INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

22nd November 2013

Subject ST6 – Finance and Investment B

Time allowed: Three Hours (10.15* – 13.30 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***
2. **** You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You have then three hours to complete the paper.***
3. ***You must not start writing your answers in the answer sheet unless instructed to do so by the supervisor.***
4. ***The answers are not expected to be any country or jurisdication specific. However, if Examples/illustrations are required for any answer, the country or jurisdication from which they are drawn should be mentioned.***
5. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
6. ***Mark allocations are shown in brackets.***
7. ***Please check if you have received complete Question paper and no page is missing. If so, kindly get a new set of Question paper from the Invigilator.***

AT THE END OF THE EXAMINATION
Please return your answer book and this question paper to the supervisor separately.

Q.1) Give brief answers to the following:

i) Compare the price of a discrete barrier option to the price of a vanilla option and the price of a continuous barrier option. (1.5)

ii) Compare the price of an Asian option to the price of a vanilla option with the same strike and maturity. (1.5)

iii) Construct an arbitrage portfolio where the price of a knock-in option plus the corresponding knockout option is not equal to the price of the corresponding vanilla option. (1.5)

iv) Suppose an asset value follows a Brownian motion and there are no interest rates. What can be said about the relative prices of out – of – the money American and European digital calls? (1.5)

v) What happens to the price of a derivative if it has a negative vega and the volatility increases? (1.5)

vi) Show that if the current spot price is S_0, and the continuous compounding rate is "*r*" then a call and a put both struck at S_0e^{rT} and expiring at time "T" are of equal value. (1.5)

vii) Consider a single period binomial model with two risky assets S_1 and S_2 and a riskless bond. In the next step, there are only two states for the risky assets, (S_1u_1, S_2u_2) and (S_1d_1, S_1d_2). Show that this model does not admit a risk neutral probability for certain u_1, u_2, d_1, d_2 and the expected rate of return R. (1.5)

viii) Show that the gamma of the call option (i.e. the second derivative of the call option price with respect to the spot) is the same as the gamma of the put option (i.e. second derivative of the put option price with respect to the spot) with the same set of parameters. (1.5)

ix) Assume that a stock price moves from S to Su (up move value) or Sd (down move value) in one period and that there is a continuous dividend yield of the stock to be q on the stock. Under what condition will the early exercise of an American call be optimal? (1.5)

x) Let $Y(t) \equiv e^{X(t)}$, where $\{X(t), t \geq 0\}$ is a Brownian motion with drift μ and variance σ^2. Show that

$$E\{Y(T)|Y(s)\} = Y(s)e^{(t-s)(\mu+\frac{\sigma^2}{2})}.$$

(1.5)

[15]

Q. 2) Let St denote the price of a non dividend paying asset at time t, satisfying the stochastic equation

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

where the drift μ and the volatility σ are both assumed constant and W_t is the wiener process. Let C_t be the price of a derivative based on this asset satisfying

$$dC_t = \mu_t^C C_t dt + \sigma_t^C C_t dW_t.$$

i) If at time *"t"* a trader is long ϕ units of the asset, and short one unit of the derivative, what is the value of the trader's position? Show that the position is risk free only if

$$\phi_t = \frac{\sigma_t^C C_t}{\sigma S_t}$$
(3)

ii) Assuming that the short term interest rate has the constant value r, show that the principle of no arbitrage implies that

$$\frac{\mu_t^C - r}{\sigma_t^C} = \frac{\mu - r}{\sigma}.$$

Comment on the financial significance of this relationship. (3)

iii) Assuming that the derivative price can be expressed in the form C_t=C(S,t), where the function C(S,t) has a continuous second derivative, use Ito's lemma to show that

$$C_t\mu_t^C = \frac{\partial C}{\partial t} + \mu S\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 C}{\partial S^2}$$

and

$$C_t\sigma_t^C = \sigma S\frac{\partial C}{\partial S}.$$
(4)

iv) Show that the no arbitrage condition implies that C(s, t) satisfies

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 C}{\partial S^2} = r\left(C - S\frac{\partial C}{\partial S}\right).$$

What is the significance of the fact the parameter μ does not appear in this relation? (4)

[14]

Q. 3) A fund manager purchases a payer swaption with option terms of 15 years and swap tenor of 5 years with a swap strike rate of 4%, a principal amount of Rs. 1000 Crores priced at Rs. 42 Crores.

The zero coupon bond prices for the interest rate are specified as:

Base	Zero Coupon Bond Prices
1	0.99
2	0.97
3	0.96
4	0.94
5	0.92
6	0.90
7	0.87
8	0.85
9	0.82
10	0.79
11	0.76
12	0.74
13	0.70

14	0.68
15	0.65
16	0.62
17	0.59
18	0.56
19	0.53
20	0.49

i) Define moneyness of an option. State whether the above option is in the money, at the money or out of money. (3)

ii) Estimate the implied volatility of the swaption with confidence interval of ± 2%. The market implied volatility of a 15 year swaption is between 20% - 30%. (4)

iii) The market yield curve has changed and the new zero coupon bond prices are as given below. Estimate the new price for this swaption.

Base	**Zero Coupon Bond Prices**
1	1.00
2	1.00
3	0.99
4	0.98
5	0.96
6	0.94
7	0.91
8	0.88
9	0.84
10	0.81
11	0.76
12	0.72
13	0.68
14	0.64
15	0.60
16	0.55
17	0.51
18	0.46
19	0.43
20	0.38

(5)

iv) The fund manager has priced the option after the change in the market curve to be Rs. 100 Crores. What are the principle causes for difference (if any) between the fund manager's price and your price and how can you allow for this difference in your model? (4)

[16]

Q. 4) An insurance company sells single premium products such that returns on a certain block of assets (called Growth Fund) are used to increase the policyholder payouts (policyholder share); i.e. if growth fund experiences a total return of 10% then the policyholder payouts (hence insurance company's liability) will also grow by 10%. The rest of the company's money is in a Matching Fund which is invested in either government bonds or cash.

There are also guaranteed payouts of premium at maturity, which if more than the policyholders' share at maturity will lead to a guarantee cost.

Balance sheet at 31/03/2013 for the company is

Balance Sheet at t = 0	
Assets	
Growth Fund	1000
Match Fund	600
Total Assets	1600
Liabilities	
Policyholders share	1200
Guarantee Costs	300
Total Liability	1500
Free Capital	100

Asset allocation of the growth fund is

Initial Asset Mix	
Equity	30%
Corporate Bond	30%
Government Bond	30%
Property	10%

i) Explain how the cost of these guarantees can be valued by application of a closed form solution. (2)

ii) Explain how Monte – Carlo simulations can be used to price these. Explain the Key steps, Mathematical principles & assumptions and inputs. (4)

iii) Which risk drivers would you recommend for a Monte – Carlo simulation. Suggest some models and inputs required to model the above recommended risk drivers and explain how you will allow for interaction between the risk drivers. (4)

iv) Construct the balance sheet allowing for a 50% fall in equity markets & 50% rise in equity market, so that the delta for the cost of the guarantee with respect to the change in the policyholder share is -12%. State for which of these scenarios the free capital would be lower and why. (6)

v) Assuming equities in the growth fund target BSE, construct a hedging strategy such that the balance sheet is immune to the above stresses. State all the information required, and the risks to be considered to have these hedges in place. (3)

vi) The fund manager expects property prices to fall in coming future months. As the company has physical property holding, it is not possible to sell it in such short time. As a solution, one actuarial student suggested creating a notional property exposure in Growth Fund and a matching notional negative property in Matching Fund.

Scenario test (20% property up and 20% property down) the suggested solution and argue whether the proposed solution is appropriate if we expect a fall in property. State what could be the possible risk involved. (6)

[25]

Q. 5) Calculate the minimum and maximum values of European call and put options. Derive the same for American call and put options. State for which option European and American type options will have different bounds and why. **[5]**

Q. 6) Arithmetic average rate options were assumed to be newly issued, and there was no historical average of prices to deal with. Show that, in terms of pricing, no generality was lost in doing so. **[8]**

Q. 7) A firm buys $100 million par of a 3 – year floating rate bond that pays 6 month LIBOR plus 0.5% every 6 months. It is financed by borrowing $100 million for 3 years on terms requiring 10% annual interest paid every 6 months. Show how these transactions can create a synthetic interest rate swap. **[7]**

Q. 8) Show that the value of a European call option with strike price K and time to maturity m under the case of known dividend yields of δ per annum (with annual compounding) equals $(1 - \delta)^m$ times the value of a European call option with time to maturity m on a non-dividend paying stock with strike price $(1 - \delta)^{-m}$ K. **[5]**

Q. 9) A stock is currently priced at Rs. 160 and pays no dividends. The price at time "i+1" is given by S_{i+1} and can be written as $S_{i+1} = S_i + 0.5\ S_i$ for the up move or $S_{i+1} = S_i - 0.5\ S_i$ for the down move. There also exists a riskless bond with a continuously compounded interest rate of 18.232% per period. Price an American put option on this asset with a strike price of Rs. 150 with three periods to expiration. **[5]**

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