# Institute of Actuaries of India

# Subject CT5 – General Insurance, Life and Health Contingencies

# Nov 2013 Examinations

# INDICATIVE SOLUTIONS

**Introduction**

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

**Solution 1 :**

**(i)** $A_{30:\overline{25|}} = A^{1}_{30:\overline{25|}} + v^{25} \times {}_{25}p_{30}$

$= (0.16023 - (1105.41 \div 3060.13 \times 0.38950)) + (1105.41 \div 3060.13)$

$= 0.38076$

**(ii)** $\ddot{a}_{60}^{(2)} = \ddot{a}_{60} - ¼ = 13.884$

**(iii)** $(Ia)_{50} = (I\ddot{a})_{50} - \ddot{a}_{50} = 231.007 - 17.444 = 213.563$

**(3 Marks)**

**Solution 2 :**

**(i)** The two methods are:

**a)** Uniform distribution of deaths

**b)** Constant force of mortality

**(2 Marks)**

**(ii)** The survival probability is given by:

${}_{2}p_{63.25} = {}_{0.75}p_{63.25} \times {}_{1}p_{64} \times {}_{0.25}p_{65}$

**a)** Uniform distribution of deaths:

${}_{0.75}p_{63.25} = {}_{1}p_{63} \div {}_{0.25}p_{63} = 0.98519$

${}_{1}p_{64} = 1 - {}_{1}q_{64} = 1-0.02199 = 0.97801$

${}_{0.25}p_{65} = 1 - {}_{0.25}q_{65} = 1-0.25 \times 0.02447 = 0.9938825$

Hence, ${}_{2}p_{63.25} = 0.95763$

**b)** Constant force of mortality:

${}_{0.75}p_{63.25} = ({}_{1}p_{63})^{0.75} = (1-0.01965)^{0.75} = 0.98523$

${}_{1}p_{64} = 1 - {}_{1}q_{64} = 1-0.02199 = 0.97801$

${}_{0.25}p_{65} = (p_{65})^{0.25} = (1-0.02447)^{0.25} = 0.99383$

Hence, ${}_{2}p_{63.25} = 0.95762$

**(4 Marks)**

**[Total Marks-6]**

**Solution 3 :**

1. Insurance works on the basis of pooling independent homogeneous risks.
2. The central limit theorem then implies that profit can be defined as a random variable having a normal distribution.
3. This result can be used to set premium rates that ensure that the probability of loss on a portfolio of policies is at an acceptable level.
4. Life insurance risks are usually independent.
5. Risk classification ensures that the risks are homogeneous.
6. Lives are divided by risk factors.
7. Examples of risk factors are age, sex, medical history, height, weight, lifestyle.
8. More factors imply better homogeneity.
9. But the collection of more factors is implied by
    a. Cost of obtaining data
    b. Problems with accuracy of information
    c. The significance of the factors
    d. The desires of the marketing department
10. In practice, rating factors will be included if they avoid any possibility of selection against the company, subject to time and cost constraints.
11. The decision to introduce rating factors is often driven by competitive pressures.

**(7 Marks)**

**Solution 4 :**

Given that:

1. $q_{[50]} = 0.012$
2. ${}_2p_{[50]} = 0.97$
3. ${}_{2|1}q_{[50]} = 0.025$
4. ${}_{2|3}q_{[50]+1} = 0.075$

From (1), $p_{[50]} = 1-q_{[50]} = 1-0.012 = 0.988$

From (2), ${}_2p_{[50]} = {}_1p_{[50]} \times {}_1p_{[50]+1}$
Hence, $0.97 = 0.988 \times {}_1p_{[50]+1}$
Hence, ${}_1p_{[50]+1} = 0.981781$

From (3), ${}_{2|1}q_{[50]} = {}_2p_{[50]} \times q_{[50]+2}$
Hence, $0.025 = 0.97 \times q_{[50]+2}$
Hence, $q_{[50]+2} = 0.025773$

From (4), ${}_{2|3}q_{[50]+1} = {}_2p_{[50]+1} \times {}_3q_{53}$
Hence, $0.075 = (0.981781) \times (1-0.025773) \times {}_3q_{53}$
Hence, ${}_3q_{53} = 0.078413$

Thus ${}_3p_{53} = 1- 0.078413 = 0.921587$ **(4 Marks)**

**Solution 5 :**

The unit fund is the amount held in units on behalf of the policyholder at any time. It may not necessarily be the amount that the policyholder is entitled to at that time.

For example, if the policy is surrendered, the policyholder may receive only a proportion of the full bid value of the units.
On death, maturity or surrender, the units held will be used to pay the benefit. Any excess/shortfall in the unit fund will give rise to a positive/negative cash flow in the non-unit fund.
The non-unit fund is the net result of the life office's cash flows.
These will arise from the following sources:
- premium less cost of allocation, *ie* the difference between the premium paid by the policyholder and the amount invested in the unit fund on the policyholder's behalf
- expenses incurred by the life office
- interest earned/charged on the non-unit fund
- management charges taken from the unit fund
- extra death or maturity costs (if the benefit payable on death or maturity is greater than the value of the units held at the time of death or maturity)
- profit on surrender (if the benefit payable on surrender is less than the value of the units held at the time of surrender)

**(4 Marks)**

**Solution 6 :**

Let P be the annual premium.

Expected Value of premium = P × $(\ddot{a})_{[40]:\overline{25|}}$ = 15.887P

Expected Value of maturity benefit = 25,000 × ($D_{65}/D_{[40]}$) = 8,394.842

Expected Value of death benefit = 2P × $(IA)^{1}_{[40]:\overline{25|}}$

= 2P × { $(IA)_{[40]}$ - ($D_{65}/D_{[40]}$) ×[ $(IA)_{65}$ + 25$A_{65}$]}

= 2P × { 7.95835- (689.23/2052.54) ×[7.89442 + 25 × 0.52786]}

= 1.752304P

So the equation of value is given by:

15.887P = 8,394.842+ 1.752304P
Hence, P = Rs. 593.92

**(5 Marks)**

**Solution 7 :**

**(i)** The expected present value of the past service benefit is:

$$= \frac{15}{100} \times 4,000,000 \times \frac{{}^{z}M_{40}^{ia} + {}^{z}M_{40}^{ra}}{s_{39}D_{40}}$$

$$= \frac{15}{100} \times 4,000,000 \times \frac{58,094 + 128,026}{(7.623)(3,207)}$$

= 4,567,929.73

The expected present value of the future service benefit is:

$$= \frac{1}{100} \times 4,000,000 \left[ \frac{{}^{z}\overline{R}_{40}^{ia} + {}^{z}\overline{R}_{40}^{ra}}{s_{39}D_{40}} \right]$$

$$= \frac{1}{100} \times 4,000,000 \times \frac{887,117 + 2,884,260}{(7.623)(3,207)}$$

= 6,170,708.91

The expected present value of the total pension benefit is therefore Rs. 10,738,638.64

**(4 Marks)**

**(ii)** Let *k* be the required contribution rate (as a percentage of salary). Then:

$$4,000,000k \frac{{}^{s}\overline{N}_{40}}{s_{39}D_{40}} = 6,170,708.91$$

$$\Rightarrow 4,000,000k \frac{363,573}{(7.623)(3,207)} = 6,170,708.91$$

$$\Rightarrow k = 10.37\%$$

**(2 Marks)**

**[Total Marks-6]**

**Solution 8 :**

**(i)** The prospective reserves are calculated as the expected present value of future outgo less expected present value of future income.

The retrospective reserves are calculated as the accumulated value allowing for interest and survivorship of the premiums received to date less the accumulated value allowing for interest and survivorship of the benefits and expenses paid to date.

The conditions for their equality are:

A. the retrospective and prospective reserves are calculated on the same basis
B. this basis is the same as the basis used to calculate the premiums used in the reserve calculation

**(4 Marks)**

**(ii)** Prospective reserve is given by = $A_{x+t} - P_x\ddot{a}_{x+t}$------------------------------------(1)

Retrospective reserve is given by = $P_x \times \ddot{s}_{x:\bar{t}|} - \frac{(1+i)^t}{{}_tp_x} A^1_{x:\bar{t}|}$-------------------------(2)

We know that

${}_{t|}\ddot{a}_x = {}_tp_x \times v^t \times \ddot{a}_{x+t}$--------------------------------------------------------------------(3)

and $\ddot{s}_{x:\bar{t}|} = \ddot{a}_{x+t} \times \frac{(1+i)^t}{{}_tp_x}$--------------------------------------------------------------------(4)

Now we will start with equation (1),

$= A_{x+t} - P_x\ddot{a}_{x+t}$

$= A_{x+t} - P_x\ddot{a}_{x+t} - P_x \times \ddot{s}_{x:\bar{t}|} + P_x \times \ddot{s}_{x:\bar{t}|}$

$= A_{x+t} - P_x(\ddot{a}_{x+t} + \ddot{s}_{x:\bar{t}|}) + P_x \times \ddot{s}_{x:\bar{t}|}$

$= A_{x+t} - P_x( {}_{t|}\ddot{a}_x \times \frac{(1+i)^t}{{}_tp_x} + \ddot{a}_{x+t} \times \frac{(1+i)^t}{{}_tp_x}) + P_x \times \ddot{s}_{x:\bar{t}|}$----------------------using (3) and (4)

$= A_{x+t} - P_x * \frac{(1+i)^t}{{}_tp_x} * ( {}_{t|}\ddot{a}_x + \ddot{a}_{x+t}) + P_x \times \ddot{s}_{x:\bar{t}|}$

$= A_{x+t} - P_x * \frac{(1+i)^t}{{}_tp_x} * \ddot{a}_x + P_x \times \ddot{s}_{x:\bar{t}|}$

$= A_{x+t} - A_x * \frac{(1+i)^t}{{}_tp_x} + P_x \times \ddot{s}_{x:\bar{t}|}$----------------------------------------------since $A_x = P_x * \ddot{a}_x$

$= P_x \times \ddot{s}_{x:\bar{t}|} - \frac{(1+i)^t}{{}_tp_x} A^1_{x:\bar{t}|}$

**(4 Marks)**

**(iii)** Let P be the monthly premium.
Let S denote the sum assured which is 100,000.

EPV premium = $12P\ddot{a}^{(12)}_{[40]:\overline{20|}}$

EPV commission = 0.75 × 12P + 0.025 × $12P\ddot{a}^{(12)}_{[40]:\overline{20|}}$

EPV expenses = 500 + 100 × ($\ddot{a}_{[40]}$ - 1) + $200\bar{A}_{[40]}$

EPV death benefits = 0.95 × S × $\bar{A}_{[40]}$ + 0.05 × S × $I\bar{A}_{[40]}$

Where

$\ddot{a}^{(12)}_{[40]:\overline{20|}} = (\ddot{a}_{[40]} - \frac{11}{24}) - \frac{D60}{D[40]} \times (\ddot{a}_{60} - \frac{11}{24})$

$= (20.009 - \frac{11}{24}) - \frac{882.85}{2052.54} \times (14.134 - \frac{11}{24}) = 19.551 - 5.8823 = 13.6687$

$\bar{A}_{[40]} = (1.04)^{0.5} \times 0.23041 = 0.23497$

$I\bar{A}_{[40]} = (1.04)^{0.5} \times 7.95835 = 8.11596$

Hence,

EPV premium = 164.0244P

EPV commission = 13.1006P

EPV Expenses = 500 + 100 × (20.009- 1) + 200 × 0.23497 = 2447.894

EPV Death Benefits = 0.95 × S × 0.23497 + 0.05 × S × 8.11596 = 22,322.15 + 40,579.8 = 62,901.95

Since,

EPV premium = EPV commission + EPV Expenses + EPV Death Benefits
164.0244P = 13.1006P + 2,447.894 + 62,901.95
P = 65,349.844/150.9238 = Rs. 432.999 **(5 Marks)**

**(iv)** Reserve at the end of 20th year is given by

= EPV benefits + EPV expenses + EPV commission – EPV premiums

(Here, premium and commission payouts are zero post 20 years)

EPV benefits (including accumulated bonuses at 5%)

$= 0.96 \times S \times \bar{A}_{60} + 0.04 \times S \times I\bar{A}_{60} + 20 \times 0.05 \times 100{,}000 \times \bar{A}_{60}$

$= 0.96 \times S \times (1.04)^{0.5} \times 0.45640 + 0.04 \times S \times (1.04)^{0.5} \times 8.36234 + 46{,}543.8501$

= 44,682.0961 + 34,111.7879 + 46,543.8501

= 125,337.7341

EPV expenses

$= 200\bar{A}_{60} + 100\ddot{a}_{60}$

$= 200 \times 0.45640 \times (1.04)^{0.5} + 100 \times 14.134 = 1{,}506.4877$

Reserve at the end of 20th year = 125,337.7341 + 1,506.4877 = Rs. 126,844.2218

**(4 Marks)**

**[Total Marks-17]**

**Solution 9 :**

**(i)**

**<u>For Endowment Assurance:</u>**

Annual premium is calculated as:

$= 100{,}000\ (A)_{[45]:\overline{20|}} / (\ddot{a})_{[45]:\overline{20|}}$

= 100,000 × 0.46982/13.785 = 3,408.1973

At the end of 2012, reserve is calculated as:

$= 100{,}000\ (A)_{53:\overline{12|}} - P(\ddot{a})_{53:\overline{12|}}$

= 100,000 × 0.63460 – 3,408.1973 × 9.5

= 31,082.1256

Death strain at risk = $S - {}_{t+1}V$ = 100,000 – 31,082.1256 = 68,917.8745

**<u>For Pure Endowment Assurance:</u>**

Annual premium is calculated as:

$= 75{,}000 \times (D60/D[45]) / (\ddot{a})_{[45]:\overline{15|}}$

= 75,000 × (882.85/1677.42) / 11.390

= 3,465.6336

At the end of 2012, reserve is calculated as:

$= 75{,}000\times \left(\frac{D60}{D53}\right) - P(\ddot{a})_{53:\overline{7}|}$

$= 75{,}000 \times (882.85/1204.65) – 3465.6336 \times 6.166$

$= 33{,}596.0384$

Death strain at risk = $S - {}_{t+1}V$ = 0 – 33,596.0384 = – 33,596.0384

**For temporary immediate annuity:**

At the end of 2012, reserve is calculated as:

$= 15{,}000 \times (\ddot{a})_{53:\overline{12}|}$

$= 15{,}000 \times 9.5$

$= 142{,}500$

Death strain at risk = $S - {}_{t+1}V$ = 0 – 142,500 = -142,500

Total Death strain at risk during 2012 =68,917.8745 – 33,596.0384 -142,500
= Rs. -107,178.1639

**(6 Marks)**

**(ii)**

**For Endowment Assurance:**

ADS = 68,917.8745 × 12 = 827,014.4940

EDS = (5000-56) × q52 × 68,917.8745
= (5000-56) × 0.003152 × 68,917.8745
= 1,073,980.870

Mortality Profit = EDS – ADS = 246,966.38

**For Pure Endowment Assurance:**

ADS = – 33,596.0384 × 7 = -235,172.2689

EDS = (3000-43) × q52 × (– 33,596.0384)
= (3000-43) × 0.003152 × (– 33,596.038416)
= -313,130.6665

Mortality Profit = EDS – ADS = -77,958.40

**For temporary immediate annuity:**

ADS = -142,500 × 6 = -855,000

EDS = (2500-30) × q52 × (-142,500)
= (2500-30) × 0.003152 × (-142,500)
= -1,109,425.2

Mortality Profit = EDS – ADS = -254,425.20

Total Mortality profit = 246,966.38 -77,958.40 -254,425.20 = Rs. -85,417.22

**(6 Marks)**

**(iii)**

| **Line of Business** | **Actual number of deaths in 2012** | **Expected number of deaths in 2012** |
|---|---|---|
| | | |
| Endowment Assurance | 12 | 4944×q52 = 15.5835 |
| Pure Endowment Assurance | 7 | 2957×q52 = 9.3205 |
| Temporary Immediate Annuity | 6 | 2470×q52 = 7.7854 |

[1.5 marks for the above 3 values for expected number of deaths, no marks for actual number of deaths]

In all the three line of business, actual numbers of deaths are lower than expected number of deaths. However, mortality profit has arisen only in endowment assurance business. Pure endowment and annuity business are showing mortality loss.

The reason is that in endowment assurance business, higher than expected no. of deaths is a risk since death benefit becomes payable. While in other two lines of businesses lower than expected no. of deaths is a risk since maturity benefit or annuity benefits are to be payable to more number of policyholders.

**(3 Marks)**
**[Total Marks-15]**

**Solution 10 :**

**(i)** First, note that at points outside the first quadrant, the d.f. will be 0.

Next, divide the first quadrant in the following four regions:

Region 1: $0 < s \le 10, 0 < t \le 10$

Region 2: $0 < s \le 10, t > 10$

Region 3: $s > 10, t > 10$

Region 4: $s > 10, 0 < t \le 10$

The d.f. at a point in Region 1 where both s and t are between 0 and 10 is:

$$F_{T_xT_y}(s,t) = P(T_x \le s, T_y \le t)$$

$$= \int_{-\infty}^{s} \int_{-\infty}^{t} f_{T_xT_y}(u,v)\, dv\, du$$

$$= \int_{0}^{s} \int_{0}^{t} 0.01\, dv\, du$$

$$= 0.01st$$

Since the joint p.d.f is 0 in regions 2, 3 and 4, we have:

In region 2: $F_{T_xT_y}(s,t) = F_{T_xT_y}(s,10) = 0.1s$

In region 4: $F_{T_xT_y}(s,t) = F_{T_xT_y}(10,t) = 0.1t$

In region 3: $F_{T_xT_y}(s,t) = F_{T_xT_y}(10,10) = 1$ **(2 Marks)**

**(ii)** The d.f. of $T_x$ is:

$$F_{T_x}(s) = F_{T_xT_y}(s,\infty)$$

$$= F_{T_xT_y}(s,10)$$

$$= 0 \qquad \text{if } s \le 0$$

$$= 0.1s \qquad \text{if } 0 < s \le 10$$

$$= 1 \qquad \text{if } s > 10$$

The p.d.f of $T_x$ is:

$$f_{T_x}(s) = F_{T_x}{}'(s)$$

$$= 0.1 \qquad \text{if } 0 < s \le 10$$

$$= 0 \qquad \text{otherwise}$$

**(2 Marks)**

**(iii)** The d.f. of $T_{\overline{xy}}$ is:

$$F_{T_{\overline{xy}}}(t) = \text{P}[\max(T_x, T_y) \le t]$$

$$= \text{P}[T_x \le t, T_y \le t]$$

$$= F_{T_x T_y}(t,t)$$

Using the result of (i), we get for $0 < t \le 10$

$$F_{T_{\overline{xy}}}(t) = 0.01\, t^2$$

For $t \le 0$ and $t > 10$, the values of $F_{T_{\overline{xy}}}(t)$ are 0 and 1 respectively.

Taking derivative of the d.f., we obtain the p.d.f.:

$$f_{T_{\overline{xy}}}(t) = 0.02t \qquad \text{if } 0 < t < 10$$

$$= 0 \qquad \text{otherwise}$$

**(2 Marks)**

**(iv)**

Let $T_{xy} = \min(T_x, T_y)$

Note that for each outcome, $T_{xy}$ equals either $T_x$ or $T_y$ and $T_{\overline{xy}}$ equals the other.

Thus, for all joint distributions of $T_x$ and $T_y$, the following relationship holds:

$$a^{T_{xy}} + a^{T_{\overline{xy}}} = a^{T_x} + a^{T_y} \quad \text{for } a > 0$$

In particular, $v^{T_{\overline{xy}}} + v^{T_{xy}} = v^{T_x} + v^{T_y}$ where $v$ is the discount rate.

Define the following random variables:

$$Z_1 = \begin{cases} v^{T_{\overline{xy}}} & \text{if } T_{\overline{xy}} < n \\ 0 & \text{otherwise} \end{cases}$$

$$Z_2 = \begin{cases} v^{T_{xy}} & \text{if } T_{xy} < n \\ 0 & \text{otherwise} \end{cases}$$

$$Z_3 = \begin{cases} v^{T_x} & \text{if } T_x < n \\ 0 & \text{otherwise} \end{cases}$$

$$Z_4 = \begin{cases} v^{T_y} & \text{if } T_y < n \\ 0 & \text{otherwise} \end{cases}$$

Note that $Z_1 + Z_2 = Z_3 + Z_4$ since:

$$T_x < n, T_y < n \Rightarrow Z_1 = v^{T_{\overline{xy}}}, Z_2 = v^{T_{xy}}, Z_3 = v^{T_x}, Z_4 = v^{T_y} \Rightarrow Z_1 + Z_2 = Z_3 + Z_4$$

$$T_x < n, T_y \geq n \Rightarrow Z_1 = 0, Z_2 = v^{T_x}, Z_3 = v^{T_x}, Z_4 = 0 \Rightarrow Z_1 + Z_2 = Z_3 + Z_4$$

$$T_x \geq n, T_y < n \Rightarrow Z_1 = 0, Z_2 = v^{T_y}, Z_3 = 0, Z_4 = v^{T_y} \Rightarrow Z_1 + Z_2 = Z_3 + Z_4$$

$$T_x \geq n, T_y \geq n \Rightarrow Z_1 = 0, Z_2 = 0, Z_3 = 0, Z_4 = 0 \Rightarrow Z_1 + Z_2 = Z_3 + Z_4$$

$$\therefore E(Z_1) + E(Z_2) = E(Z_3) + E(Z_4)$$

$$\Rightarrow \bar{A}^1_{\overline{xy}:\overline{n}|} + \bar{A}^1_{xy:\overline{n}|} = \bar{A}^1_{x:\overline{n}|} + \bar{A}^1_{y:\overline{n}|}$$

**(4 Marks)**

**(v)** From the result of part (ii), we know that the p.d.f of $T_x$ is given by

$$f_{T_x}(t) = 0.1 \qquad \text{if } 0 < t \leq 10$$

Therefore,

$$\bar{A}^1_{x:\overline{5}|} = 0.1\int_0^5 e^{-\delta t} dt$$

Define $C_n = \int_0^5 e^{-\delta t} t^n dt$

$$C_0 = \int_0^5 e^{-\delta t} dt = \frac{1-e^{-5\delta}}{\delta} = 4.5317$$

For $n \geq 1$, integrating by parts, we get

$$C_n = \left[ \frac{t^n e^{-\delta t}}{-\delta} \right]_0^5 - \int_0^5 n t^{n-1} \left( \frac{e^{-\delta t}}{-\delta} \right) dt = \frac{1}{\delta}\left( nC_{n-1} - 5^n e^{-5\delta} \right)$$

Recursively, we get:

$$C_1 = 10.9519$$

$$\therefore \bar{A}^1_{x:\overline{5|}} = 0.1\, C_0 = 0.4532$$

Note the symmetry of the joint p.d.f of $T_x$ and $T_y$, which implies that $T_x$ and $T_y$ are identically distributed and hence:

$$\bar{A}^1_{y:\overline{5|}} = \bar{A}^1_{x:\overline{5|}} = 0.4532$$

From the result of part (iii), we know that the p.d.f of $T_{\overline{xy}}$ is given by

$$f_{T_{\overline{xy}}}(t) = 0.02t \text{ if } 0 < t \leq 10$$

Therefore,

$$\bar{A}^1_{\overline{xy}:\overline{5|}} = \int_0^5 e^{-\delta t}(0.02t)dt$$

$$= 0.02C_1$$

$$= 0.2190$$

Hence, $\bar{A}^1_{xy:\overline{5|}} = \bar{A}^1_{x:\overline{5|}} + \bar{A}^1_{y:\overline{5|}} - \bar{A}^1_{\overline{xy}:\overline{5|}}$

$= 2(0.4532) – 0.2190$

$= 0.6874$ **(5 Marks)**

**[Total Marks-15]**

**Solution 11 :**

**(i)** The calculations are shown in the following tables:

| Year | Premium (1) | Expenses (2) | Interest (3) | Mortality rate (4) | Death benefit + termination expenses (5) | Expected death cost (6) |
|---|---|---|---|---|---|---|
| 1 | 50,000 | 39,500 | 840.0 | 0.001971 | 10,000,500 | 19,710.99 |
| 2 | 50,000 | 2,600 | 3,792.0 | 0.002732 | 10,000,500 | 27,321.37 |

| Year | Dependent lapse rates (7) | Survival probability (8) |
|---|---|---|
| 1 | 0.0998029 | 0.8982261 |
| 2 | 0.0498634 | 0.9474046 |
| 3 | 0 | 0.9968480 |

| Year | Net cash flow (9) | Reserve at start of year (10) | Interest on reserve (11) | Reserve required at end of year (12) | Profit vector (13) | Survival Probability (14) |
|---|---|---|---|---|---|---|
| 1 | (8,370.99) | 0 | 0 | 22,455.65 | (30,826.64) | 1 |
| 2 | 23,870.63 | 25,000 | 2,000 | 47,370.23 | 3,500.40 | 0.8982261 |
| 3 | 19,665.02 | 50,000 | 4,000 | 0 | 73,665.02 | 0.8509835 |

| Year | Profit signature (15) | NPV profit (16) | NPV premium (17) |
|---|---|---|---|
| 1 | (30,826.64) | (28,543.18) | 50,000.00 |
| 2 | 3,144.15 | 2,695.61 | 41,584.54 |
| 3 | 62,687.72 | 49,763.53 | 36,479.06 |
| Total | | 23,915.96 | 128,063.60 |

$$\text{Profit margin} = \frac{\text{NPV profit}}{\text{NPV premium}} = \frac{23{,}915.96}{128{,}063.60} = 18.68\%$$

**(11 Marks)**

**(ii)** The profit margin will remain unchanged despite the change in reserve calculations.

Although the emergence of profit will be delayed by increasing the reserves, as the reserves earn interest at the same rate as that used for discounting the profits, the net present value of profits is not sensitive to the reserves. **(2 Marks)**

**[Total Marks-13]**

**Solution 12 :**

We have:

$$(aq)_x^{\alpha} = \int_0^1 {}_t(ap)_x (a\mu)_{x+t}^{\alpha} dt$$

Since ${}_t(ap)_x = {}_t p_x^{\alpha} \, {}_t p_x^{\beta}$ and $(a\mu)_{x+t}^{\alpha} = \mu_{x+t}^{\alpha}$ :

$$(aq)_x^{\alpha} = \int_0^1 {}_t p_x^{\alpha} \, {}_t p_x^{\beta} \mu_{x+t}^{\alpha} dt$$

Using the assumption that both decrements are uniformly distributed over each year of age in the single decrement table, for integer ages $x$ and $0 \leq t \leq 1$:

$${}_t p_x^{\beta} = 1 - {}_t q_x^{\beta} = 1 - tq_x^{\beta}$$

and:

$${}_t p_x^{\alpha} \mu_{x+t}^{\alpha} = \text{constant}$$

$$\Rightarrow q_x^{\alpha} = \int_0^1 {}_t p_x^{\alpha} \mu_{x+t}^{\alpha} dt = {}_t p_x^{\alpha} \mu_{x+t}^{\alpha} \ \ \forall t, 0 \leq t \leq 1$$

$$\Rightarrow (aq)_x^{\alpha} = q_x^{\alpha} \int_0^1 (1 - tq_x^{\beta}) dt = q_x^{\alpha} \left[ t - \tfrac{1}{2} t^2 q_x^{\beta} \right]_0^1 = q_x^{\alpha} (1 - \tfrac{1}{2} q_x^{\beta})$$

**(5 Marks)**

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