INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

13th November 2013

Subject CT4 – Models

Time allowed: Three Hours (10.30 – 13.30 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception***

2. ***Mark allocations are shown in brackets.***

3. ***Attempt all questions, beginning your answer to each question on a separate sheet. <u>However, answers to objective type questions could be written on the same sheet</u>.***

4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION
Please return your answer book and this question paper to the supervisor separately.

Q. 1) Compare and contrast the following models in terms of the statistical properties of the maximum likelihood estimates in the various models. For each of the models, clearly state whether the MLE is unbiased and consistent.

- Multiple state model
- Poisson model
- Binomial model

Based on the above, which model would you consider most appropriate for modelling human mortality? **[4]**

Q. 2) **i)** Write down the Chapman-Kolmogorov equations for a continuous-time Markov process with discrete state space. (1)

ii) Derive from first principles the forward version of the Kolmogorov differential equations:

$$\frac{\partial}{\partial t}P(s,t) = P(s,t)A(t)$$

Where A is the matrix of transition rates. (4)

[5]

Q. 3) You are a consultant advising a life insurance company on model development. Your client is a little confused about stochastic and deterministic models and has asked you to describe the instances in which using deterministic models may be more appropriate than using stochastic models. Please describe in brief what points you could cover in the response to client's query. **[4]**

Q. 4) Each morning Samit leaves his house and goes for a jog. He is equally likely to leave either from his front or back door. Upon leaving the house, he chooses a pair of sports shoes (or goes for a jog barefoot if there are no shoes at the door from which he left). On his return he is equally likely to enter, and leave his sports shoes, either by the front or back door.

i) Write down the transition matrix of the process X_n, where X_n represents the number of shoes at the front door. (2)

ii) If he owns a total of x pairs of sports shoes, what proportion of the time does he jog barefooted? (5)

[7]

Q. 5) The occurrence of hurricanes in a county can be modeled as a Poisson process. Twenty hurricanes have touched down in a county within the last twenty years. If there is at least one hurricane occurring in a year, that year is classified as a ‘hurricane year’.

i) What is the probability that next year will be a ‘hurricane year’? (2)

ii) What is the probability that there will be two ‘hurricane years’ within the next three years? (3)

iii) On the average over a ten-year period,

a) How many hurricanes are expected to occur? (1)

b) How many “hurricane years" are expected to occur? (1)

[7]

Q. 6) In a game similar to “Snake and Ladders”, there are 5 levels. Level 1 is the lowest and level 5 is the highest.

Hari starts at the lowest level.

Each time, a fair coin is tossed. If it turns out heads, the player moves up one level. If it turns up tails, the player moves down to the lowest level. Once at the top level, the player moves to the lowest level if the coin turns out to be tails, and stays at the highest level if it turns out to be heads.

i) Find the transition probability matrix. (1)

ii) Find the two-step transition probability matrix. (2)

iii) Find the steady-state distribution of the Markov chain. (4)

[7]

Q. 7) For a simple survival model, let $T_x$represent the random future lifetime after age x. Assume that $T_x$is a continuous random variable taking values in the range$[0, \omega - x]$.

i) Define in strict mathematical terms:

a) The distribution function of T_x.

b) The survival function of T_x

(2)

ii) Provide the actuarial notation for each of the above and explain in words with a suitable example what this represents. (2)

iii) Given your definition of the distribution function of $\mathrm{T}x$, derive an appropriate expression for the probability density function, $f_x(t)$. (3)

iv) A small population of humans inhabit the treacherous slopes of a rocky mountain in a village called Edgestone. It is reported that nearly all deaths in this village are accidental, majority of which occur due to the deceased falling off the cliff. A traveler visited Edgestone over two consecutive Christmases. On his first visit he made many friends but on his return the subsequent year he was saddened to find that nearly one in every eight of his friends had died in the past one year. On a cheerer note he also learnt that one of his friends gave birth to twin girls.

Based on the information provided above, calculate the probability that at least one of the twin girls will live to celebrate her 18th birthday. Clearly state all assumptions you make (4)

[11]

Q. 8) **i)** Clearly explain what is meant by the census approximation of E_x^c. What is the principal assumption underlying the census approximation? (2)

A mortality investigation bureau has collected the following information on number of policies in-force each year from different companies:

	Company A	**Company B**	**Company C**
Age = 44			
2010	5,868	3,928	9,176
2011	5,883	3,946	9,176
2012	5,909	3,938	9,187
Age = 45			
2010	5,928	3,939	9,198
2011	5,920	3,921	9,148
2012	5,911	3,930	9,136
Age = 46			
2010	5,977	3,969	9,237
2011	5,993	3,973	9,252
2012	5,988	4,018	9,240

However, the bureau did not specify its data requirement precisely and consequently has received inconsistent submissions from different companies:

- Company A has provided in-force policy data as at the beginning of each year (i.e. 1^{st} January) using age nearest birthday;
- Company B has provided in-force policy data as at the financial year closing date (i.e. 31^{st} March in each year) using age last birthday; and
- Company C has provided in-force policy data as at the end of each year (i.e. 31^{st} December) using age next birthday.

ii) Based on the information above, determine the contribution to central exposed to risk for aged 45 last birthday for the calendar year 2011 from the data available from each of the companies. (8)

[10]

Q. 9) You are provided with the following set of graduated rates, compared against the rates from the standard table Indian Assured Lives Mortality (2006-08).

Age	**Graduated rates**	**Standard table**
40	0.001504	0.001803
41	0.001674	0.001959
42	0.001874	0.002140
43	0.002106	0.002350
44	0.002378	0.002593
45	0.002696	0.002874
46	0.003066	0.003197
47	0.003494	0.003567
48	0.003985	0.003983
49	0.004539	0.004444
50	0.005154	0.004946

Carry out the following tests to determine whether the graduated rates conform to the standard mortality table rates:

i) Standardised deviations test

ii) Chi-square test

iii) Grouping of signs test

You may assume that the exposure at all ages is constant at 100,000. Clearly state your conclusions and any observations based on your tests. **[10]**

Q. 10) A computer manufacturer offers a standard two year replacement warrantee against all technical defects and is planning to launch a new version of its best-selling line of laptops. Pre-launch, the manufacturer has carried out extensive testing on both the old and new versions over a 3 year period and has summarised the observed number of technical failures from an initial sample of 50 units of each version:

Number of technical failures:		
Month / Version	**Old**	**New**
1	1	0
2	0	1
3	2	1
7	1	0
8	1	0
9	0	1
15	1	0
17	0	1
18	2	0
21	0	1
22	1	0
25	1	1
26	0	2
28	2	0
29	0	1
31	1	1
36	0	0

However, the security at the manufacturer's testing facility is rather lax and it is reported that two laptops are stolen each month – one old version and one new version. You can assume that the laptops stolen are one of each old and new versions.

i) Based on a Kaplan-Meir estimate of the survival function for the above data, estimate the proportion of laptops that might need replacement during the warranty period for each version. (6)

ii) The Chief Engineer of the company has commented that the new version of the computers are expected to last up to 20% longer than the previous version and therefore the company should offer a 20% discount for the expected savings from fewer replacement warranties. Comment on the Chief Engineer's statement and explain how you would determine the level of discount that the company could offer? (3)

iii) The manufacturer wishes to change its pricing strategy and make the warranty optional for the customer. The cost of warranty is based on a 95% confidence interval of the technical failure rates observed from the testing. The retail price of the new range is expected to be Rs 50,000 (without any warranty) and the replacement cost to the manufacturer of this is Rs 35,000. Calculate the percentage mark-up that should be applied to the base retail price for customers choosing to opt for the two year warranty. (6)

[15]

Q. 11) Ajoy is an avid cricket fan and he wants to watch the next India vs. Australia to be held in his city. He has Rs. 300 and the minimum price of ticket to the stadium for that particular match is Rs. 800. One of his friends has agreed to make a series of bets with him with Rs X at stake. If Ajoy bets *Rs X*, he wins *Rs X* with probability 0.45 and loses *Rs X* with probability 0.55.

i) Find the probability that he wins Rs. 800 before losing all of his money if

a) he bets *Rs 100* each time. (5)

b) he bets, each time, as much as possible but not more than necessary to bring his capital up to Rs.800. (6)

ii) Which option gives Ajoy the better chance of getting a ticket and watching the match? (1)

iii) Unfortunately, the friend agreed to enter in to a bet backed out at the last minute and Ajoy had to look for new friends for the bet. He couldn't find a single friend who is ready to bet up to Rs.500 so that Ajoy could reach the target sum of Rs 800.

Ajoy could persuade two of his friends (friend A and friend B who are relatively inexperienced in the game) to get them agreed to play up to maximum of Rs 300. In other words, each friend has agreed for a maximum loss of Rs 300.

In order to agree them to place the bets, Ajoy had decided to alter the stake in a manner such that Ajoy would receive Rs 100 upon winning the bet but would lose Rs 150 to his friend upon losing the bet. The probability of winning against friend A is 0.8 and against friend B is 0.9. He starts by placing bet against friend A.

Additionally, A and B had also put a condition not to play more than 5 and 3 rounds respectively subject to sufficient money available to place bets. Evidently, Ajoy had an option not to continue the bets if he doesn't see any chances of reaching the desired sum (including not playing against B at all).

Find the probability that he reaches the desired sum (i.e. Rs. 800) before the game concludes due to reasons stated above. (8)

[20]

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