INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

19th November 2013

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (10.30 – 13.30 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***

2. ***Mark allocations are shown in brackets.***

3. ***Attempt all questions, beginning your answer to each question on a separate sheet. <u>However, answers to objective type questions could be written on the same sheet</u>.***

4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) Describe the cash flows by representing on a timeline:

i) For an organization which provides a 3-year annuity certain, with level payments made at the end of each year

ii) For a borrower of a 3-year fixed interest-only loan, with repayments made at the end of each year

[3]

Q. 2) Calculate the time in years for an investment to **double** at:

i) An effective rate of interest of 10% p.a. (1)

ii) A nominal discount rate of 10% p.a. convertible monthly (2)

[3]

Q. 3) The force of interest $\delta(t)$ at time t is $x+2yt^2$ where x and y are constants. An amount of £700 invested at time t=0 accumulates to £1000 at time t=4 and £1400 at time t=10.

i) Determine x and y. (5)

ii) Calculate the constant force of interest that would give rise to the same accumulation from time t = 0 to time t = 10. (2)

iii) At the force of interest calculated in (ii), calculate the present value of a continuous payment stream of 50exp (0.09t) paid between from time t = 0 to time t = 10. (3)

[10]

Q. 4) An employee proposes to purchase an annuity which is payable monthly in arrears for 10 years. The payments increase at each anniversary by 2% per annum. The annuity is to commence in exactly 8 years at an initial rate of £20,000 per annum. Calculate the amount required in 8 years' time to purchase the annuity assuming an interest rate of 9% p.a. effective.

[4]

Q. 5) An investor takes out a loan to be repaid in installments annually in arrears. The first instalment is £500, the second £550 and so on with the payments increasing by £50 per annum until the end of the 5th year after which the instalments would not increase, with no further payments after 10 years. The rate of interest charged by the lender is 9% per annum effective.

i) Calculate the amount of the loan. (5)

ii) Calculate the interest and capital components of the second instalment. (3)

iii) Calculate the amount of capital repaid in the instalment at the end of the eighth year. (3)

[11]

Q. 6) A life insurance company makes the following investments:

Date	Amount in □ Billions
1st April, 2010	150
1st October, 2010	225
1st April, 2011	130
1st April, 2012	175

The rates of return earned on money invested in the fund were as follows:

Period	Rate of Return
1st April, 2010 to 30th September, 2010	6%
1st October, 2010 to 31st March, 2011	10%
1st April, 2011 to 31st March, 2012	9.5%
1st April, 2012 to 31st March, 2013	9%

It may be assumed that 1st April to 30th September and 1st October to 31st March are precise half year periods.

i) Calculate the linked internal rate of return per annum over the three years from 1st April, 2010 to 31st March, 2013, using half-year sub-intervals. (3)

ii) Calculate the time weighted rate of return per annum over the three years from 1st April, 2010 to 31st March, 2013. (2)

iii) Calculate the money weighted rate of return per annum over the three years from 1st April, 2010 to 31st March, 2013. (5)

iv) Explain the relationship between answers to (a), (b) and (c) above. (2)

[12]

Q. 7) Development of a new car by a motor car company begins on 1st January 2013. The development costs are □6 million payable on 1st January 2013 and □24 million payable continuously during 2014. From 1st January 2015, the car will be ready for production and it is expected that a net income of □8 million per annum will be received quarterly in arrears.

Calculate the discounted payback period at an effective rate of interest of 12% per annum.

[7]

Q. 8) **i)** Give a definition of swap (1)

ii) Describe briefly the main features of interest and currency swap contracts (2)

iii) Discuss the risk characteristics to counterparties of a swap contract (2)

[5]

Q. 9) A five year index-linked security is issued at time zero. The security pays coupons at the rate of 4% p.a. per 100 nominal in arrears and is redeemed at par. The coupons and redemption amount are linked to the index in the following manner:

Real Coupon payment rate = nominal coupon rate* I(t)/I(0) where I(t) represents index at time 't' and I(0) represents index at time zero. The index is

Time	0	1	2	3	4	5
Index	125	128	135	142	145	148

i) Calculate the real yield to a non-tax payer if the price of stock is quoted at 104/- per 100 nominal (4)

ii) Calculate the price at which an investor who pays tax of 20% on coupons only shall purchase the above security to get the same real yield as the non-tax payer. (3)

[7]

Q. 10) **i)** Explain what is meant by the "no arbitrage" assumption in financial mathematics. (2)

A person bought a 3-year forward contract on 1st August 2013 to buy 100 nominal of stock that pays coupons of 4%p.a payable quarterly on 30th June, 30th September, 31st December and 31st March each year for ever. The stock also pays 50% nominal on 31st March 2015. The stock is priced such that it gives an effective yield of 5.5% p.a.

ii) Calculate the forward price of the contract, given that the risk free rate of interest is 6.5% p.a (5)

iii) Determine the value of the forward contract on 1st December 2014 when the stock price is 150% nominal given that the risk free rate of interest is 6.5% p.a (5)

[12]

Q. 11) The prices for the 100 nominal zero-coupon bonds of various terms are given below:

Term in years	1	5	10	15
Price	93	75	45	35

Calculate

i) The continuous time spot rate of interest Y_t for the terms of 10 years and 15 years (2)

ii) The continuous time forward rate of $F_{5,10}$ and $F_{10,5}$. (2)

[4]

Q. 12) An investor has to pay a lump sum of 35000/- at the end of 13 years from now and an annuity certain of 15000/- p.a payable half-yearly in advance for 10 years starting from 10 years from now.

i) Calculate the present value of these two liabilities at an effective rate of interest of 6% p.a. (3)

ii) Calculate the Discounted Mean Term of the liabilities (6)

The investor wishes to immunise his fund against small movements in the rate of interest by investing the cash in two zero coupon bonds, Bond *X* maturing in 7 years' time and Bond Y maturing in 10 years. The market prices of both bonds are calculated at an effective rate of interest of 6% per annum.

iii) Calculate the money invested in X and Y. (5)

[14]

Q. 13) An investor invested an amount of 1/- at the end of each of the years 1, 2 and 3. Let i_t be the effective rate of interest in the year't'. Find the expected value and variance of accumulated value of this investment at the end of 3 years given the following information:

- $E(i_1) = 5\%$; Standard deviation of $i_1 = 0.5\%$
- $E(i_2) = 5.5\%$; Standard deviation of $i_2 = 0.7\%$
- $E(i_3) = 6\%$; Standard deviation of $i_3 = 0.9\%$

You may assume that the interest rates in different years are independent of one another.

[8]

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