Institute of Actuaries of India

Subject ST7 – General Insurance : Reserving & Capital Modelling

November 2013 Examinations

INDICATIVE SOLUTIONS

Solution 1 :

a) Equity returns too volatile – too risky for insurance business
b) Become more volatile in uncertain economic conditions, like high-inflation scenario
c) Need to also match by term, currency, also
d) Regulation may not allow
e) Inflation-linked bonds might be better
f) May invest a lower proportion in equities and a higher proportion in inflation-linked bonds

(3 Marks)

Solution 2 :

Data can vary between classes for a number of different reasons, principally due to the different nature of risk which leads to the following:

- big variations in claim frequency between classes
- the length of the tail of some classes means that it takes considerable time to collect the necessary claims data. This is particularly true of classes that are subject to significant delays in claim notification.
- for some classes the underwriting is subjective, so it is difficult to store the details of the risk.The range and volume of data held will be greater for a class where underwriting has been largely based on statistics than for a class where the underwriting has been largely subjective

(4 Marks)

Solution 3 :

At the end of the three-year period, the managing agent would usually close the fund by estimating the value of the outstanding liabilities and reinsuring them into the subsequent open year of that syndicate. The reinsurance premium paid for this is known as reinsurance to close (RITC).

The RITC premium should cover all of the remaining liabilities of the syndicate that is closing. This includes liabilities from previous years' syndicates that had been reinsured into that syndicate. It will include amounts in respect of

- outstandings: all claims notified to the syndicate that have not yet been paid, or have not yet been quantified
- IBNR: claims incurred but not reported - an estimate of the claims that the syndicate believes that it is likely to receive in the future
- claims handling (that is, administrative) costs.

(4 Marks)

Solution 4 :

The Actuary is correct in using a stochastic method to come up with the 75^{th} percentiles for each line of business.
The implicit assumption behind adding the 75^{th} percentile is that the two lines of business are highly positively correlated (perfect rank correlation).
This is unreasonable as Motor and Health portfolio claims are driven by different causes
This will lead to overestimation of 75^{th} percentile of the aggregate distribution
Better approach will be to produce the aggregate distribution of the two lines
Analytical methods for aggregation are available but are usually difficult
The dependency can be modelled using copulas and correlation matrix
The user must specify:

1. Underlying loss distributions for the classes of business;
2. A two-way correlation matrix between all distributions; and
3. The form of the copula.

The copula approach maintains rank correlation.
The simplest form of copula is the Gaussian (normal) copula. This is often criticised for not giving enough dependency in the tail, hence failing to model extreme events. Other copulas such as the Gumbel copula and t-copula remedy this.

(5 marks)

Solution 5 :
Economic Scenario Generator is a model that generates values for economic variables (such as inflation, gilt yields and equity returns). It defines the forms the variables may take and the relationships between them.

ESG can be a very complex model, the building, parameterisation and running of which is often outsourced.

The ESG will give a joint probability distribution of outcomes for the economic variables (for example, equity returns, yield curve shifts, credit spread shifts, credit defaults and so on) and a point is chosen from the distribution that reflects the desired confidence level. The point will have been generated by a particular scenario.

Uses in ALM
The different economic scenarios used as inputs to an ALM may be obtained using an ESG
An ESG will typically take the form of a specialised asset model that stochastically models the performance and interactions of various asset classes.
The output of this model will be the performance for each economic variable (for example, inflation / asset classes) at each future projection point, for several simulations.

This table of simulation outputs will then be used within the main ALM as if the ESG was a part of the ALM.

(5 Marks)

Solution 6 :
The following measures could be considered:

- making the policy wording as tight as possible, and reviewing it regularly in the light of market and judicial changes
- devising contracts which minimise the risks, eg minimum indexed sums insured for household contents policies. Excess and deductibles could also reduce the chances of fraud
- working together with others to try to identify and punish persistent offenders
- random spot checks on claims, even smaller ones
- having repairs done by a small number of approved firms (rather than at the choice of the claimant)
- insisting on the police being involved before paying out on a theft claim
- publicity to advise against it, eg “It’s a crime to ...” and “Look what happened to this fraudster ...”.
- Analytical techniques / preditive analytics to detect fraud.

(5 marks)

Solution 7 :

- Customers are clearly explained the policy terms and conditions on how premiums could get adjusted over time to reflect changes in driver risk profiles and how their driving behavior could impact premiums
- More complex and large volumes of data – database management / IT system
- Complexity indata analysis for decision making
- Operational costs and risks. In other words, a cost-benefit analysis of gains through better risk estimation vs. cost from the perspective of both the insurer and the customer.
- Data protection - customers need to trust insurers to treat them fairly and protect their personal information. Any other regulatory requirements.

(5 Marks)

Solution 8 :

(i) Correlation is a measure of dependency; commonly used 'Pearson Correlation coefficient quantifies the extent of linear relationship between two random variables.

Dependency between two random variables means that there is some link between them, i.e. the random variables are not entirely independent.

Perfect dependence does not necessarily mean a correlation of 1/-1 (for ex., X and Y=X^2 where X~Z(0,1) have a correlation of 0)

Also, correlation does not imply causation which means that a correlation between two variables does not necessarily imply that one causes the other or any dependence.

(2 Marks)

(ii) Advantages of Copulas over Variance-Covariance approach:

- More flexibile than the use of a variance-covariance matrix
- Easily simulated using Monte-Carlo methods
- Possible to allow for non-linearities and other higher order dependencies
- Enable user to build models that better reflect reality e.g. heavy tails
- Range of different copulas can be used

(2 Marks)

(iii) (a) Independent
(b) Perfect positive dependence
(c) Perfect negative dependence

(3 Marks)

Solution 9 :

(i) Solvency margin of an Insurance company can be improved by:

- Increasing the value of assets
- Decreasing the value of liabilities
- Decreasing the regulatory minimum difference between assets and liabilities

Increasing the value of assets

Where a block of renewing business which is producing regular profits is identified, capital can be found for the insurer to bolster the free asset position by reinsuring this portfolio of profitable policies. The reinsurer would pay an initial commission in return for which the reinsurer would be entitled to the future surplus of premiums over claims for as long as the arrangement remained in place.
The arrangement mentioned here is identical to that described above except that it's for a different reason. The reinsurer still loans some money to the insurer and the loan is repaid out of future profits. Because the insurer has no liability to repay the loan unless a surplus has been made, it does not have to reserve for the future payments. So it has increased its assets by the amount of the loan but not increased its liabilities and hence has improved its free asset position.
However, the extent to which this is possible will depend on the precise requirements of the supervisory regime concerned.

Decreasing the value of the liabilities
By reinsuring the business, the insurer is reducing the value of its liabilities (as some if its liabilities are ceded to the reinsurer). Therefore, as a result of the reinsurance, it will hold smaller reserves, and so the solvency position of the insurer will improve, although of course assets will be reduced by the size of the reinsurance premiums paid.

Decreasing the regulatory minimum difference
The required solvency level is often calculated with reference to the proportion of business reinsured. In other words, more reinsurance means a lower solvency requirement, and therefore a stronger solvency position. However, this reduction may be subject to a limit, since there may be a legal requirement for an insurance company's free reserves to exceed a Required Minimum Margin (RMM).

(3 Marks)

ii) a) The two possible reinsurance options are:

1. Excess of Loss reinsurance with an attachment at 50,000 and a limit of 4.5 lakhs.
2. Surplus insurance with a maximum retention of 50,000. So, in this case the percentage ceded to the reinsurer for each risk will be:
    - Building 1: 50%
    - Building 2: 75%
    - Building 3: 83%
    - Building 4: 88%
    - Building 5: 90%

Since each loss will be split in these proportions, the reinsurer will have more payments and hence the Surplus reinsurance will cost more. **(3 Marks)**

b) Since the XOL premiums will depend on distribution of losses etc, it's easier to calculate the premium for the Surplus lines as it is based on the proportion ceded to the reinsurer. Here are the calculations:

Sum-insured	Premium	Reinsurance Proportion	Reinsurance Premium
1,00,000	5,000	50%	2,500
2,00,000	10,000	75%	7,500
3,00,000	15,000	83%	12,500
4,00,000	20,000	88%	17,500
5,00,000	25,000	90%	22,500

Total Reinsurance Premium: 62,500 **(2 Marks)**

[Total Marks-8]

Solution 10 :

Since all of them are 3-month policies, assuming that the policies are written uniformly over the month, we can create the following table:

Month	Premium written	Premium Earned	Loss Ratio	Incurred Losses	Commissions	UPR	Other Expenses	DAC	Paid Claims	OSLR as of 31/12/13	IBNR as of 31/12/13
1	50,000	8,333	55%	4,583					4,583	-	-
2	50,000	25,000	55%	13,750					13,750	-	-
3	50,000	41,667	55%	22,917					22,917	-	-
4	50,000	50,000	55%	27,500					27,500	-	-
5	50,000	50,000	55%	27,500					27,500	-	-
6	50,000	50,000	55%	27,500					27,500	-	-
7	50,000	50,000	60%	30,000					30,000	-	-
8	50,000	50,000	60%	30,000					30,000	-	-
9	50,000	50,000	60%	30,000					30,000	-	-
10	50,000	50,000	60%	30,000					30,000	-	-
11	50,000	50,000	60%	30,000					15,000	15,000	-
12	50,000	50,000	60%	30,000					-	15,000	15,000
Total	**6,00,000**	**5,25,000**		**3,03,750**	**1,80,000**	**75,000**	**50,00,000**	**22,500**	**2,58,750**	**30,000**	**15,000**

We made the following simplifying assumption that the exchange rate for payments is based on the time of occurrence of the loss. Another reasonable assumption will be that the exchange rate is based on time of loss payments. In that case, 50% of incurred losses for May and 100% of incurred losses for June will have to be adjusted for the exchange rate change.

Profit & Loss Account	
Premiums Earned	5,25,000
Claims Incurred	-3,03,750
Expenses Paid	-51,80,000
Increase in DAC	22,500
Underwriting Profit / (Loss)	-49,36,250
Investment income	50,00,000
Total Income	63,750
Tax	-19,125
Net Profit	44,625

Balance Sheet at 31/12/13	
Assets at 1/1/13	50,00,00,000
Premiums Written	6,00,000
Claims Paid	-2,58,750
Expenses Paid	-51,80,000
Investment Income	50,00,000
Taxes Paid	-19,125
Assets at 31/12/13	50,01,42,125
Liabilities	
Unearned Premium Reserve	75,000
DAC	-22,500
OS Reported	30,000
IBNR	15,000
Total Liabilities	97,500
Shareholders' Funds	50,00,44,625
Increase in Shareholders' Funds	44,625

(9 Marks)

Solution 11 :

(i) $EPD = Pr(X > C) * E[X - C \mid X > C]$

$= Pr(X > VaR_{@}) * E[X - VaR_{@} \mid X > VaR_{@}]$

$= (1 - @) * \{ E[X \mid X > VaR_{@}] - E[VaR_{@} \mid X > VaR_{@}] \}$

$= (1 - @) * [TVaR_{@} - VaR_{@}]$ **(4 Marks)**

(ii) Shareholders are interested in the value of the business as a going concern. They are potentially concerned with any scenario wherein the actual loss exceeds the expectations. They may be concerned about overall profitability rather than the insolvency risk only. In the case of insolvency, they are unconcerned about how bad any resulting policyholders shortfalls may be.

From policyholder's perspective, the only risk that matters is insurer insolvency. Policyholders are also interested in the extent of insolvency; how bad any policyholder shortfalls may be. Events that do not threaten ruin are of little interest to them.

Policyholders – probability of ruin, VaR, TvaR, EPD etc
Shareholders – beta from the CAPM model specific to the insurer, risk-adjusted return on capital etc

(6 Marks)
[Total Marks-10]

Solution 12 :

i. In NCD, the premium adjustment is made to the premium of the renewing policy and not the current policy. **(1 Mark)**

ii. Retrospective rating plans are more appropriate for employers;

- Where there is uncertainty in the number of employees; although the retrospective adjustments in premiums due to actual exposure being different from declared is common in some policies with or without the premium adjustments for loss experience.
- With large workers compensation premium;
- With proven safety practices, better than average claim experience
- Who are stable and established;
- Who are financially stable and sound;
- Who have experienced some claim frequency;
- Who have valid, consistent claim data available for analysis;
- Who have had better than average claim experience;

(5 Marks)

iii. Advantages:

- Could be less expensive option for securing workers compensation coverage;
- Flexible rating options and plan design
- Create strong loss control incentive
- Provide an excellent cash flow possibility

Disadvantages:

- Could be an expensive option if alternatives are limited or if loss experience is poor during the retro period
- The functions of the plan are not understood, lack of understanding how the adjustments work and creates a problem for an employer's accounting and budgeting
- Uncertain annual cost of risk fluctuations
- Premium adjustments could occur over several years
- Poor handling of claims or case reserving practices can cause higher cost

(4 Marks)

iv. Factors to consider in economic capital modeling:

- Reserve Risk: Loss variability reduces due to offsetting movements in adjustment premiums, Expense Risk associated with unpaid claims
- Underwriting Risk: Loss variability reduces, Expense Risk, Premium volume uncertainty
- Credit Risk: Insured may go bankrupt and may not pay the adjustment premiums
- Reputation Risk: Possible litigations by insured on additional premium calculations
- Operational Risk: More complex than usual, especially for the IT system

(5 Marks)
[Total Marks-15]

Solution 13 :

i. Recreating the Actuarial Analyst's calculations:

Incremental Paid

AY	12	24	36	48	60
2008	100	100	100	30	10
2009	300	2,100	100	30	
2010	100	100	130		
2011	200	30			
2012	200				

Cumulative Paid

AY	12	24	36	48	60
2008	100	200	300	330	340
2009	300	2,400	2,500	2,530	
2010	100	200	330		
2011	200	230			
2012	200				

Link Ratios

AY	12-24	24-36	36-48	48-60
2008	2.000	1.500	1.100	1.030
2009	8.000	1.042	1.012	
2010	2.000	1.650		
2011	1.150			

Average	3.288	1.397	1.056	1.030
Cumulative	4.998	1.520	1.088	1.030

	Paid		Ultimate		
AY	**@31/12/12**	**LDF**	**Losses**		
2008	340	1.000	340		
2009	2,530	1.030	2,607		
2010	330	1.088	359		
2011	230	1.520	350		
2012	200	4.998	1,000		
				OSLR	**IBNR**
Total	3,630		4,655	500	525

(4 Marks)

ii. Peer Review Points:

- **a.** Used only one method. Bornhuetter-Ferguson, at least, should be tried given this is a long-tailed line with significant uncertainties
- **b.** Used only paid losses. Should try to get incurred losses and do Chain-ladder and Bornhuetter-Ferguson on that
- **c.** Large loss needs to be removed from triangle before analysis and treated separately – it can bias the development factors and the calculated ultimate loss

d. A tail factor should be used, as we can see development of losses till 60 months and this is a long-tailed line.

e. The loss payments seem to be going up in the last 2 years. Is this an increase in claim size or just a quickening of claim payments? If it's the latter, then a Berquist-Sherman adjustment is required to calculate IBNR in an unbiased manner

(4 Marks)

iii. Additional data will be:

a. Incurred loss triangle or additional data that lets us create one

b. Incurred movement on large loss

c. Additional information on large loss

d. Exposure data (earned premium, earned car years)

e. Data to analyse speed of claim closure

(2 Marks)

iv. Revised Calculation:

Assumptions:

- The larger payments in 2011-2012 is due to large-sized claims and not faster payment or there is no change in speed of payment of claims
- There is no large IBNR / Reported but not paid claim in the recent Accident Years
- The exposure size has remained constant, say Rs 500 EP in each Accident Year (possible assumption to do a Bornhuetter-Ferguson) – not used in the example below
- Reasonable assumptions on value of tail factor or method of calculating tail factor.

Methodology:

a. Remove large loss and use Paid Chain-ladder on attritional losses

b. No need to add additional reserves for Large Loss, as the large claim is closed

c. Look at 2-3 different averages of link ratios (weighted average, last 3-year average, etc.) and select appropriately

d. Use a tail factor. I used a decay of 70% to calculate a tail of 1.072 [=1.03^(0.7/(1-0.3))]

Incremental Paid

AY	12	24	36	48	60
2008	100	100	100	30	10
2009	100	100	100	30	
2010	100	100	130		
2011	200	30			
2012	200				

Cumulative Paid

AY	12	24	36	48	60
2008	100	200	300	330	340
2009	100	200	300	330	
2010	100	200	330		
2011	200	230			
2012	200				

Link Ratios

AY	12-24	24-36	36-48	48-60
2008	2.000	1.500	1.100	1.030
2009	2.000	1.500	1.100	
2010	2.000	1.650		
2011	1.150			

Average	1.788	1.550	1.100	1.030	**Tail**
Cumulative	3.367	1.883	1.215	1.105	1.072

AY	Paid @31/12/12	LDF	Ultimate Losses		
2008	340	1.000	340		
2009	330	1.105	365		
2010	330	1.215	401		
2011	230	1.883	433		
2012	200	3.367	673		
				OSLR	**IBNR**
Total	1,430		2,212	500	282

Since the Large loss is closed, we do not need any additional IBNER for that. So, the total estimated IBNR is 282.

(10 Marks)
[Total Marks-20]

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