

Institute of Actuaries of India

Subject ST8 – General insurance : Pricing

November 2013 Examination

INDICATIVE SOLUTIONS

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable

Solution 1 :

Reasons:

- age is considered to not usually be a major determinant of risk of accidents until old age is reached, and this cover is not normally sold to the elderly
- Popularly sold as group insurance therefore administratively convenient

(2 Marks)

Solution 2 :

- i. Balance of a reinsurance treaty: The ratio of the total premiums receivable by a reinsurer under a surplus treaty to the reinsurer's maximum liability for any one claim, based on estimated (or expected) maximum loss (EML).
- ii. Bordereau: A detailed list of premiums, claims and other important statistics provided by ceding insurers to reinsurers, so that payments due under a reinsurance treaty (or delegated authority schemes in direct insurance) can be calculated.
- iii. Facultative reinsurance: A reinsurance arrangement covering a single risk as opposed to a treaty arrangement; commonly used for very large risks or portions of risk written by a single insurer.
- iv. Rate on line: For non-proportional reinsurance, the total premium charged (ignoring reinstatement premiums) for the reinsurance divided by the width of the layer covered.

(6 Marks)

Solution 3 :

- i. Let the office premium calculated be 100. Then 35 is the margin and 65 is the pure risk premium. 5% increase in commission will increase the margin to 40. If the margin is increased to 40% then the premium will be $65/(1-0.40) = 108.33$. So, 5% increase in commission increases the premium rate by 8.33%.

(1 Mark)

- ii. The pure risk premium of the new benefit is 5. So the new office premium will be $=70/(1-0.35) = 107.7$.

(1 Mark)

- iii. Solution

- a. The new benefit is tariff based
- b. Management discretion to make this product attractive by offering additional benefit at lower additional premium
- c. Existing market rates suggest a lower additional premium

- d. Cost of servicing the additional benefit might be lower than the other benefits
- e. There may be negative correlation in the earlier benefits and the new benefit

(2 Marks)

Solution 4 :

- i. Burning Cost = Total Amount of Claim / Total Exposure
 = Total Amount of Claim / Number of Claims * Number of Claims / Total Exposure
 = Average Claim Severity * Claim Frequency

(1 Mark)

ii. Solution

- a. Changes in propensity to claim due to economic change
- b. Change in accident frequency due to safety measures
- c. Change in legislation
- d. Change in policy terms and conditions
- e. Fraud control measures by the company

(2 Marks)

iii. Solution

- a. Inflation
- b. Change in legislation
- c. Change in policy terms and conditions
- d. Technological changes
- e. Changes in economic environment

(2 Marks)

[Total Marks-5]

Solution 5 :

Practical considerations of RI pricing

- iv. Data requirement
 - a. Format of data which will be required for experience pricing
 - b. How many past years to be used

Threshold of claims for reporting such that all claims are reported even for pricing the lowest layer

- v. Which method to use for pricing
 - a. Experience for low layers
 - b. Exposure for higher layers
 - c. Blending of the two methods
 - d. Stochastic simulation for complex layers

- iv. Terms and conditions of the contract such as annual aggregate deductible, reinstatements etc
- v. Which curve to use for exposure rating
- vi. Whether discounting of expected claims should be done as it will have impact on pricing due to Liability being a long tail business.
- vii. What severity trends to use
- viii. Any changes in the business for applying trends in frequency
- ix. Expense loading
- x. Profit loading
- xi. Investment income
- xii. Retrocession charges
- xiii. Market rate / insurance cycle

(7 Marks)

Solution 6 :

A captive insurance company is an insurer that is wholly owned by an industrial or commercial enterprise and set up with the primary purpose of insuring the parent or associated group companies and retaining premiums and risk within the enterprise

Reasons:

The usual reasons for setting up captives:

1. To fill gaps in insurance cover that may not be available from the traditional insurance market.
2. To manage the total insurance spend of large companies or groups of companies.
3. To enable the enterprise to buy cover directly from the reinsurance market rather than direct insurers.
4. To focus effort on risk management.
5. To gain tax and other legislative or regulatory advantages.

(4 Marks)

Solution 7 :

$$P(N = r) = P_r = \binom{r+k-1}{r} (1-p)^k p^r = \frac{(r+k-1)!}{r!(k-1)!} (1-p)^k p^r$$

i.

$$= \frac{p(r+k-1)}{r} \binom{r-1+k-1}{r-1} (1-p)^k p^{r-1} = \left(p + \frac{p(k-1)}{r}\right) P_{(r-1)}$$

So , a = p and b = p(k-1)

(2 Marks)

ii. $P(S < 200) = P(S=0) + P(S = 50) + P(S = 100) + P(S = 150)$ (1)

$$P(S=0) = P(N=0) = (1-p)^k = 0.9^2 = 0.81 \quad (0.5)$$

$$P(S = s) = \sum_{j=1}^s (a + bj/s) P(X = j) P(S = s - j) \text{ for } S > 0$$

$P(S=x) = 0$ for $0 < x < 50$ as the minimum claim size is 50.

$$P(S=50) = 0.1 * (1 + 1*50/50) * P(X = 50) * P(S=0) = 0.1 * 2 * 0.8 * 0.81 = 0.1296 \quad (0.5)$$

$$P(S=100) = 0.1 * (1 + 1/2) * P(X=50) * P(S=50) + 0.1 * (1+1) * P(X=100) * P(S=0) = 0.1 * 1.5 * 0.8 * 0.1296 + 0.1 * 2 * 0.2 * 0.81 = 0.047952 \quad (0.5)$$

$$P(S=150) = 0.1 * [(1 + 1/3) * P(X=50) * P(S=100) + (1 + 2/3) * P(X=100) * P(S=50)] = 0.1 * [1.33 * 0.8 * 0.047952 + 1.66 * 0.2 * 0.1296] = 0.009435 \quad (0.5)$$

$$P(S < 200) = 0.996987$$

(3 Marks)

iii.

Following would be the steps to derive the probability using simulation method

- a. Generate number of claims using Negative Binomial Distribution. This may be done using the probabilities of 0,1,2 claims etc for NB and apply the method for simulating discrete random variables. i.e.
 1. Suppose p_0, p_1, p_2 etc are probability of 0,1,2 etc claims respectively
 2. Calculate the cumulative probabilities as c_0, c_1, c_2 etc.
 3. Generate a uniform (0,1) random variate, say u .
 4. If the variate $u \leq c_0$, then NB variate is 0. If $c_i < u \leq c_{(i+1)}$, the claim number variate is i .
- b. Or use any other valid method to simulate from NB
- c. Simulation of claim amount will be needed for each claim generated in the above step. For example, no claims if NB generated is 0, 1 claim if NB generate is 1, 2 claim amount variates if NB generated is 2 and so on.

- d. Each claim amount variate should be generated using the method for generating discrete random variates. i.e.,
 1. Generate a uniform (0,1) random variate.
 2. If the variate is >0.8 , the claim amount variate is 100, otherwise 50
- e. Sum up the claim amount variates for the simulation to get the total claim for the simulation.
- f. Repeat the simulation for 10,000 times. Calculate the count of simulations where total claim is less than 200.
- g. The count / 10000 will give an estimate of the required probability.

(4 Marks)

[Total Marks-9]

Solution 8 :

Explain that though it may seem strange policies with exclusions are better than those without them

Exclusions are necessary to accurately define what is and what is not covered. The broader the basic coverage, the more exclusions will be built into the basic policy form. Consider ‘All risk policies’ ,The principle of “all risks” policies is to cover all perils not specifically excluded. Such policies also transfer the responsibility to prove what caused the loss from the insured to the insurer. That means that if no one knows what caused the loss but the property definitely had a loss and was covered on an “all risks” type policy, coverage applies.

Another reason why exclusions are in various policies is because not every customer has the same exposures and it isn't fair to charge someone needlessly for exposures they don't have. There may also be many exclusions when a policy is customized for a particular exposure to be insured. (For example, some cleaners take customer's goods to their location to clean them, but other cleaners never take customer's goods away from their customer's premises. So the Commercial General Liability has what is known as the “care, custody and control” exclusion so that no charge has to be made for an exposure most don't have.)

Another reason could be that policies have exclusions is that some exposures just aren't or shouldn't be insurable.

- For example, war, nuclear reaction and pollution are so serious loss types that most policies don't cover them.
- Additionally, certain types of exposures shouldn't be insured because it works against public policy. For example if someone could insure against faulty workmanship, many businesses wouldn't use care in their work and just turn faulty workmanship claims over to be paid by insurers. Then everyone's insurance would go up and you wouldn't have good quality either.

So as disagreeable as the idea of having exclusions is, they are in and of themselves not bad ideas. Just make sure that you understand what is excluded and why and if you need to buy special coverage for exposure you have, it's probably available under a separate policy.

(6 Marks)

Solution 9 :

- i. Credibility theory is an approach to rate setting that allows for consideration of actual experience as well as external information.

(1 Mark)

- ii. The basic formula for calculating credibility-weighted estimates is:

$$\text{Estimate} = Z \times (\text{Observation}) + (1 - Z) \times (\text{Other information}), 0 \leq Z \leq 1$$
 The question lies in deciding what value to place on Z .
 Z is called the credibility assigned to the observation. $1 - Z$ is generally referred to as the complement of credibility.

(1 Mark)

- iii. The practical considerations are:

- Simplicity -The first is that experience ratings are done frequently and so simplicity is of paramount importance
- visibility –as it is very visible to the individual insured due consideration to what the insured perceives to be fair.
- goodness of fit (ie accuracy versus simplicity)- The success of the application of credibility theory depends upon how closely the model represents reality.
- level of grouping versus accuracy -The problem is that if the data is grouped into too many sub-divisions, there will not be enough data in each group to allow a credible analysis.
- source of data more years / more locations / national data etc-whether the broader base actually applies, ie is the extra data relevant? Has there been a change over time
- stability of data (eg weightings based on numbers, not amounts)- give more weight to more stable phenomena
- use of partial premiums-It will give more weighting to the more stable claim types.
- choice of credibility complement (accuracy / bias / independence from base data / availability / ease of calculation / relationship to risk)
- the need to use considerable judgement when considering how to allow for large claims, trends and differing opinions of the correct rate.

(5 Mark)

[Total Marks-7]

Solution 10 :**i.**

Proportional reinsurance likely to be arranged on a policies-incepting basis.

With proportional reinsurance the direct writer cedes a specified portion of a risk to the reinsurer. It is natural that they provide cover in the defined proportions throughout the duration of the risk. That way both the division of cover into different accounting periods and the dates when any claims are reported become irrelevant.

(2 Marks)**ii.**

- a.** The cedant and reinsurer will have proportionately the same overall underwriting experience on the business included in the treaty, apart from differences in expenses and commission. The reinsurer will therefore be concerned at the outset to establish:
- the nature of the business being offered
 - the cedant's attitude to underwriting and claims settlement
 - any previous experience of this business.

(2 Marks)

b. The measures /conditions are :

- The level of commission written into the treaty is linked/will reflect the expectation of the experience of the business being reinsured
- The reinsurer might also negotiate for part of the commission payment to the cedant to be a profit commission that is payable only if the business ceded meets specified profitability criteria.
- The reinsurer may reserve the right in the treaty to be involved in the approval and settlement process for claims above a certain size.
- Since the capacity of the reinsurer to write such business may be limited, the treaty may specify a limit on the amount of business that may be ceded to the treaty. This limit will normally be expressed in terms of the original gross premium income of the cedant for that business

(4 Marks)

iii. Claim experience:

With surplus reinsurance the cedant can choose which individual risks it wishes to cede and in what proportion. The cedant may retain varying proportions of each risk. Therefore, depending on the claims outcome the cedant and reinsurer will have different outcomes. (The smaller risks may not reach the reinsurer because they are within the retention. The cedant and reinsurer will not therefore share the same portfolio of risks. The risks that are ceded to the reinsurer will vary with the proportion of risk reinsured. Hence the underwriting experience of the risks to which the treaty applies will vary between the two parties).

(1 Mark)**[Total Marks-9]**

Solution 11 :

- i. Let d be each observation's contribution to the deviance defined by:

$$d(Y_i; \mu_i) = 2\omega_i \int_{\mu_i}^{Y_i} \left(\frac{Y_i - t}{V(t)} \right) dt$$

Where ω_i is the weight and $V(\cdot)$ is the variance function.

This compares the observed value Y_i to the fitted value μ_i with allowance for the weights and assigning higher importance to errors where the variance should be small.

Using the above definition of deviance residual, the total deviance for a model is defined as the sum of those:

$$D = \sum_{i=1}^n d(Y_i; \mu_i)$$

Scaled deviance

$$D' = \frac{D}{\phi}, \text{ where } \phi \text{ is the scale parameter.}$$

(2 Marks)

- ii. The scaled deviance is adjusted with the scale parameter ϕ to give a standardised measure of deviance that can be compared to other models.

(1 Mark)

- iii. Matrix for a GLM with Age, Car Type and Interaction term, including an intercept:

Let age Young and car type One be the base value. The following will be parameters to estimate β_1 – Intercept, β_2 – age Medium, β_3 – age Old, β_4 – car type Two, β_5 – interaction for age Medium and car type Two, β_6 – interaction of age Old and car type Two. The following will be the matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{bmatrix}$$

The rows correspond to the following combination of factors:

Row	Age	Car Type
1	Young	One
2	Young	Two
3	Medium	One
4	Medium	Two
5	Old	One
6	Old	Two

(4 Marks)

iv. Model validation ways

a. Residual study

1. Tests
2. Plots
3. Cook's distance and leverage

b. Out of sample test

c. Out of Time test

d. Population stability test

e. Actual vs expected

f. Lift curves

g. Gain curves

(4 Marks)

[Total Marks-11]

Solution 12 :

i.

- a. Claims with higher limits should also be considered.
- b. Even in the claims limited at above 1 Crore, there may be claims below 1 Crore which should be considered.
- c. Claims above 1 Crore should also be considered after capping them at 1 Crore. The reason is that even if the actual claim is above 1 Crore, had the limit been 1 Crore, only 1 Crore would have been paid.

(2 Marks)

ii.

- a. Both the methods will give a correct estimate for the ILF factor.
- b. However, to estimate the average claim at limit 5, only those claims should be considered which could have given a claim upto 5 Crore or higher. Therefore, claims with limit 1 should not be considered.

(2 Marks)

iii.

- a. The claims with limit 1 Crore should not be considered as they can never hit the 4 Crore xs 1 Crore layer.
- b. All claims with limit more than 1 Crore should be considered even if their size was below 1 Crore. This is because all these claims have the 'potential' to be in the layer 4 Crore xs 1 Crore.
- c. Out of the claims with limit more than 1 Crore, if we consider only the claims above 1 Crore, it will be the CONDITIONAL expected value of claims GIVEN they are above 1 Crore.

(2 Marks)

iv.

Calculation Table

Group	Lower	Upper	FGU Amount (A)	Amount Limited at 1 (B)	Amount in layer 1 to 5 (C)	Number for limit 1 (D)	Number for layer 1 to 5 (E)
1	0	1	202.0	202.0	0.0	352	0
2	0	1	75.0	75.0	0.0	300	300
2	1	2.5	225.0	150.0	75.0	150	150
2	2.5	5	270.0	78.0	192.0	78	78
3	0	1	125.0	125.0	0.0	500	500
3	1	2.5	375.0	250.0	125.0	250	250
3	2.5	5	300.0	100.0	200.0	100	100
3	5	10	200.0	30.0	120.0	30	30

B = A if Upper is 1 or D*1 if Upper is > 1

C = A-B is Upper is till 5, (5-1)*D is Upper is 10

D = Number of claims in the summary table

E = number of claims. It is 0 for Group 1 as those claims would not have contributed to losses to layer 4 Xs 1.

Average loss to layer 4 Xs 1 = Sum of C / Sum of E = 712 / 1408 = 0.50682 Crore

(3 Marks)

v.

Average loss for limit 1 = Sum of B / Sum of D = 1010 / 1760 = 0.57386

ILF (5) = (Average with limit 1 + Average for 4 Xs 1) / Average with limit 1

= (0.57386 + 0.50682) / 0.57386 = 1.88

(1 Mark)

vi.

- a. For which market was the ILF created
- b. For what type of risks the ILF was created
- c. Whether the ILF includes ALAE / ULAE or not
- d. Is the ILF on per risk or per event basis

(2 Marks)

[Total Marks-12]

Solution 13 :

(i)

1. All Risk Insurance Cover -:

All Loss or Damage due to theft, fire, storm and other catastrophic risks, electric surges including machinery breakdown/mechanical derangement to Solar Panels and Ancillary Equipment at the premises.

Cover extends to include damage to Panels and Ancillary Equipment while being temporarily removed from the premises for cleaning, renovation, repair or other similar purposes and while in transit by road rail or inland waterway.

2. Business Interruption Insurance/ Loss of Revenue Insurance: Business interruption insurance protects against the financial consequences of loss of or damage to the insured property. The cover specifically covers loss of revenue from income streams. (If the solar power system is unable to feed electricity due to physical damage, the business interruption insurance covers revenues lost on power feed-in for the duration of the breakdown)
3. Reduced Yield Insurance- Cover against sun irradiation .Transfers the risk of being short on power to an insurance company. Very simply it is insuring the output of a solar farm. (Reduced yield insurance is a unique type of extended cover. This comes into force if the annual energy yield predicted in the yield forecast is more than x% below the forecasted amount due to lower insolation.)
4. Panel Warranty Insurance- Provides solar panel replacement coverage, is a Contractual Liability Insurance Product (CLIP) coverage and provides equipment protection that is designed to replace or wrap around the manufacturer's warranty
5. Insurance that protects solar park operators in the event that their panel provider goes bankrupt. The insurance in effect picks up where the manufacturer's warranty leaves off after bankruptcy voids it.

6. Environmental Risk Insurance: Environmental damage coverage indemnifies solar system owners of the risk of either environmental damage done by their development on the development site.

(5 Marks)

ii.

- Reinsurer or broker data
- Competitor rates for similar solar parks
- Aggregate market statistics (if these are available)
- Industry/scientific studies e.g. by environmental groups, energy institutes etc
- Research reports
- Publicly available data curves
- Catastrophe model vendors

(2 Marks)

iii.

External data may be:

- out of date
- of poor quality
- not detailed enough
- not representative of the type of business you intend to price
- expensive
- sparse
- not developed, as the cover is new and fast-evolving

There will be heterogeneity due to:

- companies operate in different geographical or socio-economic sections
- of the market- data coming from solar parks in different countries, each with different exposure to storms, earthquakes etc.
- the policies sold by different companies are not identical- differing levels of covers or different limits, deductibles, excesses
- the companies will have different practices; for example, underwriting, claim settlement and outstanding claim reserving policies making it difficult to compare claims amounts
- Different insurers are likely to have different reserving philosophies
- Different loadings for expenses and profit in different insurer's premiums
- Data stored by different companies may not always be at the level of granularity desired
- inconsistent coding of data - the coding used for the risk factors may vary from company to company

(4 Marks)

iv.

Rating factors:

Solar Panels:

- specification of the panels
- Structural implications - mounting surface

- ease of access for repairs e.g. is it necessary to build a road big enough to take a crane before repairs can be carried out
- manufacturer of the solar panel-may affect quality, may have guarantees that kick in before the insurance
- model of panels
- age of the panel-this will also reflect the level of technology (which is changing fast)
- plans for upgrading panels in the future
- safety features/procedures within the panels

Solar park:

- territory in which the solar parks is located
 - as this affects the exposure to catastrophic risks
 - also affects legal environment for liability claims
 - and exchange rates and local inflation rates will affect the cost of repair
 - may affect possible compensation claims eg proximity to a highly populated area
 - risk of theft
- size of plant/number of panels covered
- expansion plans in the future
- value of plant (sum insured/ EML)

Solar park operations

- Quality of management of the operator– levels of monitoring, frequency and quality of maintenance and servicing procedures, staff training
- Security of the site– affects theft & vandalism & liability
- Turnover/profit– for business interruption cover
- Size of workforce/payroll– relevant to employers' liability

Premium and Claim History

- Premium by relevant parameters
- Numbers of losses (whether claimed for or not – may give an idea of the likelihood of future losses, together with any actions taken to prevent similar claim events happening in the future)
- Cause/peril/type of losses for each one
- Exposure details to match claims history
- rating factor details to match past claims– since upgradations may have taken place meaning that past risks may no longer be likely
- dates
- claim status
- amounts & estimates
- currency

(7 Marks)

[Total Marks-18]
