solutions

Q.1)

a)

(i) By memory less property of the exponential distribution, we mean that the inter-event time is independent of the absolute time. In other words it means that the time until the next event has the same distribution, irrespective of the time since the last event or the number of events that have already occurred. **[1]**

(ii) When the aggregate claims are a Compound Poisson Process, the adjustment coefficient R is defined in terms of the Poisson parameter (l), the moment generating function of individual claim amounts ($M_X(R)$) and the premium income (c) per unit time. R is defined as the unique positive root of **:** $\lambda M_X(R) = \lambda + cR$

Substituting (1+ q) m_1 l for c this reduces to $M_X(R) = 1 + (1+q)m_1 R$ **[1]**

(iii) In continuous time the probability of ruin is defined as follows.

Given the initial surplus U,

(1) Ultimate ruin probability ?(U)=P [U(t)<0 for some t , 0<t< ∞]

(2) The probability of ruin within time t =? (U, t)=P[U(τ)<0 for some τ , 0<τ? ?t]

In discreet time the probability of ruin is defined as follows

$$\Psi_h(U) = P[U(t) < 0], \text{ for some } t \qquad t = h, 2h, 3h, \ldots\ldots$$

$$\Psi_h(U,t) = P[U(t) < 0], \text{ for some } t \qquad t = h, 2h, 3h, \ldots\ldots t - h, t$$

Where h is a given interval of time and t is an integer multiple of h. **[3]**

b)

(i) With usual notations the claim amounts X ~ EXP(α) $\therefore$ $m_1 = \alpha = 2000$

MGF= $M_X(r) =$ $M_X(r) = \dfrac{1}{1-a\ R}$ **;** loading= q =0.25

R is given by $M_X(r) = 1 + (1+q)aR$

ie $\dfrac{1}{1-a\ R} = 1 + (1+q)aR$ $\therefore$ $1 = (1-a\ R)\{1+(1+q)aR\}$

Expanding $1 = 1 + (1+q)aR - aR - (1+q)a^2R^2$

Which reduces to $qaR - (1+q)a^2R^2 = 0$ *ie* $aR\{q - (1+q)aR)\} = 0$

$\therefore$ $R = \dfrac{q}{(1+q)a}$ $[\because R \neq 0]$

[3]

(ii)

Substituting, $q = 0.25$ and $a = 2000$ in the above result

$$R = \frac{0.25}{1.25 \times 2000} = 0.0001$$

[1]

(iii) Let U denote the initial surplus and ? (U) denote the probability of ultimate ruin

From Lundberg's inequality, we get an upper bound for ? viz.

$y(U) \leq e^{-RU} = e^{-.0001 \times 100000} = e^{-10} =$ 0.0000454 ?

? ??

(iv) The formula for R is independent of the Poisson parameter l for the claim number distribution. Hence The value of R will not change **[1]**

(v) The Poisson parameter speeds up the process. Therefore claims arise at a faster rate. This in turn would mean that the ruin will happen relatively soon, if at all it were to happen. But it does not affect the probability of ruin while considering ruin at any time in the future **[2]**

(vi)

$$R = \frac{q}{(1+q)m_1} = \frac{0.2}{1.2 \times 2000} = .000833$$

Now $e^{-RU} = \Psi = .0000454$

so that $U = \frac{1}{R}\log(\frac{1}{\Psi}) = \frac{1}{.000833}\log(\frac{1}{0.0000454}) = 120048$ say 120000

The initial surplus U required will be INR 120000

[2]

Q.2)

i) EBCT Model 1

With usual notations for model 1

Insurer	Year 1	Year 2	Year 3	Year 4	$\overline{X}_i$	$\frac{1}{3}\sum_j \left(X_{i,j} - \overline{X}_i\right)^2$
A	11	11.81	9.68	22	13.62	31.96
B	250	373.13	262.5	250	283.91	3572.79
C	500	550	515	545	527.50	575.00
						[2]

$\overline{X}$ = (13.623+283.908+527.5)/3 = 275.010 [0.5]

E [s^2(q)] = (31.9628+3572.7909+575)/3 = 1393.25 [0.5]

Var [m(q)]= $\frac{1}{N-1}\sum_i (X_i - \overline{X})^2$ - $\frac{1}{n}E[s^2(q)]$

= (1/2) [(13.623-275.01)^2+(283.908-275.01)^2+(527.5-275.01)^2] - (1/4) x 1393.251

= (1/2)[68323.1638+79.1744+63751.2001] - (1/4) x 1393.251 = **65728.46** **[1]**

Credibility Factor = $\frac{n}{n + \frac{E[s^2(q)]}{\text{var}[m(q)}}$ = $\frac{4}{4 + \frac{1393.251}{65728.4564}}$ = 0.9947

EBCT _1 Credibility Premium for the year 5 for Insurer A =

0.9947 x 13.623 + (1- 0.9947) x 275.01 = INR **15.01** m **[1]**

ii) EBCT Model 2

With usual notations for model 2

$\overline{P}_1$? ? 2000 + 2250 + 1800 + 4000 = 10050

$\overline{P}_2$ = 50000 + 75000 + 52500 + 50000 = 227500

$\overline{P}_3$ = 100000 + 110000 + 103000 + 109000 = 422000

$\overline{P}$ = 10050 + 227500 + 422000 = 659550 **[1]**

P* = $$P^* = \frac{1}{11}\left[10050(1-\frac{10050}{659550}) + 227500(1-\frac{227500}{659550}) + 422000(1-\frac{422000}{659550})\right]$$

= <u>28265.13</u> **[1]**

$\overline{X_1}$ = (11 + 11.81 + 9.68 + 22) /10050 = 0.005422

$\overline{X_2}$ = (250 + 373.13 + 262.5 + 250) /227500 = 0.00499

$\overline{X_3}$ = (500 + 550 + 515 + 545) /422000 = 0.00500

$\overline{X}$ = $\frac{(10050 \times .005422 + 227500 \times .00499 + 422000 \times .005)}{(10050 + 22750 + 422000)}$ = **0.005004** **[1]**

E[m(q)] = $\overline{X}$ = **0.005004** **[0.5]**

E [s²(q)] = $$\frac{1}{N}\sum_{i=1}^{N}\frac{1}{n-1}\sum_{j=1}^{n}P_{ij}(X_{ij} - \overline{X}_i)^2$$

= (1 / 3*3) * (0.000107 + 0.000031 + 0) = **0.000015** **[0.5]**

Var[m(q)] = $$\frac{1}{P^*}\left[\frac{1}{(Nn-1)}\sum_{i=1}^{N}\sum_{j=1}^{n}P_{ij}(X_{ij} - \overline{X})^2 - \frac{1}{N}\sum_{i=1}^{N}\frac{1}{(n-1)}\sum_{j=1}^{n}P_{ij}(X_{ij} - \overline{X}_i)^2\right]$$

= $$\frac{1}{28265.133}\left[\frac{1}{11}(0.001866 + 0.000063 + 0.000005) - \frac{1}{3 \times 3}(.000107 + 0.000031 + 0)\right]$$

= **0.000000010** **[1]**

$$Z_1 = \frac{\sum_{j=1}^{n} P_{1,j}}{\sum_{j=1}^{n} P_{1,j} + \frac{E[s^2(q)]}{\text{var}[m(q)]}} = \frac{\overline{P}_1}{\overline{P}_1 + \frac{E[s^2(q)]}{\text{var}[m(q)]}} =$$

= 10050 / { 10050 + (0.000015 / 0.00000001) } = **0.870** **[2]**

Credibility premium for A = 0.87013 x 0.005422 + (1 -0.87013) x 0.005004
= 0.0054

For risk volume of 4000 it is = INR 21.47 M . **[1]**

iii) Credibility premium for B = 10050 / { 10050 + (0.000015 / 0.00000001) }
= 0.993

Credibility premium for C = 422000 / {422000 + (0.000015 / 0.00000001)}
= 0.996 **[2]**

iv) The volume is relatively very small for insurer A.

The credibility factor is 0.87 which is less than those for B & C . this is to be expected

One would normally expect the credibility premium to be more than the figure of INR 21.69 m on account of greater variability due to small volume.

But the EBCT Model 2 produces a figure of only 21.47

It may be observed that the value of $\overline{X_1}$ is greater than that of $\overline{X}$. It would be of interest to examine why the cost of claims per policy for A is much higher than those of B and C.

[3]

Q.3)

a)

i)

- A poorer claim experience
- A cautious policy towards reserving
- Slower speed of settling claims due to litigations, inexperienced staff etc
- Happening of a catastrophic event in the recent past
- Differences in reinsurance arrangements
- Lower level of free reserves **[0.5 x 6]**

ii) Case estimates might be more suitable than statistical methods in some circumstances List the major ones.

- Inadequate volume of past data
- Low claim frequency and small number of outstanding claims
- Wide variations between individual claims necessitating individual assessment
- High variance in claim amounts
- A new class of business
- Non-availability of suitable statistical method **[0.5 x 6]**

iii)

The four main classes into which statistical methods for reserving may be classified are

- chain ladder methods
- average cost per claim methods
- loss ratio methods
- blends

[0.5 x 4]

iv) You have a run-off triangle of paid claims split by accident year. Suggest possible reasons for:
(a) a row of figures that is unusually high
(b) a column of figures that is unusually high
(c) a diagonal of figures that is unusually low. **[3]**

a) A row in the run of triangle represents progression of claims pertaining to a particular accident year.Unusually high values in a particular row indicate a year of heavy claims. For example A year which witnessed a natural calamity such as flood, storm, or earthquake which resulted in loss of lives and/ or material damage to property or a bad winter etc.

b) A column in a run-off triangle gives the payments made in a particular development year ie in the nth year after the year of accident. Many "damage to property' claims are settled in the early years and then there is a fall in payments before liability claims are paid.

c) A diagonal in the run-off triangle represents the payments made in a particular calendar year. A low diagonal represents deterioration in payments. Possible reasons for this are a postal/ employee strike, implementation of a new system or loss of staff.

Q.4)

i)

Claim ratio or loss ratio is defined as
(incurred claims amount) / (earned premiums) **[0.5]**

Usually it is calculated net of reinsurance but may be calculated gross to assess underwriter's performance. Net of reinsurance, premiums will be net of outwards reinsurance premiums and claims will be net of reinsurance recoveries. **[0.5]**

The ratio gives a basic measure of level of claims. High level may need company's attention on adequacy of premiums, underwriting standards or other contributing factors. **[0.5]**

Solvency ratio is calculated as
(free reserves) / (net written premiums) **[0.5]**

This is a basic measure of financial strength. **[0.5]**

Most countries have legislation requiring maintenance of a statutory minimum solvency margin (SMSM) **[0.5]**

It is normal market practice to maintain solvency margin at a level higher than SMSM **[0.5]**

Length of tail is
(claims paid) / (outstanding claims reserve) at the end of the year **[0.5]**

Length of tail reflects the mix of classes of business **[0.5]**

Total [5]

ii) All the figures given in the question are assumed to be net of reinsurance. **[0.5]**

Claims ratio: The ratios are

2002	72%
2003	87%
2004	97%

[0.5]

This is a clear deterioration in the ratio. **[0.5]**

The following points need examination.
Any change in key underwriting personnel and/ or underwriting standards? **[0.5]**

Increase in written premiums might mean poor quality business. **[0.5]**
Big increase in premiums in 2004 might be due to reduction in premium rates. **[0.5]**

Any large claims or CAT events in 2003 and/ or 2004? If so, reinsurance review needed **[0.5]**

Is the market at the wrong end of the insurance cycle? **[0.5]**

Considering years 2000 and 2001 claims paid high but increase in outstanding is negative. One large claim with over estimation in the reserve could have led to this feature. **[1]**

Solvency ratio:

The ratio at the beginning of each year:

2001	71%
2002	78%
2003	48%
2004	25%

Clear trend of worsening from 2002 **[0.5] + [0.5]**

Premium growth is not matched by growth in shareholders' funds. **[0.5]**

Dividend pay out is more liberal than what can be sustained by growth in profits **[0.5]**

Reduction in solvency could lead to future insolvency and regulatory intervention. Also reduction in public confidence. **[1]**

Length of tail:

The calculation as defined in (i) is

2002	36%
2003	26%
2004	24%

[0.5]

More likely trend of change in business mix with increasing proportion of liability type of business with length of tail of business at say 3 years or more. **[1]**

Some chance that higher growth rate of business gave rise to the observed experience. **[0.5]**

Total [10]

Q.5)

i) All business risks give rise to uncertainty and possible cost to insurer. **[0.5]**

Failure of third parties leads to loss of monies due. For example

- Brokers delaying premium remittances reduces investible funds **[0.5]**
- Default of third party leads to loss of full cost **[0.5]**
- Staff may make irregular claim payments, falsify accounts, etc. **[0.5]**
- External suppliers of goods and services may default causing need for further funds [0.5]

Timing risk:
Delays in payment of premiums by policyholders or brokers. **[0.5]**
Delays in recoveries from reinsurers **[0.5]**
Claims occurrences earlier than projected **[0.5]**

Competition:

- Premium rates appear high to customers/ policyholders **[0.5]**
- Administration and/ or distribution channels follow obsolete methods or are costly **[0.5]**
- Pricing assumptions unrealistic **[0.5]**
- Loss of major commercial lines business to competitors **[0.5]**

Consequences of insurance cycle, i.e. profitability in different classes tends to go in cycles, driven by market forces of supply and demand. **[1]**

New business and lapse risks:

- Maintaining to write adequate quantity of new business and also ability to retain business which is profitable **[1]**

Total [8]

ii)
Failure of third parties:

- New company particularly exposed to risk of failure of brokers and policyholders to pay premiums on time since money available to meet claims and expenses is reduced. **[0.5]**

- Similarly, inefficiency/ dishonesty of staff may lead to inaccurate accounts, irrecoverable losses or higher than anticipated claim ratios. **[0.5]**
- If expansion is associated with any changes to IT systems for new classes, failure of service provider could be disastrous. **[1]**

Timing risk:

- Recoveries from reinsurer particularly for new classes may get delayed at least in an initial period. **[0.5]**
- Inefficiency/ willingness of brokers to remit premiums promptly for new classes **[0.5]**

Competition:

- Products under new classes may not appeal to customers, lowering performance levels and shortfall over budgeted premiums **[0.5]**
- High expenses connected with new products implying higher prices and lower volumes [0.5]
- Similarly, low on underwriting and claim settlement standards **[0.5]**

Insurance cycle:
Expansion coinciding with a downswing? (Less likely if company has considered this) **[0.5]**

NB and lapse risks:

- Entering into new classes at less than economic premiums is a high risk strategy. Need to monitor premium adequacy promptly and regularly. **[1]**
- Failure to retain at expected level of persistency leads to higher unit costs of procurement and make business expansion objective misplaced. **[1]**

Overall: Need to monitor regularly aspects of price adequacy, growth rate, systems controls and retention levels. **[1]**

[7]

Total [15]

Q.6) General points:

Item	Company A	Company B
Solvency margin	Modest and reducing	High and maintained
Reinsurance cover	High and increasing	Low and maintained
Company size and growth	Small and growing fast	Large and modes growth
Expense ratio	Somewhat high and increasing	Modest and maintained
Dependency on brokers and agents	Possibly high and increasing	Low and maintained
Business mix	Reasonable but increase in length	Reasonable and maintained

½ each, max [2]

The above points will be used for answers to both (i) and (ii). In both companies matching of assets and liabilities would be a prime objective. **[0.5]**

i) Investment policy of Company A would be conservative with greater weight in the portfolio towards fixed interest and liquid assets. **[0.5]**

Equities and other investments providing growth would be built up only gradually and with caution. **[0.5]**

Review of policy and rebalancing of portfolio would be both more frequent than otherwise- say quarterly review and monthly rebalancing. **[0.5]**

Investment policy of Company B would be more aggressive with a high content of equities and similar investments. **[0.5]**

Review of policy could be half yearly and for rebalancing greater importance will be given to changes in investments based on "timing" opportunities. **[1]**

Total [3]

ii)

Suggested portfolio:

Investment category	Company A		Company B	
	2005	2008	2005	2008
FI Govt bonds	45%	45%	25%	20%
Other FI	20%	15%	20%	20%
Index linked gov bonds	5%	10%	10%	10%
Property	0%	0%	10%	10%
Equities	20%	20%	30%	35%
Cash	10%	10%	5%	5%

[2]

Suggested percentages are central figures. Actual could vary within a narrow range of say 2% to 4% for Company A and 4% to 8% for Company B, in each category. **[0.5]**

Suggested percentages reflect broadly the requirements in each case:

- Business mix and hence length of claim tail
- Level of solvency margin and hence freedom to take risks
- Dependence on reinsurance and company size
- Likely proportions of investible funds **[2]**

In the case of Company A, review of investment policy should in particular consider whether any large claims and catastrophes have impacted the portfolio. **[0.5]**

Total [10]

Q.7)

i)

- Professional fee charged
- Cost of building and property
- Number of floors and floor area
- Sum insured **[2]**

ii)

- Insurer has no previous experience and no published market data may be available
- Each risk unique and engineering experts advice may be required for assessment
- Trends in building design and construction keep changing and risk profile changing continuously
- Suitable criteria to establish professional competence of firms may not be available.
- Establishing liability is a unique exercise in each case, often involving long delays and legal actions.
- Design faults come to light much after occurrence of damage/ claims, implying latent IBNR and settlement delays.
- Claim amounts and harsh court awards unpredictable.
- Appropriate spread of business by size and competence of firms may not be possible. Too few firms cornering too high a proportion of the profession’s business is common.
- Reinsurer’s terms may be stringent.
- Apart from difficulty in establishing claim costs, fixing charges for procurement and other expenses is a difficult exercise.

1 each [10]

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