

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

20th November 2013

**Subject CT5 – General Insurance, Life and
Health Contingencies**

Time allowed: Three Hours (10.30 – 13.30 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
2. *Mark allocations are shown in brackets.*
3. *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*
4. *Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

- Q. 1)** Calculate:
- i)** $A_{30:\overline{25}|}$
 - ii)** $\ddot{a}_{60}^{(2)}$
 - iii)** $(Ia)_{50}$
- Basis: Mortality AM92 Ultimate
Interest rate 4% per annum
- [3]**
- Q. 2)**
- i)** Name the two approximate methods used to compute probabilities involving non-integer ages or duration. (2)
 - ii)** Estimate the probability that a male with age 63 years and 3 months exactly will be alive for next two years using both the above methods.
- Basis: ELT15 (Males) mortality at integer ages (4)
- [6]**
- Q. 3)** Explain the use of risk classification in life insurance business. [7]
- Q. 4)** A mortality table has a select period of 3 years with the following probabilities:
- $$q_{[50]} = 0.012$$
- $${}_2p_{[50]} = 0.97$$
- $${}_{2|1}q_{[50]} = 0.025$$
- $${}_{2|3}q_{[50]+1} = 0.075$$
- Calculate ${}_3p_{53}$. [4]
- Q. 5)** Explain the terms “unit fund” and “non-unit fund” in the context of a unit-linked life assurance contract, listing the various items that make up the non-unit fund. [4]
- Q. 6)** A man at age 40 exact buys a special 25-year endowment policy that pays Rs 25,000 on maturity. If the policyholder dies before age 65, then twice all premiums paid so far are returned without interest at the end of the year of death. Level premiums are payable annually in advance for 25 years or until earlier death. Calculate the annual premium.
- Basis: Mortality AM92 Select
Interest rate 4% per annum
Expenses NIL [5]

- Q. 7)** A pension scheme provides a pension on retirement, for any reason, of $1/100^{\text{th}}$ of Final Pensionable Salary for each year of pensionable service, with fractions counting proportionately. Final Pensionable Salary is defined as the average salary earned in the three years before retirement.

A member of the scheme is currently aged 40 years exact with exactly 15 years past service and salary for the year before the calculation date equal to Rs. 4,000,000. Using the functions and symbols defined in the Example Pension Scheme Table in the Actuarial Tables and the assumptions underlying this Table, calculate:

- i) the expected present value of the member's total pension (4)
- ii) the contribution rate required as a percentage of salary to fund the future service element of the pension (2)

[6]

- Q. 8)** i) Explain what is meant by prospective reserves and retrospective reserves in the context of a life insurance contract. State the two required conditions for equality of prospective and retrospective reserves. (4)
- ii) For a whole life assurance contract issued to a life aged x , where death benefits are payable at the end of the year of death, show that the prospective reserves and retrospective reserves at duration t years are equal on the assumption the equality conditions are met. (4)
- iii) A life insurance company issues a with profit whole life assurance policy to a life aged 40 exact with a limited premium paying term of 20 years. The sum assured is Rs. 100,000 together with any attaching bonuses and is payable immediately on death. Level premiums are payable monthly in advance.

Simple annual bonuses are added at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year of death).

The company calculates the premium on the following basis:

Mortality	AM92 select
Interest	4% per annum
Initial Expenses	Rs 500
Renewal Expenses	Rs 100 at the start of the second and subsequent policy years and payable until death
Terminal Expenses	Rs 200 on death
Initial Commission	75% of the total first year premium at outset
Renewal Commission	2.5% of the monthly premium (from 1 st year onwards)
Bonus	Simple bonus of 5% of basic sum assured per annum

Calculate the monthly premium for this policy. (5)

- iv) For the first 20 years, the company has declared a simple bonus of 5% of the basic sum assured per annum attached at the end of the year and expects this to be 4% of the basic sum assured per annum for future years.

Calculate the reserve at the end of 20th policy year. (4)

[17]

Q. 9) A life insurance company issues the following 3 types of policy:

- 20-year endowment assurance with a sum assured of Rs 100,000 payable at maturity or at the end of the policy year of death if earlier. Level premiums for this contract are paid annually in advance.
- 15-year pure endowment assurance with a sum assured of Rs 75,000 payable at maturity. Level premiums for this contract are paid annually in advance.
- 20-year single premium temporary immediate annuity with an annual annuity payable in advance of Rs 15,000.

On Jan 01, 2005, the company sold 5,000 endowment assurance policies, 3,000 pure endowment assurance policies and 2,500 temporary immediate annuity policies, all to lives aged 45 exact.

Basis: Mortality AM92 select

Interest rate 4% per annum

Expenses NIL

i) Calculate the death strain at risk during 2012. (6)

ii) During the first 7 years, the company has experienced 56 deaths from endowment policies, 43 deaths from pure endowment policies and 30 deaths from immediate annuity policies.

During 2012, there were 12 deaths from the endowment assurance policies, 7 deaths from pure endowment policies and 6 deaths from the immediate annuity policies.

Calculate total mortality profit/loss during 2012. (6)

iii) Compare the expected number of deaths and actual number of deaths in all the three line of business during 2012 and comment on the mortality profit from these lines of businesses. (3)

[15]

Q. 10) For two lives (x) and (y), the joint probability density function (p.d.f.) of their future lifetimes, T_x and T_y , (in years) is:

$$f_{T_x T_y}(s, t) = \begin{cases} 0.01 & ; 0 < s < 10, 0 < t < 10 \\ 0 & ; \text{otherwise} \end{cases}$$

i) Prove that the joint distribution function (d.f.) of T_x and T_y is:

$$F_{T_x T_y}(s, t) = \begin{cases} 0.01st & ; 0 < s \leq 10, 0 < t \leq 10 \\ 0.1s & ; 0 < s \leq 10, t > 10 \\ 0.1t & ; s > 10, 0 < t \leq 10 \\ 1 & ; s > 10, t > 10 \\ 0 & ; \text{otherwise} \end{cases} \quad (2)$$

ii) Prove that the p.d.f. of T_x is:

$$f_{T_x}(s) = \begin{cases} 0.1 & ; 0 < s < 10 \\ 0 & ; \text{otherwise} \end{cases} \quad (2)$$

iii) Prove that the p.d.f. of $T_{\overline{xy}} = \max(T_x, T_y)$ is:

$$f_{T_{\overline{xy}}}(t) = \begin{cases} 0.02t & ; 0 < t < 10 \\ 0 & ; \text{otherwise} \end{cases} \quad (2)$$

iv) Prove that

$$\overline{A}_{\overline{xy:n}}^1 + \overline{A}_{\overline{xy:n}}^1 = \overline{A}_{\overline{x:n}}^1 + \overline{A}_{\overline{y:n}}^1$$

where $\overline{A}_{\overline{xy:n}}^1$ is the expected present value of an n-year term insurance paying a death benefit of 1 at the moment of the last death of (x) and (y) if this death occurs before n and $\overline{A}_{\overline{xy:n}}^1$ is the expected present value of an n-year term insurance paying a death benefit of 1 at the moment of the first death of (x) and (y) if this death occurs before n. (4)

v) Use the above formula to calculate the expected present value of a 5-year term insurance paying a death benefit of 1 at the moment of the first death of the two lives (x) and (y), if this death occurs within the 5-year term, given that the force of interest, $\delta = 0.04$ per annum. (5)

[15]

Q. 11) A special 3-year level term assurance policy pays the sum assured at the end of the year of death. Level annual premiums are required to be paid at the start of each year. Policies may lapse at the end of each policy year without any benefits payable.

The premium basis is as follows:

Interest:	8% per annum	
Mortality:	AM92 Select	
Lapse rates:	10% of all policies in force at the end of year 1 5% of all policies in force at the end of year 2	
Expenses:	Initial Fixed	Rs. 2,000
	Initial Variable	75% of annual premium
	Renewal Fixed	Rs. 100 at start of year 2 Rs. 105 at start of year 3
	Renewal Variable	5% of annual premium at start of years 2 and 3
	Termination	Rs. 500 per death
Reserves:	50% of the total premiums paid till date.	

The policy and life assured details of a particular contract are as follows:

Sum Assured:	Rs 10,000,000
Annual Premium:	Rs 50,000
Age at entry:	50 years (exact)

- i) Calculate the profit margin for this contract according to the premium basis, where the profit margin is measured as the net present value of the profit divided by the net present value of the premiums and the discount rate used in the net present values is the same as the interest rate in the premium basis. Assume that death cost occurs at the end of each policy year. (11)
- ii) State with reasons what the effect on the profit margin for this contract would be, if in the premium basis, reserves for in force policies are calculated as 100% of the total premiums paid till date. (2)
- [13]

Q. 12) A population is subject to two modes of decrement, α and β , both of which are uniformly distributed over each year of age in the single decrement table. Prove that for integer ages x :

$$(aq)_x^\alpha = q_x^\alpha (1 - \frac{1}{2} q_x^\beta)$$

[5]
