# INSTITUTE OF ACTUARIES OF INDIA

# EXAMINATIONS

## 21st November 2013

## Subject CT8 – Financial Economics

## Time allowed: Three Hours (10.30 – 13.30 Hrs.)

## Total Marks: 100

### INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
2. *Mark allocations are shown in brackets.*
3. *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*
4. *Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

| AT THE END OF THE EXAMINATION |
|---|
| Please return your answer book and this question paper to the supervisor separately. |

**Q.1)** Suppose that the price of a given stock, $X_t$ at time t, follows a geometric Brownian motion with parameters $\mu$ and $\sigma$

i.e. $dX_t = \mu X_t dt + \sigma X_t dW_t$, where $W_t$ is Wiener process.

Let $Z_t = X_t^{\lambda}$, $0 < \lambda < 1$

**i)** Using Ito's lemma, show that $Z_t$ follows geometric Brownian motion with respect to the filtration on $W_t$ (5)

**ii)** Find $E(Z_T)$ (2)

**iii)** For what relation between $\mu$, $\sigma$ and $\lambda$ would $Z_t$ be a martingale? (3)

**[10]**

**Q. 2)** $\{N_t, t>=0\}$ is a Poisson process with rate $\lambda$. Show that the following processes are all martingales with respect to the filtration of $\{N_t\}$

**i)** $N_t - \lambda t$ (1)

**ii)** $[N_t - \lambda t]^2 - \lambda t$ (3)

**iii)** $\exp[\alpha N_t - \lambda t(\exp(\alpha)-1)]$ (4)

**[8]**

**Q. 3)** **i)** What is the difference between an American and a European option? (1)

**ii)** Prove that the lower bound for an American put option is greater than the lower bound for a European put option on a non-dividend paying share. Define all terms /notations used. (5)

**[6]**

**Q. 4)** **i)** Ganesh's investment portfolio consists of security X and half as much security Y. Return from security X is equally likely to be 4% or 8% per annum. Return from security Y is equally likely to be 8% or 16% per annum. Calculate the expected return and variance of return on his portfolio assuming the coefficient of correlation is 1. (4)

**ii)** Re-do your calculations if the coefficient of correlation is -1. (1)

**iii)** Comment on the results of part ii and iii above. (2)

**iv)** For what proportion invested in each security would Ganesh achieve minimum variance of return of the resulting portfolio should the coefficient of correlation be zero. (2)

**[9]**

**Q. 5)** Below is the updating equation for the real yield R(t) from Wilkie's model

$$\ln[R(t)] = \ln(RMU) + RA.\{\ln[R(t-1)] - \ln(RMU)\} + RBC.CE(t) + RE(t)$$

**i)** Define all components of the above equation. (2)

**ii)** Write down an expression for the total return on an equity from time t to t+1 in terms of the equity dividend yield at end of year t and the rate of dividend growth, continuously compounded, during year t. (4)

**[6]**

**Q. 6)** **i)** Provide an expression for the return on a security in the context of a single-index model of security returns. Define all terms/notations used. (2)

**ii)** Prove the following:

**a)** $V_i = \beta_i^2 . V_M + V_{\varepsilon i}$

**b)** $C_{ij} = \beta_i \beta_j . V_M$

where,

$V_i$ is the variance of return on security $i$
$C_{ij}$ is the covariance of returns between securities $i$ and $j$; $i \neq j$ (6)

**[8]**

**Q. 7)** **i)** State Ito's lemma. (2)

**ii)** Let $X_t$ be an Ito process defined by the equation $dX_t = \alpha dt - dW_t$ where $W_t$ is a standard Brownian motion and $\alpha$ is a constant. Apply Ito's lemma to derive the stochastic differential equation for the process $R_t$ defined as $1/X_t$ , $t > 0$ (3)

**iii)** Define a mean reverting process and check if $R_t$ as defined above is mean reverting. (3)

**[8]**

**Q. 8)** An investor has a quadratic utility function $U(w) = w + \alpha w^2$

**i)** Over what constraints does this satisfy the requirements of non-satiation and risk aversion? (1)

**ii)** Using the Arrow-Pratt measures of risk aversion determine if this investor exhibits increasing or declining risk aversion. (2)

**iii)** Show the expected utility of the investor can be (1) expressed as a linear combination of only the first two moments of the distribution of wealth.

**[4]**

**Q. 9)** **i)** Define 'risk-neutral probability'. (1)

**ii)** Show that in a one-step binomial tree model of the price of a non-dividend paying share, the risk-neutral probability $q$ of an upward movement is given by $q = \frac{e^{r\delta t} - d}{u - d}$ where, r is the risk-free rate of return compounded continuously, u is the proportionate change in the price of the share if it goes up and d if it goes down and δt is the length of the one-step time period. State any assumptions that you may have made. (5)

**iii)** Show the real-world probability of an upward movement is more than the risk-neutral probability if and only if $e^{r\delta t} < 1 + \lambda$ where $\lambda$ is the expected real-world one-step rate of return. (3)

**[9]**

**Q. 10)** Three risky securities A, B and C are currently available in the market with market capitalizations of Rs. 25, 50 and 25 crores.

The rates of returns on these 3 securities under different states of the economy have been tabulated below: The prevailing risk free rate in the market is 5%.

| States | Probability | Security A | Security B | Security C |
|---|---|---|---|---|
| 1 | 0.2 | 40% | 20% | 10% |
| 2 | 0.3 | 20% | 5% | 20% |
| 3 | 0.5 | 10% | 10% | 7% |

**i)** Assuming CAPM holds, determine the market price of risk. (3)

**ii)** Which security has the highest market risk? (2)

**iii)** Which security has the highest unsystematic risk? (2)

**iv)** Break down the variance of the market portfolio into systematic and unsystematic components? Is the market portfolio well diversifies? (3)

**[10]**

**Q. 11)** The term structure for zero-coupon bonds is currently:

| **Maturity (years)** | **Yield to Maturity (YTM)** |
|---|---|
| 1 | 4% per annum |
| 2 | 5% per annum |
| 3 | 6% per annum |

Next year at this time, you expect it to be:

| **Maturity (years)** | **Yield to Maturity (YTM)** |
|---|---|
| 1 | 5% per annum |
| 2 | 6% per annum |
| 3 | 7% per annum |

All the rates are quoted with annual compounding.

**i)** What do you expect the rate of return to be over the coming year on a 3-year zero-coupon bond? (2)

**ii)** What do you expect the rate of return to be over the coming year on a 2-year zero-coupon bond? (2)

**iii)** What should be the current price of a 3-year maturity bond with a 8% coupon rate paid annually? If you purchased it at this price, what would your total expected rate of return be over the next year (coupon plus price change)? Ignore taxes. The face value of the bond is Rs. 1,00,000 (3)

**[7]**

**Q. 12)** You being the part of the credit risk assessment team, have been assigned the task of credit risk evaluation of the following available corporate bonds.

The one year rating transition matrix has been provided as below:

| **Ratings** | **AAA** | **AA** | **A** | **BB** | **B** |
|---|---|---|---|---|---|
| AAA | 95% | 5% | 0% | 0% | 0% |
| AA | 5% | 85% | 10% | 0% | 0% |
| A | 1% | 2% | 85% | 7% | 5% |
| BB | 0% | 0% | 2% | 80% | 11% |
| B | 0% | 0% | 0% | 5% | 70% |

**i)** Reconstruct the transition matrix with the default probabilities (2)

**ii)** Evaluate the worst possible outcome and estimate the corresponding probability (3)

**iii)** Calculate the VAR at 95% for the portfolio constituting the bonds underlying the worst scenario over the period of one year just before the first coupon payment. Assume all the bonds pay annual coupon of 10% (compounded annually) with term of 5 years (7)

The forward rate matrix after one year is as given below

| **Ratings/Time** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|
| AAA | 8% | 8.5% | 9.0% | 9.5% |
| AA | 8.5% | 9.0% | 9.5% | 10.0% |
| A | 9.0% | 9.5% | 10.0% | 10.5% |
| BB | 9.5% | 10.0% | 10.5% | 10.75% |
| B | 10% | 10.5% | 10.75% | 11.0% |

**[12]**

**Q. 13)** Consider a portfolio that has a delta of -1000, a gamma of -10,000 and a vega of - 4000. The options shown in the table below can be traded.

| | **Delta** | **Gamma** | **Vega** |
|---|---|---|---|
| Option 1 | 0.7 | 1.0 | 1.0 |
| Option 2 | 0.6 | 1.6 | 0.6 |

How could the portfolio be made delta, gamma, and vega neutral?

**[3]**

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