Institute of Actuaries of India

Subject CT1 – Financial Mathematics

November 2013 Examinations

INDICATIVE SOLUTIONS

Solution 1 :

i. Cashflows +x -y -y -y

Cashflows	+x	-y	-y	-y
Time	0	1	2	3

ii.

Cashflows	+p	-q	-q	-p-q
Time	0	1	2	3

(3 Marks)

Solution 2 :

i. $2 = (1+i)^n$

$n = \frac{\ln(2)}{\ln(1.1)} = 7.272$ years **(1 Mark)**

ii. $d^{(12)} = 10\%$; $1+i = (1-\frac{d^{(p)}}{p})^{(-p)}$

$i = 10.563\%$ p.a

Thus, as in part (a), $n = 6.902$ years **(2 Marks)**

[Total Marks-3]

Solution 3 :

i. $1000 = 700 \exp\{\int_0^4 (x+2yt^2)dt\} = 700 \exp(4x+42.667y)$

$1400 = 700 \exp\{\int_0^{10} (x+2yt^2)dt\} = 700 \exp(10x+666.667y)$

$\ln(1.428) = 4x+42.667y$

$\ln(2) = 10x+666.667y$

Solving gives $x = 0.09281$ and $y = -0.0003525$ **(5 Marks)**

ii. $700 \exp(10\,\delta) = 1400$

Solving gives $\delta = 0.06931$ **(2 Marks)**

iii. Present Value $= \int_0^{10} 50e^{0.09t}\ e^{-0.06931t}\ dt$

$= \int_0^{10} 50e^{0.02069t}\, dt$

$= 50 \left[\dfrac{e^{0.02069t}}{0.02069} \right]_0^{10}$

$= 555.34$ **(3 Marks)**

[Total Marks-10]

Solution 4 :

In 8 years' time the amount required, say X is

$X = 20000 \{a_{\overline{1}|}^{(12)} + 1.02\, a_{\overline{1}|}^{(12)} v + (1.02)^2 a_{\overline{1}|}^{(12)} v^2 + \ldots\ldots + (1.02)^9 a_{\overline{1}|}^{(12)} v^9\}$

$= 20000\, a_{\overline{1}|}^{(12)} \{ 1 + \dfrac{1.02}{1.09} + \dfrac{(1.02)^2}{(1.09)^2} \ldots\ldots + \dfrac{(1.02)^9}{(1.09)^9} \}$

$= 20000\, a_{\overline{1}|}^{(12)} \{ \dfrac{1 - (1.02 / 1.09)^{10}}{1 - (1.02 / 1.09)} \}$

$a_{\overline{1}|}^{(12)} = v * i / i^{(12)} = \dfrac{1.040608}{1.09} = 0.9546$

$X = 20000 \times 0.9546 \times \dfrac{0.4850}{0.0642}$

X = □144,230.84

(4 Marks)

Solution 5 :

i. Amount of loan $= 500v + 550v^2 + \ldots\ldots + 700v^5 + 700v^6 + \ldots.. + 700v^{10}$

$= 450v + 450v^2 + \ldots\ldots + 450v^5 + 50 (v + 2v^2 + \ldots\ldots + 5v^5) + 700 (v^6 + v^7 + \ldots\ldots + v^{10})$

$= 450\, a_{\overline{5}|} + 50\, (Ia)_{\overline{5}|} + v^5\, 700\, a_{\overline{5}|}$

Using Tables –

$= 450 \times 3.8897 + 50 \times 11.0007 + 0.64993 \times 700 \times 3.8897$

= □4070.02 **(5 Marks)**

ii. Interest component in first instalment is 0.09 x 4070.02 = 366.30

So the capital component is 500 – 366.30 = 133.70

Capital outstanding after first instalment is
4070.02 – 133.70 = 3936.32

Interest component in second instalment is 0.09 x 3936.32 = 354.27
So the capital component is 550 – 354.27 = 195.73 **(3 Marks)**

iii. At the end of seventh year, the capital outstanding is

$$700\,v + 700\,v^2 + 700\,v^3 = 700 \times 0.91743 + 700 \times 0.84168 + 700 \times 0.77218$$

$$= 1771.903$$

So the interest component in the eighth instalment is
0.09 x 1771.903 = 159.47

The capital component in the eighth instalment is
700 – 159.47 = 540.53 **(3 Marks)**

[Total Marks-11]

Solution 6 :

i. If i_1 is the linked internal rate of return, then

$$(1+ i_1)^3 = 1.06 \times 1.1 \times 1.095 \times 1.09$$

Thus, $i_1 = 0.11646$ or 11.646% **(3 Marks)**

ii. For TWRR, we require the value of the fund at the time of payments:

Size of the fund as on 1st October, 2010 is – 150 x 1.06 = 159
Size of the fund as on 1st April, 2011 is - (159 + 225) x 1.1 = 422.4
Size of the fund as on 1st April, 2012 is – (422.4 + 130) x 1.095 = 604.88
Size of the fund as on 31st March, 2013 is
– (604.88 + 175) x 1.09 = 850.07

The TWRR is i_t, where:

$$(1+ i_t)^3 = (159 / 150) \times \{ 422.4 / (159+225) \} \times \{ 604.88 / (422.4+130) \} \times \{850.07 / (604.88+175) \}$$

$$(1+ i_t)^3 = 1.06 \times 1.1 \times 1.095 \times 1.09$$

Giving $i_t = 11.646\%$ **(2 Marks)**

iii. For MWRR, we require the size of the fund at the end of the period.

From the data given,
MWRR (i_m) is solution to:

$150 (1+i_m)^3 + 225 (1+i_m)^{2.5} + 130 (1+i_m)^2 + 175 (1+i_m) = 850.07$

By using linear interpolation, starting with $i_m = 12\%$

$i_m = 12\%$ gives LHS of the above equation as 868.50
$i_m = 11\%$ gives LHS of the above equation as 851.63
$i_m = 10.5\%$ gives LHS of the above equation as 843.28

Interpolate between 10.5% and 11%.

$i_m = 0.105 + 0.005 \text{ x } (850.07 – 843.28) / (851.63 – 843.28)$
$= 0.10906 = 10.906\%$ **(5 Marks)**

iv. LIRR and TWRR are same because there are no cash flows within sub-periods to cause any deviation. The MWRR is lower than the other LIRR and TWRR because the fund size is smaller in the beginning of the period when rates of return are higher.

(2 Marks)

[Total Marks-12]

Solution 7 :

Working in millions at 12%, we have

Present Value of liabilities = $6 + v\ 24\ \bar{a}_{\overline{1|}}$

$= 6 + v\ 24\ (i / \delta)\ v$

$= 6 + 24 \text{ x } 0.892857^2 \text{ x } 1.058867$

$= 26.26$

Present Value of assets, considering up to ‘p’ years

$= 8\ v^2\ a_{\overline{p|}}^{(4)}$

$= 8\ v^2\ (i / i^{(4)})\ a_{\overline{p|}}$

$= 8 \text{ x } 0.79719 \text{ x } 1.043938 \text{ x } a_{\overline{p|}}$

$= 6.6577\ a_{\overline{p|}}$

With p = 6, PV of assets = 6.6577 x 4.1114

$= 27.37$

Present value of last net income part 2 million received at time 8

= 2 x v^8 = 2 x 0.40388 = 0.80776

Thus, present value of net income up to time 5 years and 9 months
= 27.37 – 0.80776 = 26.56

With PV of assets (26.56) higher than PV of liabilities (26.26), the discounted payback period = 7 years and 9 months.

(7 Marks)

Solution 8 :

i. A swap is a contract between two parties under which they agree to exchange a series of payments according to a prearranged formula.

(1 Mark)

ii. Interest rate swaps:
In the most common form of interest rate swap, one party agrees to pay to the other a regular series of fixed amounts for a certain term. In exchange, the second party agrees to pay a series of variable amounts based on the level of a short-term interest rate. Both sets of payments are in the same currency.

Currency swaps
A currency swap is an agreement to exchange a fixed series of interest payments and a capital sum in one currency for a fixed series of interest payments and a capital sum in another.

(2 Marks)

iii. Each counterparty to a swap faces two kinds of risk:

- *Market risk* is the risk that market conditions will change so that the present value of the net outgo under the agreement increases. The market maker will often attempt to hedge market risk by entering into an offsetting agreement.

- *Credit risk* is the risk that the other counterparty will default on its payments. This will only occur if the swap has a negative value to the defaulting party so the risk is not the same as the risk that the counterparty would default on a loan of comparable maturity.

(2 Marks)

[Total Marks-5]

Solution 9 :

Time	0	1	2	3	4	5	Price
Index	125	128	135	142	145	148	
Cashflow		4	4	4	4	104	
Indexed cashflow		=4*128/125	=4*135/125	=4*142/125	=4*145/125	104*148/125	
	1mark	4.096	4.32	4.54	4.64	123.14	
@6%	1mark	3.864	3.845	3.812	3.675	92.02	107.21
@7%	1mark	3.828	3.773	3.706	3.54	87.79	102.63
The real yield = 6.7% -- 1mark							
With tax @20%-coupns	1mark	=0.8*4.096=3.2768	=0.8*4.32=3.456	=0.8*4.54=3.632	=0.8*4.64=3.712	=0.8*4*(148/125)+100*148/125=3.7888+118.40	
v-value @6.7%	1mark	0.9372	0.8783	0.82318	0.7715	0.72303	
Price	1 mark	3.0710	3.0354	2.989	2.8638	88.346	100.30
Total marks	7 marks						

[7 Marks]

Solution 10 :

i. No arbitrage means that arbitrage opportunities do not exist. Arbitrage is a risk-free trading profit, which occurs when an investor can make a deal that gives them:

- an immediate profit, with no risk of future loss;

or:

- no initial cost,
- no risk of future loss, and
- a non-zero probability of future profit. **(2 Marks)**

ii. The price of stock is equal to

$S_0 = v^{(2/12)} (4\, \ddot{a}_{\overline{\infty|}}^{(4)} + 50v^{(3/2)})$ @5.5%

$d^{(4)} = p[1-(1+i)^{(-1/p)}] = 0.053184$

$S_0 = 120.27$

The forward price is given by

$K0 = (S0 - I)(1+i)^3$, where I is the present value of income from the stock during the term and i = 0.065 . Calculating the necessary values:

$d^{(4)} = 4(1-1.065^{(-0.25)}) = 0.062482$; $\ddot{a}_{\overline{3}|}^{(4)} = 2.7555$

$I = v^{(2/12)} (4\ \ddot{a}_{\overline{3}|}^{(4)} + 50v^{(3/2)})$ @6.5%

= 0.93896^(2/12)(4*2.7555 + 50* 0.93896^1.5)

=0.98955 *(11.022+45.49)

= 55.92

K_0 = (120.27 – 55.92)(1.065)^3 = 77.73 **(5 Marks)**

iii. The value of the forward contract is $(K_r-K_0)*e^{-20/12}$ where K_r is the forward price for a contract entered on 1st December 2014 for the remaining period of 20months.

$Kr = (Sr - I')(1+i)^{\wedge(20/12)}$, where I' is the present value of income from the stock during outstanding term 20 months and i = 0.065 .

Calculating the necessary values:

$\ddot{a}_{\overline{1.75}|}^{(4)} = 1.6702$

$I' = v^{(1/12)} * 4\ \ddot{a}_{\overline{1.75}|}^{(4)} + 50v^{(4/12)}$ @6.5%

= 0.93896^(1/12)(4*1.6702) + 50* 0.93896^(4/12))

= 55.61 --

K_r = (150 – 55.61)(1.065)^(20/12) = 104.83

Value of forward contract = (104.83-77.73)(1.065)(-20/12) = 24.40 **(5 Marks)**

[Total Marks-12]

Solution 11 :

i. $Y_t = \frac{-1}{t}\log P_t$ where P_t is the price of unit zero coupon bond.

=> Y_{10} = -(1/10)log $_e$ (45/100) = 7.985%

$Y_{15} = \frac{-1}{15}\log_e 0.35 = 7\%$ **(2 Marks)**

ii. $F_{t,r} = \frac{1}{r}\log_e(\frac{P_t}{P_{t+r}})$

⇨ $F_{5,10} = \frac{1}{10}\log_e(\frac{o.75}{0.35})$ =7.62%

$F_{10,5} = \frac{1}{5}\log_e(\frac{0.45}{0.35})$ = 5.02%

(2 Marks)

[Total Marks-4]

Solution 12 :

i. Present value of liabilities = $35000 * v^{13} + v^{10} * \ddot{a}_{\overline{10}|}^{(2)} * 15000$ @ 6%

=35000* 0.46884 + 0.55839*7.3601*1.044782*15000
=80817.17 **(3 Marks)**

ii. DMT of liabilities:
$13 * 35000 * v^{13}$ @ 6%+ $7500 * v^{10} * [10 + 10.5 * v^{0.5} + \ldots\ldots + 19 * v^{9} + 19.5 * v^{9.5}]$ @6%

The value in square brackets can be written as:

$9.5\ddot{a}_{\overline{20}|} + 0.5 \times I\ddot{a}_{\overline{20}|}$ @2.956% **(6 Marks)**

= 9.5*15.379 + 0.5 * 146.67 = 219.44

DMT of liabilities = (213322.20+919014.72)/80817.17 = 14.01

iii. As the portfolio is immunized, the present value of liabilities is equal to present value of assets
$X * v^{7} + Y * v^{10}$ =80817.17 ---(1)
0.66506X + 0.55839Y = 80817.17
DMT of assets =
$(7 * v^{7} * X + 10 * Y * v^{10})/80817.17 = 14.01$
4.65542x + 5.5839y = 1132248.55 ---- (2)

Solving equations 1 and 2
X = -162432 ; Y= 338194 **(5 Marks)**

[Total Marks-14]

Solution 13 :

The accumulated value of amounts at the end of 3yrs is
$(1+i_2)(1+i_3) + (1+i_3) + 1$

The expected value of accumulated amount is
$E[(1+i_2)(1+i_3) + (1+i_3) + 1]$

As the interest rates are independent in different years
$= (1+ E(i_2))(1+E(i_3)) + (1+ E(i_3)) + 1$

= 1.055 * 1.06 + 1.06 + 1 = 3.1783

Variance of accumulated amount:

$Var[(1+i_2)(1+i_3) + (1+i_3) + 1]$

$= Var[(1+i_3)(1+i_2+1)]$ as $Var(X+K) = Var(X)$

$= Var(1+i_3)(2+i_2)$ -

Now writing variance in terms of expected values

$Var(1+i_3)(2+i_2) = E[(1+i_3)^2(2+i_2)^2] – (E[(1+i_3)(2+i_2)])^2$

$= [var(1+ i_3) + [E(1+i_3)]^2][var(2+ i_2) + [E(2+i_2)]^2] –[E(1+i_3)]^2[E(2+i_2)]^2$

$= [0.009^2 +1.06^2][0.007^2+2.055^2] –[(1.06)(2.055)]^2$

$= 0.000398$

Standard deviation = 2%

(8 Marks)

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