

Actuarial Society of India

EXAMINATIONS

21st November 2005

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (10.30 – 13.30 pm)

INSTRUCTIONS TO THE CANDIDATES

1. *Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.*
2. *Mark allocations are shown in brackets.*
3. *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*
4. *Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.*
5. *In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.*

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor.

Q.1

- I) Explain the “law of one price”. (2)
- II) Write down the Equation of Value for an Insurance Policy. (1)
- III) An insurance company accepts an obligation to pay Rs.10,000 at the end of each year for 2 years. The insurance company purchases a combination of the following two bonds at a total cost of X in order to exactly match its obligation:
- 1-year 4% annual coupon bond with a yield rate of 5%
 - 2-year 6% annual coupon bond with a yield rate of 5%.
- Calculate X . (4)
- IV) Sania, Jaya, and Lisa each borrow Rs.5000 for five years at a nominal interest rate of 12%, compounded half-yearly. Sania has interest accumulated over the five years and pays all the interest and principal in a lump sum at the end of five years. Jaya pays interest at the end of every six-month period as it accrues and the principal at the end of five years. Lisa repays her loan with 10 level payments at the end of every six-month period. Calculate the total amount of interest paid on all three loans. (5)
- Total [12]**

Q.2

- I) Prove

$$(Ia)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}.$$

(3)

- II) A loan is repayable by an increasing annuity payable annually in arrears for 15 years. The repayment at the end of the first year is Rs.3,000 and subsequent payments increase by Rs.200 each year. The repayments were calculated using a rate of interest of 8% per annum effective.
- Calculate the original amount of the loan. (3)
 - Construct the capital/interest schedule for years nine (after the eighth payment) and ten, showing the outstanding capital at the beginning of the year, the interest element and the capital repayment. (6)
 - Immediately after the tenth payment of interest and capital, the interest rate on the outstanding loan is reduced to 6% per annum effective. Calculate the amount of the eleventh payment if subsequent payments continue to increase by Rs.200 each year, and the loan is to be repaid by the original date, i.e. 15 years from commencement. (5)

Total [17]

Q.3

On 1st April 2002 a large number of nominal Rs.1 zero-coupon bonds were issued, to be redeemed at par on 1st April 2007. The issue price was Rs.0.67 per Rs.1 bond, and the table gives the prices at 1 April for each year until redemption in 2007. The prices are used for both the sale and purchase of bonds, and fractions of bonds may be bought and sold.

Date	1/4/2002	1/4/2003	1/4/2004	1/4/2005	1/4/2006	1/4/2007
	67p	78p	81p	86p	95p	100p

Consider two investors A and B. Investor A invests Rs.100 in bonds on 1 April each year from 2002 to 2006 inclusive; and investor B buys 100 bonds on 1 April each year from 2002 to 2006 inclusive.

For both investors construct a schedule to show the value of the fund at 1 April each year from 2002 to 2007 inclusive. From this calculate for the period from 1 April 2002 to 1 April 2007 for both investors:

- (i) the money weighted rate of return per annum
- (ii) the time weighted return per annum

Total [12]

Q.4

A pension fund expects to make payments of Rs.100,000 per annum at the end of each of the next five years. It wishes to immunise these liabilities by investing in two zero coupon bonds which mature in five years and in one year respectively. The rate of interest is 5% per annum effective.

- (i)
 - (a) Show that the present value of the liabilities is Rs.4,32,948/-.
 - (b) Show that the duration of the liabilities is 2.9 years. (6)
- (ii) Calculate the nominal amounts of the two zero coupon bonds which must be purchased if the pension fund is to equate the present value and duration of assets and liabilities. (6)
- (iii)
 - (a) Calculate the convexity of the assets.
 - (b) Without calculating the convexity of the liabilities, comment on whether you think Redington's immunisation has been achieved. (6)

Total [18]

Q.5

An investor purchased a bond with exactly 20 years to redemption. The bond, redeemable at par, has a gross redemption yield of 6%. It pays annual coupons, in arrears, of 5%. The investor does not pay tax.

(i) Calculate the purchase price paid for the bond. (3)

(ii) After exactly ten years, immediately after payment of the coupon then due, this investor sells the bond to another investor. That investor pays income and capital gains tax at a rate of 30%. The bond is purchased by the second investor to provide a net rate of return of 6.5% per annum.

(a) Calculate the price paid by the second investor.

(b) Calculate the annual effective rate of return earned by the first investor during the period for which the bond was held. (10)

Total [13]

Q.6

I) Explain briefly the shape of the yield curve by reference to the liquidity preference theory. (2)

II) An annuity pays 1 at the end of each year for n years. Using an annual effective interest rate of i , the accumulated value of the annuity at time $(n + 1)$ is 13.776. It is also known that $(1 + i)^n = 2.476$. Calculate n . (3)

III) Bhanu deposits Rs.100 into a bank account. Her account is credited interest at a nominal rate of interest i convertible half-yearly. At the same time, Priya deposits Rs.100 into a separate account. Priya's account is credited interest at a force of interest of \ddot{a} . After 7.25 years, the value of each account is 200. Calculate $(i - d)$. (3)

Total [8]

Q.7

I) State what is meant by a forward contract. Your answer should include reference to the terms short forward position and long forward position. (2)

II) A 3-month forward contract is issued on 1 February 2001 on a stock with a price of Rs.150 per share. Dividends are received continuously and the dividend yield is 3% per annum. In addition, it is anticipated that a special dividend of Rs.30 per share will be paid on 1 April 2001. Assuming a risk-free force of interest of 5% per annum and no arbitrage, calculate the forward price per share of the contract. (3)

Total [5]

Q.8

A company has just borrowed Rs.100, 000 at a fixed rate of 8% pa. The loan is to be repaid at par value in five years and interest must be paid at the end of each year. In order to reduce its exposure to fixed rate borrowing, it arranges an interest rate swap to exchange the 8% pa fixed rate for a variable rate that is $\frac{1}{2}\%$ higher than the central bank's base rate at the time when the interest payments are due. Over the next five years, the base rates turn out to be:

($7\frac{1}{2}\%$, $6\frac{1}{2}\%$, $7\frac{1}{2}\%$, $8\frac{1}{2}\%$, $9\frac{1}{2}\%$)

- a) State the constant effective annual interest rate that would have been paid by the company if it had not arranged the swap.
- b) Calculate the constant effective annual interest rate that has been paid by the company as a result of taking out the swap.

Total [4]

Q.9

I) Outline the similarities and differences between deterministic and stochastic interest rate models. **(3)**

II) The rate of interest earned in the year from time $\tilde{t}-1$ to t is denoted by i_t . Assume $(1+i_t)$ is lognormally distributed. The expected value of the rate of interest is 5%, and the standard deviation is 11%.

a) Calculate the parameters of the lognormal distribution of $(1+i_t)$. **(4)**

b) Calculate the probability that the rate of interest in the year from time $\tilde{t}-1$ to t lies between 4% and 7%. **(4)**

Total [11]
