Institute of Actuaries of India

Subject CT5 – General Insurance, Life and Health Contingencies

November 2012 Examinations

INDICATIVE SOLUTIONS

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1

Required Probability (P) $= \int_0^\infty \ {}_t\mathrm{p}_\mathrm{x}\ \mu(x+t)\ {}_t\mathrm{p}_\mathrm{y}\ {}_{10}\mathrm{q}_{\mathrm{y+t}} dt = \int_0^\infty \ {}_t\mathrm{p}_\mathrm{x}\ \mu(x+t)\ {}_t\mathrm{p}_\mathrm{y}\ (1 - {}_{10}\mathrm{p}_{\mathrm{y+t}}) dt$

$\mathrm{P} = \int_0^\infty \ {}_t\mathrm{p}_\mathrm{x}\ \mu(x+t)(\ {}_t\mathrm{p}_\mathrm{y}\ - {}_{t+10}\mathrm{p}_\mathrm{y}\ )dt = \ \int_0^\infty \mathrm{e}^{-0.01\mathrm{t}}(0.01)\left(\mathrm{e}^{-0.01\mathrm{t}} - \mathrm{e}^{-0.01(\mathrm{t}+10)}\right)dt$

P = 0.0476

[4]

Solution 2

(i) q_{43} = 1.10 * q_{42} = 1.10 * 0.00232 = 0.002552

q_{44} = 1.10 * q_{43} = 1.10 * 0.002552 = 0.002807

q_{45} = 1.10 * q_{44} = 1.10 * 0.002807 = 0.003088

v = 1/(1.0825) = 0.92739

$q_{[43]}$= 0.5 * q_{43} = 0.001276

$q_{[43]+1}$= 0.5^(1/2) * q_{44} = 0.7071 * 0.002807 = 0.001985

$q_{[43]+2}$= 0.5^(1/3) * q_{45} = 0.7937 * 0.003088 = 0.002451

EPV of benefits = 10,00,000 * $q_{[43]}$* v + 9,00,000 * $p_{[43]}$ * $q_{[43]+1}$ * v^2 + 8,00,000 * $p_{[43]}$* $p_{[43]+1}$* $q_{[43]+2}$ * v^3

= 10,00,000 * 0.001179 + 9,00,000 * 0.001692 + 8,00,000 * 0.001926

= Rs 4242.6

Let P be the level annual premium, then

EPV of premiums = P * [1 + $p_{[43]}$* v + $p_{[43]}$* $p_{[43]+1}$* v^2]
= P * [1 + 0.92261 + 0.85061]

= 2.77322 * P

Using principle of equivalence

- ⇨ 2.77322 P = 4,242.6
- ⇨ P = Rs 1,529.8

Comment: Candidates who read $\boldsymbol{q_{[x-t]+t}}$ differently were not penalized and full credit was given.

(6)

(ii) if the value of future cover becomes less than the value of premium, policyholder may lapse the policy and the company may suffer a loss.

Possible solutions:

1. Short premium payment term
2. Make premiums larger in earlier years, smaller in later years

(2)

[Total Marks - 8]

Solution 3

Let superscript (f) denote “academic failure” and (w) denote “withdrawal for all other reasons”

$l_0^{(t)}$ = number of students entering Year 1

Then, $p_0^{(t)} = 1 – 0.4 – 0.2 = 0.4$

Given, $l_2^{(t)} = 10\, l_2^{(t)}\, q_2^{(f)} \rightarrow q_2^{(f)} = 0.1$

$q_2^{(w)} = 1 – 0.6 – 0.1 = 0.3$

Also given, $l_1^{(t)}\, q_1^{(f)} = 0.4 * [l_1^{(t)} * (1 – q_1^{(f)} – q_1^{(w)})]$

$q_1^{(f)} = 0.4 * (1 – q_1^{(f)} – 0.3) \rightarrow q_1^{(f)} = 0.2$

$p_1^{(t)} = 1 – q_1^{(f)} – q_1^{(w)} = 1 – 0.2 – 0.3 = 0.5$

Required Probability = $q_0^{(w)} + p_0^{(t)}\, q_1^{(w)} + p_0^{(t)}\, p_1^{(t)}\, q_2^{(w)}$

= 0.2 + 0.4*0.3 + 0.4*0.5*0.3 = 0.38

[6]

Solution 4

(i) Methods: 1. Uniform distribution of deaths (UDD)

2. Constant force of mortality (CFM)

(1)

(ii) For $0 \le s < t < 1$,

$${}_{t-s}q_{x+s} = 1 - {}_{t-s}p_{x+s} = 1 - \frac{{}_tp_x}{{}_sp_x} = 1 - \frac{1-\ {}_tq_x}{1-\ {}_sq_x} = 1 - \frac{1-tq_x}{1-sq_x} = \frac{(1-sq_x)-(1-tq_x)}{1-sq_x} = \frac{(t-s)q_x}{1-sq_x}$$

(4)

(iii) We know that ${}_tp_x$ = exp $\{-\int_0^t \mu_{x+s}\, ds\} = e^{-t\mu}$ => $p_x = e^{-\mu}$

For 0 ≤ s < t < 1, ${}_{t-s}p_{x+s}$ = exp $\{-\int_s^t \mu_{x+r}\, dr\} = e^{-(t-s)\mu}$ = ${p_x}^{t-s}$

(2)

(iv) ${}_{0.75}q_{75.75}$ = 1 - ${}_{0.75}p_{75.75}$ = 1 - ${}_{0.25}p_{75.75}$ * ${}_{0.50}p_{76}$ = 1 – (1- ${}_{0.25}q_{75.75}$)* (1 - ${}_{0.50}q_{76}$)

Now,

${}_{0.25}q_{75.75}$ = $\frac{(0.25)q_{75}}{1-0.75q_{75}}$ = $\frac{0.25*0.06197}{1-0.75*.06197}$ = 0.01625

${}_{0.50}q_{76}$ = 0.50 * q_{76} = 0.50 * 0.06777 = 0.03389

Hence, ${}_{0.75}q_{75.75}$ = 1 – (1-0.01625)*(1-0.03389) = 1 – 0.95041 = 0.04958

(4)

[Total Marks – 11]

Solution 5

(i) The directly standardised rate (DSR) is given by:

DSR = $\frac{\sum_x \mathrm{E}_{x,t}^{cs}\mathrm{m}_{x,t}}{\sum_x \mathrm{E}_{x,t}^{cs}}$

The standardised mortality ratio (SMR) is given by:

SMR = $\frac{\sum_x \mathrm{E}_{x,t}^{c}\mathrm{m}_{x,t}}{\sum_x \mathrm{E}_{x,t}^{c}\mathrm{m}_{x,t}^{s}}$

For 5 year policies:

DSR = (6.991*0.86 + …) / (6.991 + …) = 13.56053

SMR = (6.013*0.86 + …) / (6.013*1.08 + …) = 0.920149

For 10 year policies:

DSR = (6.991*2.12 + …) / (6.991 + …) = 21.06187

SMR = (0.978*2.12 + …) / (0.978*1.08 + …) = 1.424669

In each of the above, DSR is expressed as the number of deaths per 1000.

(5)

(ii) SMR is favourable. The directly standardized mortality rate requires $m_{x,t}$ to be recorded for each age group, for the 5-year and 10-year policies separately. The data may not be readily available. The SMR requires the number of deaths in each age and policy group only to be recorded: these data can be easily recorded.

(2)

[Total Marks – 7]

Solution 6

(i) The benefits may be classified into three types.

Accrued benefit is a benefit that has been earned as a result of pensionable service (or credited service) prior to the valuation date eg a pension of n / 80^{th} of final average salary where n is the number of years of past pensionable service at the valuation date.

Future service benefit is a benefit that is expected to be earned as a result of pensionable service after the valuation date eg a pension of (65 − x)/ 80^{th} of final average salary on age retirement at NPA of 65 for a member aged x at the valuation date.

Some benefits do not depend on either past or future service, although they may depend on total expected pensionable service. Such benefits are termed prospective service benefits eg a pension of m/80^{th} of final average salary at date of ill-health retirement where m is the total expected pensionable service before NPA. An example of a benefit which is prospective but which does not depend on amount of service would be a death in service benefit of 4 × salary at the date of death.

(5)

(ii) Withdrawal from a pension scheme is associated with voluntary or compulsory termination of employment (changing jobs or redundancy). If voluntary resignation is the cause this tends to select those with lighter mortality (and ill-health retirement) rates. If redundancy is the cause withdrawal rates tend to vary markedly over time as economic conditions vary.

(2)

[Total Marks – 7]

Solution 7

(i) In many life insurance contracts, the expected cost of paying benefits increases over the term of the contract – consider an endowment assurance, for example. The probability that the benefit will be paid in the first few years is small – the life is young and in good health. Later the expected cost increases as the life ages and the probability of a claim by death increases. In the final year the probability of payment is large, since the payment will be made if the life survives the term, and for most contracts (which often mature when the policyholder is in his or her 50s or 60s) the probability of survival is large.

Although on average the cost of the benefit is increasing over the term, the premiums that pay for these benefits are level. This means that the premiums received in the early years of a contract are more than enough to pay the benefits that fall due in those early years. But in the later years, and particularly in the last year of an endowment assurance policy, the premiums are too small to pay for the benefits.

It is therefore prudent for the premiums that are not required in the early years of a contract to be set aside, or reserved, to fund the shortfall in the later years of the contract. While funds are reserved, they are invested so that interest also contributes to the cost of benefits.

If the life insurance company were to spend all the premiums received, perhaps by distributing to shareholders the premiums that were not required to pay benefits, then later in the contract the company may not be able to find the money to pay for the excess of the cost of benefits over the premiums received. We set up reserves to ensure (as far as possible) that this does not happen, and that the company remains solvent.

(4)

(ii) Conditions for equality
If:
1. the retrospective and prospective reserves are calculated on the same basis, and
2. this basis is the same as the basis used to calculate the premiums used in the reserve calculation,
then the retrospective reserve will be equal to the prospective reserve.

(2)

(iii) The prospective reserve is:

A_{x+t} - $P_x\, \ddot{a}_{x+t}$

and the retrospective reserve is:

$P_x * \ddot{s}_{x:\bar{t}|} - \frac{(1+i)^t}{{}_tp_x} A^1_{x:\bar{t}|}$

The first term in the retrospective reserve is interpreted as the expected accumulation of premiums received, and the second term as the expected accumulated cost of benefits paid.
Starting with the prospective reserve:

${}_tV_x = A_{x+t} - P_x\, \ddot{a}_{x+t}$

$= A_{x+t} - P_x\, \ddot{a}_{x+t} - P_x * \ddot{s}_{x:\bar{t}|} + P_x * \ddot{s}_{x:\bar{t}|}$

Now, $_{t|}\ddot{a}_x = v^t * {}_tp_x * \ddot{a}_{x+t}$ and $\ddot{s}_{x:\overline{t}|} = \frac{(1+i)^t}{_tp_x} * \ddot{a}_{x:\overline{t}|}$

$= P_x * \ddot{s}_{x:\overline{t}|} - \frac{(1+i)^t}{_tp_x} P_x \, ({}_{t|}\ddot{a}_x + \ddot{a}_{x:\overline{t}|}) + A_{x+t}$

$= P_x * \ddot{s}_{x:\overline{t}|} - \frac{(1+i)^t}{_tp_x} P_x \, \ddot{a}_x + A_{x+t}$

$= P_x * \ddot{s}_{x:\overline{t}|} - \frac{(1+i)^t}{_tp_x} \, A_x + A_{x+t}$

$= P_x * \ddot{s}_{x:\overline{t}|} - \frac{(1+i)^t}{_tp_x} \, A^1_{x:\overline{t}|}$

which is the formula for the retrospective reserve.

(5)
[Total Marks – 11]

Solution 8

Replacing age by years of service, the probability that Varun will be an executive in 35 years is $_{35}p_0^{01}$

Which is, $_{35}p_0^{01} = \int_0^{35} {}_{\mathrm{t}}\mathrm{p}_{00} * \mu_t^{01} * {}_{35-\mathrm{t}}\mathrm{p}_{\mathrm{t}}^{11}\mathrm{dt}$

$= \int_0^{35} \mathrm{e}^{-(\mu_t^{01}+\mu_t^{02})t} * \mu_t^{01} * \mathrm{e}^{-\mu_t^{12}(35-t)}\mathrm{dt}$

Substituting the values,

$= 0.01 * \mathrm{e}^{-0.7} \int_0^{35} \mathrm{e}^{-0.05t}\mathrm{dt}$

$= 0.01 * \mathrm{e}^{-0.7} * (1 - \mathrm{e}^{-1.75})/0.05$

= 0. 08205

For Varun to retire as an executive, he will need to become an executive in Company ABC in 35 years and survive to age 65.

So required probability = 0.08205 * 0.9 = 0.0733

[5 Marks]

Solution 9

(i) GFLRV = 200 + 0.10 P + 250 * $a_{\overline{\min(K_{[50]},9)}|}$ - P $\ddot{a}_{\overline{\min(K_{[50]}+1,10)}|}$
+ (if $K_{\{50]} < 10$) $v^{K_{[50]}+1}$ * (100,000 + 10,000 $K_{[50]}$ + 200)

(4)

(ii) By equivalence principle, E{L} = 0

⇨ P * $\ddot{a}_{[50]:\overline{10|}}$ = 200 + 0.l0 * P + 250 * ($\ddot{a}_{[50]:\overline{10|}}$ - 1) + 90,000 * $\bar{A}^{1}_{[50]:\overline{10|}}$ + 10,000*$(\overline{IA})^{1}_{[50]:\overline{10|}}$ + 200 * $\bar{A}^{1}_{[50]:\overline{10|}}$

Now,
$\ddot{a}_{[50]:\overline{10|}}$ = 7.698

$A^{1}_{[50]:\overline{10|}}$ = $(1.06)^{0.5}[A_{[50]:\overline{10|}}$ - $v^{10}*\ {}_{10}p_{[50]}]$
= 1.02956 * [0.56426 – 0.55839 * $\frac{9287.2164}{9706.0977}$] = 0.03086

$(IA)^{1}_{[50]:\overline{10|}}$ = $(1.06)^{0.5}[(IA)_{[50]}$ - $v^{10}*\ {}_{10}p_{[50]}$ {10* A_{60} + $(IA)_{60}$}]
=1.02956 *[4.84789 – 0.55839 * $\frac{9287.2164}{9706.0977}$ {10 * 0.32692 + 5.46572}]
= 1.02956 * [4.84789 – 0.53429 * 8.73492]
= 0.18626

⇨ P * 7.598 = 200 + 250 * 6.698 + 90200 * 0.03086 + 10,000 * 0.18626
⇨ P * 7.598 = 6520.67
⇨ P = Rs 858

(7)
[Total Marks – 11]

Solution 10

Let *b* be the simple bonus rate (expressed as a percentage of the sum assured). Then the equation of value at 4% p.a. interest is (where *P* = 1,972):

⇨ P * $\ddot{a}^{@4\%}_{45:\overline{20|}}$ = (50,000 +250) * $A^{@4\%}_{45:\overline{20|}}$ + 500 b $(IA)^{@4\%}_{45:\overline{20|}}$ + 0.20P + 0.05P ($\ddot{a}^{@4\%}_{45:\overline{20|}}$-1) +50 ($\ddot{a}^{@\mathbf{0}\%}_{45:\overline{20|}}$-1)

Now,

$\ddot{a}^{@4\%}_{45:\overline{20|}}$ = 13.780
$A^{@4\%}_{45:\overline{20|}}$ = 0.46998

$(IA)^{@4\%}_{45:\overline{20|}}$ = $(IA)_{45}$ - $v^{20}*\ {}_{20}p_{45}$ [20* A_{65} + $(IA)_{65}$]
= 8.33628 – 0,45639 * $\frac{8821.2612}{9801.3123}$ * [20 * 0.52786 + 7.89442]
= 8.33628 – 0.41075 * 18.45162
= 0.75727

$\ddot{a}^{@\mathbf{0}\%}_{45:\overline{20|}}$ = (1 + e_{45}) - ${}_{20}p_{45}$ * (1 + e_{65}) = (1+34.271) - 0.9 * (1+16.645) = 19.3905

Hence,

- ⇨ P * 13.780 = 50,250 * 0.46998 + 500b * 0.75727 + 0.20P + 0.05P* 12.780 +50 * 18.3905
- ⇨ 12.941 P = 23616.5 + 378.6b + 919.7
- ⇨ 378.6b = 12.941* 1972 – 23616.5 – 919.7 = 983.5
- ⇨ b = 2.6

i.e. a simple bonus rate of 2.6% per annum.

[9]

Solution 11

(i) Scenario 1

Year	UF at start	Premium	Allocation Charge	UF after charges	UF after growth	FMC	UF at end
	(1)	(2)	(3)=(2)*Alloc Charge	(4)=(1)+(2)-(3)	(5)	(6)=(5)*1%	(7)=(5)-(6)
1	0	50,000	2,500	47,500	50,667	507	50,160
2	50,160	50,000	1,500	98,660	95,577	956	94,621
3	94,621	50,000	1,000	143,621	152,887	1,529	151,358

$(5)_t = (4)_t$ * (XYZ level at time t / XYZ level at time t-1) under Scenario 1

Scenario 2

Year	UF at start	Premium	Allocation Charge	UF after charges	UF after growth	FMC	UF at end
	(1)	(2)	(3)=(2)*Alloc Charge	(4)=(1)+(2) -(3)	(5)	(6)=(5)*1 %	(7)=(5)- (6)
1	0	50,000	2,500	47,500	49,083	491	48,593
2	48,593	50,000	1,500	97,093	95,526	955	94,571
3	94,571	50,000	1,000	143,571	148,278	1,483	146,796

$(5)_t = (4)_t$ * (XYZ level at time t / XYZ level at time t-1) under Scenario 2

Guaranteed Amount at maturity = 3 * Annual Premium = Rs. 150,000

Shortfall under Scenario 1 = 150,000 - FV at maturity = 150,000 – 151,358 → negative, hence zero since difference cannot be negative.

Shortfall under Scenario 2 = 150,000 - FV at maturity = 150,000 – 146,796 = 3,204

Assigning the respective probabilities, CoG = PV of (0 * 0.6 + 3204 * 0.4) = PV of 1282

= 1282 / 1.06^3 = 1076

(8)

(ii) By reducing the charges under the product which will increase the projected fund value and hence will reduce the difference between the guaranteed amount and fund value at maturity.

By changing the investment strategy by not replicating the XYZ index and investing in other available investments which offer safe and steady return or following other indices which are more stable in nature.

Comment: Other valid reasons were given full credit

(2)

(iii)

Year	UF at start	Premium	Allocation Charge	UF after charges	UF after growth	FMC	UF at end
	(1)	(2)	(3)=(2)*Alloc Charge	(4)=(1)+(2)-(3)	(5)=(4)*1.08	(6)=(5)*1%	(7)=(5)-(6)
1	0	50,000	2,500	47,500	51,300	513.0	50,787
2	50,787	50,000	1,500	99,287	107,230	1,072.3	106,158
3	106,158	50,000	1,000	155,158	167,570	1,675.7	165,895

Cash flows per policy:

Year	Allocation Charge	FMC	Expense	Commission	CoG	Interest	End of year cash flow
	(8)=(3)	(9)=(6)	(10)	(11)	(12)=2.5%*(2)	(13)=6%*[(8)-(10)-(11)]	(14)=(8)+(9)-(10)-(11)-(12)+(13)
1	2,500	513.0	500	1,000	1,250	60.0	323.0
2	1,500	1,072.3	500	1,000	1,250	0.0	(177.7)
3	1,000	1,675.7	500	1,000	1,250	(30.0)	(104.3)

Non-unit reserves required at the end of each policy year:

$V_3 = 0$, since there are no future cash flows

$V_2 = 104.3/1.06 = 98.29$

$V_1 * 1.06 = (1-0.001) * V_2 + 177.7 \rightarrow V_1 = 260.37$

Comment: Above solution assumes cost of guarantee expense at end of year. Full credit was given to candidates who assumed this expense at the beginning of the year.

(6)

(iv)

- It will increase the level of protection of the policyholders by increase in reserves
- Increase in reserves mean an increase in capital requirements
- To maintain the same profitability, the profits now have to be higher to provide the required rate of return on the capital, which would mean greater cost to the customer through higher charges
- Higher charges will make the product less attractive to the customers and hence the expected new business volumes might be low

(3)

(v) FMC reduction will tend to decrease the profitability. However, reduction in FMC will decrease the CoG, as the difference between the projected fund value and the guaranteed amount will reduce. The profitability may hence increase or decrease.

(2)

[Total Marks – 21]

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