Actuarial Society of India

EXAMINATIONS

16th November 2005

Subject CT4– Models

Time allowed: Three Hours (10.30 am – 1.30 pm)

Total Marks 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
4. ***Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.***
5. ***In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.***

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor

103 Questions

Q.A1) An automobile insurance company operates a 5 level no claims discount system as defined below:

Level	% premium charged
5	100
4	90
3	80
2	70
1	60

Insured drivers move between the levels depending on the number of claims in the previous year. For each policyholder, the number of claims per year follow a Poisson distribution with a mean of 0.25.

For those in levels 2, 3, 4 and 5 at the start of the previous year:

- If no claims were made in the previous year, the insured moves down one level (for example from level 4 to level 3)
- If one claim is made in the previous year, the insured moves up one level (except those in level 5 at the start of the previous year, who remain in level 5)
- If two claims are made in the previous year, the insured moves up two levels (except those in level 5 at the start of the previous year remain in level 5 and those in level 4 move to level 5)
- If three or more claims are made during the previous year, the insured moves to level 5

For those in level 1 at the start of the previous year, a no claims discount protection policy applies whereby they remain in level 1 if they make one claim. If they make two claims they move to level 2. If they make three or more claims, they move to level 5. If they make no claims, they remain in level 1.

i. Determine the transition matrix for the no claims discount system (you can assume that all drivers continue with their policies)

(3)

ii. Let a policyholder be in Level 3 for the first year of the policy and assume that the policy is maintained. Calculate the probability that at the start of the third year, the policyholder will be in

a. In Level 1
b. In Level 3

(3)

iii. Also

a. State the conditions under which the probability of being in a particular state after n years converges as $n \to \infty$ to some limit which is independent of the initial state
b. Verify that the conditions are satisfied in this case
c. Determine the ultimate probability that the insured will be in level 1

(8)

iv. The insurance company suspects that the model used for its calculations may be too elementary. Based on the annual data showing the number of claims per policy, broken down by discount levels, state which test would be most appropriate to test the assumption that the distribution of the number of claims per policy is Poisson with mean 0.25 **(1)**

Total [15]

Q.A2) It is the cricket series and India plays a match every week. If the Indian team suffers 3 consecutive defeats, the BCCI bows to public pressure and would fire the coach Mr. Channel. Matches are independent of each other and the probability of defeat for any match is 0.2. Last week India managed to win.

i. Formulate the process as a four state Markov chain indicating the current run of consecutive defeats or whether Mr. Channel has been fired and write down the transition matrix. **(4)**

ii. Let K be the remaining number of weeks that Mr. Channel will stay as coach for team India. Find *Pr*.(K = k) for k = 3, 4, 5, 6 and 7 **(5)**

iii. Find E(K) **(3)**

iv. In order to avoid being fired by BCCI after India suffers two consecutive defeats, Mr. Channel decides to be unscrupulous and bribe both the umpires of the next match. (a bribe costs $10,000 guarantees avoiding defeat and the coach is never caught). What is the average amount per week the manager pays for bribes in the long run. **(5)**

Total [17]

Q.A3) Patients arriving at the Accident and Emergency department at AIIMS (State A) wait for an average of one hour before being classified by a junior doctor as requiring in – patient treatment (I), out – patient treatment (O) or further investigation (F). Only 10% of the new arrivals are classified as in – patient and 50% as out – patients.

If needed, further investigation requires an average if 3 hours, after which 50% of the cases are discharged (D), 25% are sent to receive out patient treatment and 25% are admitted as in – patients.

Out – patient treatment takes an average of 2 hours to complete, in – patient treatment an average of 60 hours. Both result in discharge.

It is suggested that a time – homogeneous Markov process with states A, F, I, O and D could be used to model the progress of patients through the system with the ultimate aim of reducing the average time spent at AIIMS.

i. Write down the matrix of transition rates, $\{\sigma_{ij}: i, j = A, F, I, O, D\}$, of such a model **(2)**

ii. Calculate the proportion of patients who eventually receive in – patient treatment **(1)**

iii. Derive the expressions for the probability that a patient arriving at time t = 0 is:

a. Yet to be classified by a junior doctor at time t

b. Undergoing further investigation at time t

(4)

iv. Let m_i denote the expectation of the time until discharge for a patient currently in state i.

a. Explain in words why m_i satisfies the following equation

$$m_i = \frac{1}{l_i} + \sum_{j \in \{I,D\}} \frac{s_{ij}}{l_i} m_j \quad where\ l_i = \sum s_{ij}$$

b. Hence calculate the expectation of the total time until discharge for a newly arrived patient

(4)

v. State the distribution of the time spent in each state visited according to this model. **(1)**

vi. Note that the average times listed above may be assumed to be the sample mean waiting times derived from tacking a large sample of patients through the system. Describe briefly what additional features of the date might be used to check that this simple model matches the situation being modeled. **(2)**

vii. AIIMS management believes that replacing the junior doctor with a more senior doctor will save resources by reducing the proportion of cases sent for further investigation. Alternatively, the same resources could go towards reducing out – patient treatment time.

a. Outline briefly the calculations that would be needed to compare the options

b. Discuss whether the current model is suitable as a basis for making decisions of this nature

(4)

Total [18]

104 Questions

Q.B1) **a)** It is assumed that the future lifetime of a life aged x is expressed as

$T_x = K_x + S_x$

where K_x = curtate life time

S_x is independent of K_x, has a uniform distribution in the interval (0,1)

Show that ${}_u q_x = u.q_x \quad 0<u<1$ **(2)**

b) **(i)** Sketch the form of the force of mortality m_{20+t} for the population mortality of males of a developed country and identify the key components of the curve. **(2)**

(ii) m_{20+t} is modeled using GM(Gompetz Makeham) family of curves.

$$m_{20+t} = GM(r,s) = a_1 + a_2 t + a_3 t^2 + ... + a_r t^{r-1} + \exp\left\{a_{r+1} + a_{r+2} t + a_{r+3} t^2 + ... + a_{r+s} t^{s-1}\right\}$$

where a_1, a_2..... are constants.

Show that a GM(2,2) curve is able to explain the above key components of the m_{20+t} curve. **(3)**

Total [7]

Q.B2) **a)** State Balducci assumption. **(1)**

b) Under Balducci assumption, prove that

$${}_{b-a} q_{x+a} = \frac{(b-a)q_x}{1-(1-b)q_x} \quad \text{for } 0 \le a < b \le 1$$ **(3)**

c) You are estimating ${}_t q_x$, $0 < t < 1$, under the assumption of Udd (Uniform Distribution of deaths) and constant force of mortality separately.

State with reasoning the relative size of ${}_t q_x$ under the two assumptions. **(2)**

Total [6]

Q.B3) In a mortality investigation, q_x and E_x^c are respectively the number of deaths and central exposed to risk observed at age x last birthday.

Assuming that the force of mortality is constant over the year, express the following in terms of q_x.

a) The standard deviation of the number of deaths occurring at age x last birthday **(1)**

b) The 95% confidence interval of the A/E (actual to expected deaths) ratio.

State the assumptions that you make. **(2)**

Total [3]

Q.B4) You are investigating mortality experience for the whole of the calendar years 1999 and 2000. A life being observed in this investigation attained exact age 54 on 1-06-2000 and died on 25-10-2000.

Calculate to the nearest number of weeks, the contribution of the above life to the

a) Central exposed to risk and

b) Initial exposed to risk

for a population of lives aged 54 last birthday. **[3]**

Q.B5) You work in a small life office and have been asked to investigate the mortality experience of the whole life policyholders of the office over a 5 year period 01-01-1992 to 31-12-1996.

The following data are available to you

1. Deaths over the period 01-01-1992 to 31-12-1996 grouped by calendar year of death and by calendar year of birth.
2. The number of policies on the office's books as at 31st December of each calendar year 1991 to 1996, subdivided by calendar year of birth.

a) State the principle of correspondence. **(1)**

b) Explain how you would use the data above to estimate the non-select central mortality rates for the period as a whole. **(7)**

Explain all the steps. Match age grouping of death and exposure to risk to ensure close correspondence between them as far as possible.

Define all the symbols used and state the assumptions made.

Total [8]

Q.B6) A tiny insect in Nanoland has a probability of death of 0.15 in the 1st year of its life and a constant force of mortality of 0.02 thereafter.

(a) If we assume a constant force of mortality operating in the first year, calculate this force of mortality. **(1)**

(b) Calculate the probability of the insect dying in the first 15 years. **(2)**

(c) Find the complete expectation of life of the insect at its birth. **(4)**

(d) Would the expected life time of the insect at birth be higher or lower than that at the beginning of the second year? Give adequate reasoning for your answer. You need not calculate the latter life expectation. **(3)**

Total [10]

Q.B7) A life insurance office has carried out an investigation of its withdrawal experience. It has obtained crude rates of withdrawal in the following form

$$\hat{q}_t = \frac{w_t}{E_t}$$

w_t = number of observed withdrawals at curtate policy durations.

E_t = initial exposed to risk corresponding to w_t.

The company has graduated the rates by fitting the following formula by the weighted least square method

$$\overset{0}{q}_t = a + bt + gt^2$$

a, b, c are parameters estimated from the data.

The following data is given

t	$\hat{q}_t$	$\overset{0}{q}_t$	E_t
0	0.1515	0.1324	3706
1	0.0963	0.1051	3262
2	0.0746	0.0832	5171
3	0.0683	0.0654	3243
4	0.0553	0.0521	5147
5	0.0396	0.0438	3850
6	0.0432	0.0383	3080
7	0.0514	0.0381	2094
8	0.0381	0.0423	1083
9	0.0437	0.0515	1511

(i) Comment on the suitability of the graduation model for the underlying withdrawal experience. Analysis should be by applying the following tests

- **a)** Chi square
- **b)** Standard deviation test
- **c)** Sign test

For each test, you are required to state the Null Hypothesis, show your calculations and draw conclusions from your results. **(8)**

(ii) Comment on the suggestion that the following means be adopted to improve graduation of the data

- **a)** Refit the parameters using Minimum Chi Square method
- **b)** Fit a higher order polynomial to the data
- **c)** Perform graphical graduation. **(5)**

Total [13]

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