Actuarial Society of India

EXAMINATIONS

16th November 2005

Subject CT4 (104) – Models (104 Part)

Time allowed: One and a Half Hours (10.30 am – 12.00 noon)

INSTRUCTIONS TO THE CANDIDATES

1. ***Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
4. ***Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.***
5. ***In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.***

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpre t scripts.

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor

Q.1 **a)** It is assumed that the future lifetime of a life aged x is expressed as

$T_x = K_x + S_x$

where K_x = curtate life time

S_x is independent of K_x, has a uniform distribution in the interval (0,1)

Show that ${}_u q_x = u.q_x \quad 0<u<1$ **(2)**

b) **(i)** Sketch the form of the force of mortality m_{20+t} for the population mortality of males of a developed country and identify the key components of the curve. **(2)**

(ii) m_{20+t} is modeled using GM(Gompetz Makeham) family of curves.

$$m_{20+t} = GM(r,s) = a_1 + a_2 t + a_3 t^2 + \ldots\ldots + a_r t^{r-1} + \exp\left\{a_{r+1} + a_{r+2} t + a_{r+3} t^2 + \ldots\ldots + a_{r+s} t^{s-1}\right\}$$

where $a_1, a_2 \ldots\ldots$ are constants.

Show that a GM(2,2) curve is able to explain the above key components of the m_{20+t} curve. **(3)**

Total [7]

Q.2 **a)** State Balducci assumption. **(1)**

b) Under Balducci assumption, prove that

$${}_{b-a} q_{x+a} = \frac{(b-a)q_x}{1-(1-b)q_x} \quad \text{for } 0 \le a < b \le 1$$ **(3)**

c) You are estimating ${}_t q_x$, $0 < t < 1$, under the assumption of Udd (Uniform Distribution of deaths) and constant force of mortality separately.
State with reasoning the relative size of ${}_t q_x$ under the two assumptions. **(2)**

Total [6]

Q.3 In a mortality investigation, q_x and E_x^c are respectively the number of deaths and central exposed to risk observed at age x last birthday.
Assuming that the force of mortality is constant over the year, express the following in terms of q_x.

a) The standard deviation of the number of deaths occurring at age x last birthday **(1)**

b) The 95% confidence interval of the A/E (actual to expected deaths) ratio.
State the assumptions that you make. **(2)**

Total [3]

Q.4 You are investigating mortality experience for the whole of the calendar years 1999 and 2000. A life being observed in this investigation attained exact age 54 on 1-06-2000 and died on 25-10-2000.

Calculate to the nearest number of weeks, the contribution of the above life to the

a) Central exposed to risk and

b) Initial exposed to risk

for a population of lives aged 54 last birthday. **[3]**

Q.5 You work in a small life office and have been asked to investigate the mortality experience of the whole life policyholders of the office over a 5 year period 01-01-1992 to 31-12-1996.

The following data are available to you

1. Deaths over the period 01-01-1992 to 31-12-1996 grouped by calendar year of death and by calendar year of birth.
2. The number of policies on the office's books as at 31st December of each calendar year 1991 to 1996, subdivided by calendar year of birth.

a) State the principle of correspondence. **(1)**

b) Explain how you would use the data above to estimate the non-select central mortality rates for the period as a whole. **(7)**

Explain all the steps. Match age grouping of death and exposure to risk to ensure close correspondence between them as far as possible.

Define all the symbols used and state the assumptions made.

Total [8]

Q.6 A tiny insect in Nanoland has a probability of death of 0.15 in the 1st year of its life and a constant force of mortality of 0.02 thereafter.

(a) If we assume a constant force of mortality operating in the first year, calculate this force of mortality. **(1)**

(b) Calculate the probability of the insect dying in the first 15 years. **(2)**

(c) Find the complete expectation of life of the insect at its birth. **(4)**

(d) Would the expected life time of the insect at birth be higher or lower than that at the beginning of the second year? Give adequate reasoning for your answer. You need not calculate the latter life expectation. **(3)**

Total [10]

Q.7 A life insurance office has carried out an investigation of its withdrawal experience. It has obtained crude rates of withdrawal in the following form

$$\hat{q}_t = \frac{w_t}{E_t}$$

w_t = number of observed withdrawals at curtate policy durations.

E_t = initial exposed to risk corresponding to w_t.

The company has graduated the rates by fitting the following formula by the weighted least square method

$$\overset{0}{q}_t = a + bt + gt^2$$

a, b, c are parameters estimated from the data.

The following data is given

t	$\hat{q}_t$	$\overset{0}{q}_t$	E_t
0	0.1515	0.1324	3706
1	0.0963	0.1051	3262
2	0.0746	0.0832	5171
3	0.0683	0.0654	3243
4	0.0553	0.0521	5147
5	0.0396	0.0438	3850
6	0.0432	0.0383	3080
7	0.0514	0.0381	2094
8	0.0381	0.0423	1083
9	0.0437	0.0515	1511

(i) Comment on the suitability of the graduation model for the underlying withdrawal experience. Analysis should be by applying the following tests

- **a)** Chi square
- **b)** Standard deviation test
- **c)** Sign test

For each test, you are required to state the Null Hypothesis, show your calculations and draw conclusions from your results. **(8)**

(ii) Comment on the suggestion that the following means be adopted to improve graduation of the data

- **a)** Refit the parameters using Minimum Chi Square method
- **b)** Fit a higher order polynomial to the data
- **c)** Perform graphical graduation. **(5)**

Total [13]

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