Actuarial Society of India

EXAMINATIONS

21st November 2005

Subject CT8 – Financial Economics

Time allowed: Three Hours (02.30 – 05.30 pm)

Total Marks : 100

INSTRUCTIONS TO THE CANDIDATES

1) ***Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.***

2) ***Mark allocations are shown in brackets.***

3) ***Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.***

4) ***Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.***

5) ***In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.***

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor.

Q.1) Explain what is meant by the following terms, in the context of Mean-Variance Portfolio theory :

a) Efficient Frontier **(1)**
b) Indifference curves **(1)**
c) Optimal Portfolio **(2)**

Total [4]

Q.2) Consider two securities such that

$E_1 = 5\%$, $V_1 = [10\%]^2$

$E_2 = 10\%$, $V_2 = [20\%]^2$

Let ñ denote the correlation coefficient between the returns of the securities.

(i) Derive the equation of the minimum variance curve in E-V space **(6)**

(ii) Derive expressions for the portfolio expected return E and the portfolio proportion X_1 invested in security 1 at the point of global minimum variance. Hence explain briefly how E and X_1 vary with ñ

(6)

Total [12]

Q.3) Define Shortfall Probability. Explain with the aid of a simple numerical example its two main limitations as a basis for making investment decisions. **[6]**

Q.4) Explain what are meant by diversifiable and non-diversifiable risks of a share. How does the expected return on a share depend on each of these two types of risk under the capital asset pricing model? **[6]**

Q.5) Let S_t be a Geometric Brownian motion process defined by the equation S_t = exp [ìt+ óW$_t$], where W_t is a Standard Brownian Motion and ì and ó are constants.

i) Write down the stochastic differential equation satisfied by $X_t = \log_e S_t$ **(2)**

ii) By applying Ito's Lemma or otherwise, write down the stochastic differential equation satisfied by S_t **(4)**

iii) The price of a share follows a Geometric Brownian Motion with ì = 0.06 and ó =0.25, both expressed in annual units. Find the probability that, over a given one year period the share price will fall. **(4)**

Total [10]

Q.6)

i) What are the major difficulties involved with testing informational efficiency? **(4)**

ii) Describe three examples of events to which investment markets sometimes appear to under-react. **(2)**

Total [6]

Q.7)

(i) What is meant by saying that the process $\{Y_t\}$ is a martingale with respect to another process $\{X_t\}$? **(2)**

(ii) Let B_t (t   0) be a standard Brownian motion and let "a" and "b" be any constants. Show that there is a value of the constant c [which you should specify] such that $[a + bB_t]^2 + ct$ is a martingale with respect to B_t **(4)**

Total [6]

Q.8)

(a) Using the standard Black-Scholes call option pricing formula, calculate the price of an European call on a non-dividend paying stock with the following features:

- current price of the underlying stock : 200
- strike price : 200
- time to expiration : 1 year
- risk free rate : 6% pa [continuously compounded]
- volatility : 25% pa

State assumptions, if any

(4)

(b) Briefly discuss as to why the option prices actually quoted in the market may differ from the theoretical prices given by the Black-Scholes model.

(2)

(c) Using the Black-Scholes formula for the value of a call option on a non-dividend paying stock, show that the call price 'C' tends to the maximum of S-K exp [-r (T-t)] and zero, depending on the strike price, as σ tends to zero.

(9)

Total [15]

Q.9)

(a) Define: Delta, Gamma, Theta, Kappa (Vega), Rho and Lambda **(6)**

(b) The following table shows the portfolio of OTC [Over The Counter] options on an equity share held by an investment bank:

Type of Option	Option Delta	Option Gamma	Option Vega	Size of Position
Call	0.5	2.2	1.8	-1,500
Call	0.7	1.8	1.4	-750
Call	0.8	0.6	0.2	-750
Put	-0.4	1.3	0.7	-3,000

(i) Calculate the delta, gamma and vega of the investment bank's portfolio. **(3)**

(ii) An exchange offers a traded option on the same equity share with the following parameters:

Delta 0.6
Gamma 1.5
Vega 0.8

Calculate the position in the traded option and in the equity share to make the investment bank's position both delta – neutral and gamma-neutral.

(3)

(iii) Calculate the position in the traded option and in the equity share to make the investment bank's position both vega-neutral and delta-neutral.

(2)
Total [14]

Q.10)

(a) State the key limitations of one-factor interest rate models. **(3)**

(b) Write down the structure of the 2 – factor Vasicek model, defining the symbols you use. State the formula for calculating the zero coupon bond price B(t,T) under this model.

(3)
Total [6]

Q.11)

(a) Let c(t) denote the accumulated value at time t of 1 unit of cash invested at time 0, assuming that the cash always earns the risk-free interest rate r(t), so that:

d c(t) = r(t) c(t) dt

Prove that c(t) $= \exp \int_0^t r(u)du$ **(3)**

(b) An individual invested a sum of 10,000 at time 5 in an account that earns the risk-free rate of interest. Over the period from time 0 to time 10, the risk-free rate has increased linearly from 4% to 8%. Calculate the accumulated value of the account at time 10.

(3)
Total [6]

Q.12)

(a) The following equation represents Wilkic's updating equation for the dividend yield Õ (t) :

Y (t) = exp [YW.I(t) + ln YMU + YN (t)]

where

YN (t) = YA. YN(t-1) + YSD.YZ (t)

<u>Show that</u>

ln Y(t) = ln YMU + YW.I (t)
+ YA. [ln Y (t-1) – {ln YMU + YW.I (t-1)}]
+ YSD.YZ(t)

(3)

(b) State the reasons, as to why you would consider the resulting equation in (a), as appropriate for modeling equity dividend yields

(6)
Total [9]

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