Actuarial Society of India

Examinations

November 2005

CT5 – General Insurance, Health and Life Contingencies

Indicative Solutions

Q.1)

a)

If

$q_{[x]+t}$ = 0.02/(1.02), then

$p_{[x]+t}$ = 1 – (0.02/(1.02))

= 1/(1.02)

= v @ 2% for values of t = 0 to 4

[0.5]

Similarly

p_x = v @ 3%

[0.5]

$l_{[10]}$ is given, and as the select period is 5 years

l_{15} = 100000 X v^5 @ 2%

[0.5]

Similarly

l_{45} = l_{15} v^{30} @ 3%

= 100000 $v^5_{\ 2\%}$ X $v^{30}_{\ 3\%}$

= 100000 X 0.90573 X 0.41199

= 37315.17

[0.5]
[2]

b)

$l_{[40]+1}$ = l_{45}/($v^4_{\ 2\%}$) = 37315.17/(0.92385) = 40390.94

[1]
Total [3]

Q.2)

a)

$_tp'_x$ = exp{ -?µ'$_{x+u}$ du}

[0.5]

$$= \exp\{-\int_0^t \mu_{x+u} + k.du\}$$ **[0.5]**

$$= \exp\{-\int_0^t \mu_{x+u}\,du\}\exp\{-\int_0^t k\,du\}$$ **[0.5]**

$$= {}_tp_x \exp\{-kt\}$$ **[1.5]**

Total [3]

b) $\ddot{a}_{x:\overline{n}|} = \sum_{t=0}^{n-1} {}_tp'_x \cdot v^t$ **[0.5]**

$$= \sum_{t=0}^{n-1} {}_tp_x \cdot \exp\{-kt\} \cdot v^t$$ **[0.5]**

$$= \sum_{t=0}^{n-1} {}_tp_x \cdot (ve^{-k})^t$$ **[0.5]**

$$= \sum_{t=0}^{n-1} {}_tp_x \cdot v_j^t$$ **[0.5]**

$$= \ddot{a}_{x:\overline{n}|\,j}$$

where $(1/(1+j)) = (1/(1+i))x(1/e^k)$

$\Rightarrow j = (1+i)e^k - 1$

[1]

Total [3]

Q.3)

Bonus Methods

There are 3 methods of declaring a reversionary bonus, whereby the sum insured is increased and, once increased cannot be decreased.

- Simple reversionary bonus: the rate of bonus each year is a percentage of the initial sum insured. [0.5]
- Compound reversionary bonus: the rate of bonus each year is a percentage of the initial sum insured plus previously declared reversionary bonuses. [0.5]
- Super-compound reversionary bonus: there are 2 rates of bonus. One is applied to the basic sum insured, the other is applied to the previously declared bonuses. **[0.5]**

In addition there is the terminal bonus, where by the sum insured is increased at maturity or an earlier claim. The terminal bonus is normally a percentage of the final sum insured. **[0.5]**

Rate of distribution of surplus

Super-compound defers the distribution of surplus more than compound does (if the first rate of bonus rate, acting on the basic sum insured, is lower than the second rate, which acts on previously declared bonus).

[1]

Compound defers the distribution of surplus more than simple does.

[0.5]

Distributing terminal bonus rather than reversionary bonus will defer the distribution of surplus.

[0.5]

Total [4]

Q.4)

Use a general reasoning argument to construct Thiele's differential equation for a regular premium n-year term assurance issued to a life aged x by considering the change in reserves over a short time period. Assume that:

- interest is earned continuously at force of interest ?;
- premiums are payable continuously at rate $P_{x:n}$ per annum; and
- the Sum Assured of 1 is payable immediately on death

[5]

The standard symbol ${}_tV_{x:n}$ for the net premium at time t (where 0< t < n) is used.

As per the equation of equilibrium

Reserve at time t + interest + premium received during (t, t+h)

= Reserve for survivors at time (t, t+h) + benefits paid during (t, t+h)

In symbols this is:

$${}_tV_{x:n} + d\,h\,{}_tV_{x:n} + h\,P_{x:n} = (1 - h\,\mu_{x+t})\,{}_{t+h}V_{x:n} + h\,\mu_{x+t} + o(h)$$

[2]

o(h) term is needed because the interest and survival probabilities are only accurate to first order.

Rearranging this equation, we get,

$$({}_{t+h}V_{x:n} - {}_tV_{x:n})/h = d\,{}_tV_{x:n} + P_{x:n} + \mu_{x+t}\,{}_tV_{x:n} - \mu_{x+t} + o(h)/h$$

[1]

taking the limit as h tends to 0

$$\lim_{h ? 0} ({}_{t+h}V_{x:n} - {}_tV_{x:n})/h = d\,{}_tV_{x:n} + P_{x:n} + \mu_{x+t}\,{}_tV_{x:n} - \mu_{x+t}$$ **[1]**

i.e.

$$(?/?t)\ {}_tV_{x:n} = (\mu_{x+t} + d)\ {}_tV_{x:n} + P_{x:n} - \mu_{x+t}$$

or

$$(?/?t)\ {}_tV_{x:n} = d\ {}_tV_{x:n} + P_{x:n} - (1 - {}_tV_{x:n})\ \mu_{x+t}$$

[1]

Total [5]

Q.5)

Advantages of single figure indices

The advantage of using a single figure index is that a single figure is more easily assimilated than a set of figures. Thus you can see at a glance a summary statistic to give you a feel for the overall picture.

[1]

Some single figure indices, *eg* the crude death rate, are easy to calculate too.

[1]

Disadvantages of single figure indices

The disadvantages of using single figure indices are that some of them (*eg* the crude death rate) reflect differences between the age specific mortality rates and also other *compositional differences* between the groups being compared – in particular, differences in age and sex compositions.

[1]

Standardisation removes age differences, and sex differences can be removed by comparing sexes separately. Other differences (*eg* occupational) may remain and are confounded with true mortality differences.

[1]

Many single figure indices are heavily biased towards the mortality levels at the older ages and take very little account of mortality at the younger ages where changes are usually the most dramatic.

[1]

A single figure index will miss any abnormalities in the age-sex specific rates that may exist. Any errors are less likely to be detected.

[1]

[Total 6]

Q.6)

Expected present value of benefits

$5{,}000(D_{65}/D_{45})\{a_{5\ 4\%} + (D'_{70}/D'_{65})a_{70}\} = 19910.340$

$2053.75\ \{4.54028 + 8.7060881\} = 27204.729$

[2]

where ' = PMA92C20

assuming weekly payments are a good approximation to continuous payments

$a_{\overline{5}|4\%} = (1 - (1.04)^{-5})/\log_e(1.04) = 4.54028$ **[0.5]**

$a_{70} \approx \ddot{a}_{70} - ½ = 11.062$

$(D_{65}/D_{45}) = (689.23 / 1677.97) = 0.4107522$

$(D'_{70}/D'_{65}) = v^5 (l'_{70}/l'_{65}) = 0.8219271 \text{ x } (9238.134 / 9647.797) = 0.7870265$

[0.5]

$10{,}000\ A_{45:10} \approx 10{,}000\ (1 + i/2)((M_{45} - M_{55})\ /D_{45})$

$= 10{,}000\ (1.02)\ ((463.20 - 430.55)/1677.97) = 198.47196$

[1]

$25{,}000\ (D_{55}/D_{45})\ A_{55:10} \approx 25{,}000\ (1 + i/2)\ ((M_{55} - M_{65})\ /D_{45}) = 1014.0914$

$= 25{,}000\ (1.02)\ ((430.55 - 363.82)/ 1677.97)$
$= 1014.0914$

[1]

Expected present value of premiums, Annual Premium P.

$P\ \ddot{a}_{45:\overline{20}|} = 13.78P$

Equation of value
13.78P = 27204.729 + 198.47196 +1014.0914 = 28417.292

[1]

=> P = 2062.2128

Total [6]

Q.7)

The EPV of sickness benefit is

$50000\ [v p_{60}^{HH} + {}_2v p_{60}^{HH}]$

${}_2p_{60}^{HH} = p_{60}^{HH}\ p_{61}^{HS} + p_{60}^{HS}\ p_{61}^{SS} =$ 0.92*0.08+0.08*0.25= 0.0936

EPV of sickness benefit is
50000(0.08/1.05+0.0936/1.05^2)=8054

EPV of death benefit is

$200000(\, v\,{}_{}p_{60}^{HD} + v^2({}_{}p_{60}^{HH}\ {}_{}p_{61}^{HD} + {}_{}p_{60}^{HS}\ {}_{}p_{61}^{SD})$

=200000*(0/1.05+1/1.05^2(0.92*0+0.08*0.05)
=726

Total [7]

Q.8)

Independent rates of decrement

X	$(aq)^r_x$	$(aq)^d_x$	1-(1/2) $(aq)^d_x$	q^r_x	1-(1/2) $(aq)^r_x$	q^d_x
60	0.1291	0.0494	0.9753	0.13237	0.93545	0.05281
61	0.1790	0,0806	0.9597	0.18658	0.9105	0.08852

$q^d_x \sim \dfrac{(aq)^d_x}{1-\frac{1}{2}(aq)^r_x}$ and $q^r_x \sim \dfrac{(aq)^r_x}{1-\frac{1}{2}(aq)^d_x}$

[3.5]

After adjustment, $q'^d_{60} = 0.031686$, $q'^d_{61} = 0.053112$

[1]

The new service can be found using the formula, that is,

$(aq)'^d_x = q'^d_x(1-\frac{1}{2}\, q^r_x)$, $(aq)'^r_x = q^r_x(1-\frac{1}{2}\, q'^d_x)$

X	q'^d_x	q^r_x	1-(1/2) q'^d_x	$(aq)'^r_x$	1-(1/2) q^r_x	$(aq)'^d_x$
60	0.031686	0.13237	0.98416	0.13027	0.933815	0.029589
61	=0.053112	0.18658	0.97344	0.181624	0.90671	0.048157

[3.5]

Age (x)	**No. in service $(al)_x$**	**Deaths $(ad)^d_x$**	**Retirement $(ad)^r_x$**
60	10,000	296	1303
61	8401	405	1526
62	6470		

[1]
Total [9]

Q.9)

Let x =40 and y = 50

A payment is made at time t, if

1) x and y are both alive
2) x is alive, y having died
3) y is alive, x having died
4) both having died, under particular conditions

[1]

To understand (4), consider the following diagram

```
          a              b              c
l-------------l--------------l--------------l
0           t-20           t-10             t
```

under (4), a payment at t would NOT be made

- if x and y both died in (a), or
- if x died in (a) and y died in (b), or
- if y died in (a) and x died in (b)

[1]

So the contingencies applying to (4) regarding the times of death of x and y are

x	y
a	c
c	a
b	b
b	c
c	b
c	c

Hence, overall for a general payment at time t, we need to consider

$$v^t\,[{}_tp_{xy} + (1 - {}_tp_x)\,{}_tp_y + {}_tp_x\,(1 - {}_tp_y) + ({}_{t-20}p_x - {}_tp_x)({}_{t-20}p_y - {}_tp_y)$$

$$+ (1 - {}_{t-20}p_x)({}_{t-10}p_y - {}_tp_y) + (1 - {}_{t-20}p_y)({}_{t-10}p_x - {}_tp_x)$$

[3]

$$= v^t\,[{}_{t-20}p_{xy} + (1 - {}_{t-20}p_x)\,{}_{t-10}p_y + (1 - {}_{t-20}p_y)\,{}_{t-10}\,p_x$$

The relevant values for t are t = 20

Hence the present value

$$= 30000 \;?\; v^t\,[{}_{t-20}p_{xy} + (1 - {}_{t-20}p_x)\,{}_{t-10}p_y + (1 - {}_{t-20}p_y)\,{}_{t-10}p_x]$$

[2]

putting t = 20 + s

$= 30000v^{20}$? v^s [${}_sp_{xy}$ + ${}_{10}p_y \cdot {}_sp_{y+10}$ – ${}_{10}p_y \cdot {}_sp_{y+10\,:x}$ + ${}_{10}p_x \cdot {}_sp_{x+10}$ - ${}_{10}p_x \cdot {}_sp_{x+10\,:y}$]

$= 30000v^{20}$ [$\ddot{a}_{40:50}$ + ${}_{10}p_{50} \cdot \ddot{a}_{60}$ – ${}_{10}p_{50}\ \ddot{a}_{40:60}$ + ${}_{10}p_{40} \cdot \ddot{a}_{50}$ – ${}_{10}p_{40}\ \ddot{a}_{50:50}$]

[2]

for t = 0 to 19, the payments are certainly made, so an additional amount of $30000a_{\overline{20}|}$ should be added to the above expression.

[1]

Total [10]

Q.10)

i) Gross future loss random variable (GFL r.v.) =

$(v^{K[60]+1})\{200000 - (50000)(K_{[60]})\} + 300 + 30a_{\overline{K[60]}|}$

- $P(0.975\ddot{a}_{\overline{K[60]+1}|} - 0.225)$ for $K_{[60]} < 4$

Or $300 + 30a_{\overline{3}|} - P(0.975\ddot{a}_{\overline{4}|} - 0.225)$ for $K_{[60]} = 4$

[3]

ii)

$q_{[60]} = 0.005774$ $p_{[60]} = 0.994226$ ${}_0p_{[60]} = 1$

$q_{[60]+1} = 0.00868$ $p_{[60]+1} = 0.99132$ ${}_1p_{[60]} = 0.994226$

$q_{62} = 0.010112$ $p_{62} = 0.989888$ ${}_2p_{[60]} = 0.985596$

$q_{63} = 0.011344$ $p_{63} = 0.988656$ ${}_3p_{[60]} = 0.97563$

[1]

Year	Prem	Expense	Interest	Claim	Cash Flow	Profit Signature	NPV
1	P	0.25P + 300	0.03P – 12	1154.8	0.78P – 1466.8	0.78P – 1466.8	0.75P – 1410.38
2	P	0.025P + 30	0.039P – 1.2	1302	1.014P – 1333.2	1.008145P – 1325.5	0.932087P – 1225.5
3	P	0.025P + 30	0.039P – 1.2	1011.2	1.014P – 1042.4	0.999394P – 1027.39	0.888458P – 913.342
4	P	0.025P + 30	0.039P – 1.2	567.2	1.014P – 598.4	0.989289P – 583.817	0.845648P – 499.049

[4]

Total NPV = 3416193P – 4,048.28

So P = Rs.1,185.03

[1]

Total [6]

iii)

(a) Profit is deferred but as earned interest and risk discount rate are equal, there is no impact on NPV or premium.

[1]

(b) Profit is deferred but because the discount rate exceeds earned rate, NPV falls and premium would have to increase to satisfy the same profit criterion.

[2]

Total [3]

Total For Qn. [12]

Q.11)

Derive dependent rates from independent rates

$(aq)^d_x = q^d_x\,[\,1-1/2(q^w_x + q^i_x)+1/3(q^w_x\, q^i_x)]$

Multiple decrement table

x	(al)x	(aq)d	(aq)w	(aq)i	(ad)d	(ad)w	(ad)i
62	100000	0.016932	0.098112	0.018832	1693	9811	1883
63	86612	0.018365	0.147390	0.013740	1591	12766	1190
64	71066	0.020600	0.196715	0.008900	1464	13980	633
65	54990	0.000000	0.000000	0.000000			

[6]

(11/2 for each column of aq values and +11/2 for rest

(a) Death Benefit

Assuming death occurs on average halfway through the year EPV of Death Benefit is

$$\frac{2{,}00{,}000}{(al)_{62}}\left[v^{0.5}\,(ad)_{62} + v^{1.5}\,(ad)_{63} + v^{2.5}\,(ad)_{64}\right] \quad [1]$$

=200000/100000*(1693/(1.06)^0.5+1591/(1.06)^1.5+1464/(1.06)^2.5)
=8736

[1]

[2]

(b)

Assuming that withdrawals occur on average half way through the year the EPV of withdrawal benefit is,

$5000/100000 * [v^{1.5} (ad)^{w}_{63} + 2 * v^{2.5} (ad)^{w}_{64}]$

[1]

=5000/100000*[12766/1.06^0.5+13980/1.06^2.5]
=1224

[1]
[2]

(c)

Assuming that ill health retirements occur on average half way through the year the EPV of ill heath retirement benefit is,

$$\frac{50000\, \ddot{a}^{(12)}_{\overline{5}|}}{(al)_{62}} [v^{0.5} (ad)^{i}_{62} + v^{1.5} (ad)^{i}_{63} + v^{2.5} (ad)^{i}_{64}]$$

[1.5]

= 50000*4.4518*1.021537*(1883/1.06^0.5+1190/1.06^1.5+633/1.06^2.5)/100000
=7882

[1]

[2.5]

(d) Normal retirement benefit

$$3 * 20000 * v^{3} * (al)_{65} / (al)_{62} * (\ddot{a}^{(12)}_{\overline{10}|} + v^{10}\ l_{75}/l_{65}\ \ddot{a}^{(12)}_{75})$$

[1.5]

=60000*1/1.06^3*54990/100000*(8.1109*1.021537+1/1.04^10*8405.16/9647.17*(9.456-11/24)

=376241

[1]
[2.5]

Q.12)

Unit fund growth

Policy year	Premium allocated	Cost of allocation	With fund b/f	Fund before charge	Annual charge	Fund c/f
1	18000	17550.00	17550.00	18954.00	473.85	18480.15
2	19400	18915.00	37395.15	40386.76	1009.67	39377.09
3	19800	19305.00	58682.09	63376.66	1584.42	61792.24

[5]

Premium allocated =(1-allocation charge)* premium
Cost of allocation= (1-b/o spread)
Fund before charge= Fund c/f prev year+ cost of allocation for the yeat
Annual charge= Fund before charge*fund management charge

The final maturity benefit is : 61792.24*1.05=64881.86

b) Non unit fund development is

Policy year	Profit on allocation	Expenses	Non unit interest	Annual charge	Mort rate	Death Claims cost	Bonus cost	Profit
1	2450.00	1000	58	473.85	0.002508	79.0517838	0	1902.80
2	1085.00	500	23.4	1009.67	0.002809	29.8397459	0	1588.23
3	695.00	500	7.8	1584.42	0.003152	0	3079.87	-1292.66

[4]

Profit allocation= Premium-cost of allocation
Non unit interest = (profit on allocation-expenses)*1.04
Death claims cost (Sum assured-unit fund)*mort rate
Bonus cost= (Unit fund*0.05*(1-mort rate))

Probability of policies in force

Policy year	Mort rate	Policies in force at Beginning of yr	Deaths	Policies in force at end
1	0.002508	1	0.002508	0.997492
2	0.002809	0.997492	0.002802	0.99469
3	0.003152	0.994683	0.003135	0.991548

So the profit signature is (1903,1584,-1286)

[2]

[6]

d)

Non unit provisions end of year 1= 3%*18480= 554.40

Non unit provisions end of year 1= 3%*39377= 1181.31

[1]

Revised profit signature is

Policy year	Non unit provisions b/f	Interest on provisions	Increase in provisions	Profit ignoring provisions	Profit after provisions
1	0	0.00	553.0	1902.80	1349.78
2	554.40	22.18	623.6	1584.25	982.84
3	1181.31	47.25	-1181.3	-1285.78	-57.22

[4]

The profit signature is (1350,980,-57)

[1]

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