

Actuarial Society of India

EXAMINATIONS

16th November 2005

Subject CT4 (103) – Models (103 Part)

Time allowed: One and a Half Hours (10.30 am – 12.00 noon)

INSTRUCTIONS TO THE CANDIDATES

- 1. Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.*
- 2. Mark allocations are shown in brackets.*
- 3. Attempt all questions, beginning your answer to each question on a separate sheet.*
- 4. Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.*
- 5. In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.*

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor

Q.1) An automobile insurance company operates a 5 level no claims discount system as defined below:

Level	% premium charged
5	100
4	90
3	80
2	70
1	60

Insured drivers move between the levels depending on the number of claims in the previous year. For each policyholder, the number of claims per year follow a Poisson distribution with a mean of 0.25.

For those in levels 2, 3, 4 and 5 at the start of the previous year:

- If no claims were made in the previous year, the insured moves down one level (for example from level 4 to level 3)
- If one claim is made in the previous year, the insured moves up one level (except those in level 5 at the start of the previous year, who remain in level 5)
- If two claims are made in the previous year, the insured moves up two levels (except those in level 5 at the start of the previous year remain in level 5 and those in level 4 move to level 5)
- If three or more claims are made during the previous year, the insured moves to level 5

For those in level 1 at the start of the previous year, a no claims discount protection policy applies whereby they remain in level 1 if they make one claim. If they make two claims they move to level 2. If they make three or more claims, they move to level 5. If they make no claims, they remain in level 1.

- i. Determine the transition matrix for the no claims discount system (you can assume that all drivers continue with their policies) **(3)**
- ii. Let a policyholder be in Level 3 for the first year of the policy and assume that the policy is maintained. Calculate the probability that at the start of the third year, the policyholder will be in
 - a. In Level 1
 - b. In Level 3**(3)**
- iii. Also
 - a. State the conditions under which the probability of being in a particular state after n years converges as $n \rightarrow \infty$ to some limit which is independent of the initial state
 - b. Verify that the conditions are satisfied in this case
 - c. Determine the ultimate probability that the insured will be in level 1**(8)**

- iv. The insurance company suspects that the model used for its calculations may be too elementary. Based on the annual data showing the number of claims per policy, broken down by discount levels, state which test would be most appropriate to test the assumption that the distribution of the number of claims per policy is Poisson with mean 0.25

(1)

Total [15]

Q.2) It is the cricket series and India plays a match every week. If the Indian team suffers 3 consecutive defeats, the BCCI bows to public pressure and would fire the coach Mr. Channel. Matches are independent of each other and the probability of defeat for any match is 0.2. Last week India managed to win.

- i. Formulate the process as a four state Markov chain indicating the current run of consecutive defeats or whether Mr. Channel has been fired and write down the transition matrix. (4)

- ii. Let K be the remaining number of weeks that Mr. Channel will stay as coach for team India. Find $Pr.(K = k)$ for $k = 3, 4, 5, 6$ and 7 (5)

- iii. Find $E(K)$ (3)

- iv. In order to avoid being fired by BCCI after India suffers two consecutive defeats, Mr. Channel decides to be unscrupulous and bribe both the umpires of the next match. (a bribe costs \$10,000 guarantees avoiding defeat and the coach is never caught). What is the average amount per week the manager pays for bribes in the long run. (5)

Total [17]

Q.3) Patients arriving at the Accident and Emergency department at AIIMS (State A) wait for an average of one hour before being classified by a junior doctor as requiring in – patient treatment (I), out – patient treatment (O) or further investigation (F). Only 10% of the new arrivals are classified as in – patient and 50% as out – patients.

If needed, further investigation requires an average of 3 hours, after which 50% of the cases are discharged (D), 25% are sent to receive out patient treatment and 25% are admitted as in – patients.

Out – patient treatment takes an average of 2 hours to complete, in – patient treatment an average of 60 hours. Both result in discharge.

It is suggested that a time – homogeneous Markov process with states A, F, I, O and D could be used to model the progress of patients through the system with the ultimate aim of reducing the average time spent at AIIMS.

- i. Write down the matrix of transition rates, $\{\sigma_{ij}: i, j = A, F, I, O, D\}$, of such a model (2)

- ii. Calculate the proportion of patients who eventually receive in – patient treatment (1)

- iii. Derive the expressions for the probability that a patient arriving at time $t = 0$ is:
- Yet to be classified by a junior doctor at time t
 - Undergoing further investigation at time t

(4)

- iv. Let m_i denote the expectation of the time until discharge for a patient currently in state i .
- Explain in words why m_i satisfies the following equation

$$m_i = \frac{1}{I_i} + \sum_{j \in \{I, D\}} \frac{S_{ij}}{I_i} m_j \quad \text{where } I_i = \sum S_{ij}$$

- Hence calculate the expectation of the total time until discharge for a newly arrived patient

(4)

- v. State the distribution of the time spent in each state visited according to this model. (1)

- vi. Note that the average times listed above may be assumed to be the sample mean waiting times derived from tracking a large sample of patients through the system. Describe briefly what additional features of the data might be used to check that this simple model matches the situation being modeled. (2)

- vii. AIMS management believes that replacing the junior doctor with a more senior doctor will save resources by reducing the proportion of cases sent for further investigation. Alternatively, the same resources could go towards reducing out – patient treatment time.

- Outline briefly the calculations that would be needed to compare the options

- Discuss whether the current model is suitable as a basis for making decisions of this nature

(4)

Total [18]
