Institute of Actuaries of India

Subject ST8 — General Insurance: Pricing

November 2011 EXAMINATION

INDICATIVE SOLUTION

1.

i. **Annual Aggregate Deductible**: These are deductibles that apply to the annual aggregate (otherwise recoverable) losses.

For example, with an annual aggregate deductible of Rs 1 Crore on an excess of loss treaty, this means that if reinsurance recoveries on an excess of loss treaty total less than Rs 1 Crore (within a year), then no recovery will in fact be made. Once the recoveries exceed Rs 1 Crore, then the reinsurance structure applies on the excess over Rs 1 Crore.

ii. **Reinstatements:** Within excess of loss reinsurance, reinstatements are the restoration of full cover following a claim. Normally, the number of reinstatements, and the terms upon which they are made, will be agreed at the outset. Once agreed, they are automatic and obligatory on both parties.

Unlimited free reinstatements mean that reinstatements can continually be made, at no cost. Paid reinstatements mean that a reinstatement premium must be paid before the reinstatements go ahead.

iii. **Indexed Limits**: Where inflation has a significant effect on the cost of claims, a stability clause may be applied to the excess point. This is so that the reinsurer does not receive a higher proportion of the risks purely because of inflation. The cedant will normally be required to pay an extra premium to compensate the reinsurer for the added risk if the excess point is not indexed.

An inflation index is agreed between the reinsurer and the cedant and the retention and limits of the cover vary as per the inflation index. A particular index is applied to a claim depending on when the claim was made. However, the exact terms will depend on the cover wording.

iv. **Overriding Commission**: Commission paid by a reinsurer to an insurer ceding proportional business, as a contribution towards expenses and profit.

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2.

- GLM method of pricing makes certain assumptions about the experience:
  - The response variable should be a member of the exponential family
  - There is a certain relationship between a function of a parameter of the response variable distribution and a linear additive function of the explanatory variables with a set of parameters to be estimated.
- The parameters estimated based on the above assumptions are used to estimate the expected value of the response variable for a given combination of explanatory variables. The parameters are estimated using the maximum likelihood estimation method.
- However the fit of the assumed distribution (for example for claim amount distribution) to the actual experience data may not be good. In such a case using GLM may not give a satisfactory. It may happen that the data does not fit well to any of the claim distributions in the exponential family.
- Moreover, an additive function of the explanatory variables may not explain their effect correctly on the response variable.
- In such a scenario, a nonparametric analysis will give a better result if it is based on a sufficiently large volume of data.
- However, a nonparametric model based on the experience should only be used if the data is 'fully credible'. Alternatively, if a market level statistics is available, a credibility factor based approach may be used.
- GLM technique has several strengths which a nonparametric analysis may not have such as:

- Analysis of signification of factors,
- Analysis of standard errors,
- Modelling with offsetting,
- ease of modelling interaction terms,
- simple output format,
- Analysis for testing model appropriateness, and
- modelling with restrictions.

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3.

The aspects which should be analyzed are:

i. The amount of risk that an insurer can safely retain, having regard to its solvency position and available capital.

ii. The extent of the likely exposure to accumulations of risk should be analyzed. In case of significant exposure to risk accumulation, aggregate excess of loss cover may be appropriate.

iii. The need for catastrophe reinsurance, having regard to the insurer's exposure to both natural and human-made catastrophes, and the appropriate upper and lower limits for such cover

iv. The extent of the possible need for reinstatements to cover

v. The value for money provided by the existing reinsurance programme, and whether it meets the objectives of the business; for example, does it sufficiently reduce the volatility of the claims experience?

vi. A general assessment of the appropriateness of existing covers (proportional vs non-proportional, for example).

vii. The profitability of layers

viii. The effects on capital. In many countries the reinsurance reduces the amount of capital required. The reinsurance cost should be compared with the reduction in capital required and the resultant reduction in cost of capital.

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4.

i. Inputs required to run a CAT model are:

a. Detailed exposure database of the insured risks and their locations and structure details (age, occupancy, construction type etc.).

b. Details of the policy conditions such as limits, excess, sub-limits, coverage terms etc.

ii. The catastrophe model output would usually be the distribution of events. There are two bases for these files:

a. OEPs – an occurrence exceedance probability file, which considers the probability that the largest individual event loss in a year exceeds a particular threshold. The problem with this file is that it may ignore the possibility of multiple events.

b. AEPs – an aggregate exceedance probability file, which considers the probability that the aggregate losses from all loss events in a year exceeds a particular threshold.

iii. Two ways to load for cost of CAT XL reinsurance in the pure risk premium:

a. Loading gross cost of reinsurance: We can load the gross cost of reinsurance on the net risk premium (risk premium net of reinsurance premium)

b. Loading net cost of reinsurance: We can load the net cost of reinsurance (reinsurance premium less reinsurance recoveries) on the gross risk premium.

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5.

i. An interaction term is used where the pattern in the response variable is better modelled by including extra parameters for each combination of two of more factors.

An interaction exists when the effect of one factor varies depending on the levels of another factor.

For example, male drivers may have an x% higher risk than female drivers. Young drivers may have a y% higher risk than older drivers. However, the combination of being young and male may result in a much higher risk than ((1+x/100)(1+y/100) -1) ×100%. In this case, the effect of age depends on gender, and the effect of gender depends on age.

ii. Two ways in which interactions may be expressed:

a. Complete interaction: In this method we consider a single factor representing every combination of the two factors

Factor 1:		A	B
Factor 2:	X	0.65	0.80
	Y	0.90	1.00
	Z	0.97	1.20

In this case, the base level has been selected to be the level corresponding to Level B of Factor 1 and Level X of Factor 2, and the interaction term has 5 parameters.

b. Marginal interaction: An alternative representation of this interaction is to consider the single factor effects of Factor 1 and Factor 2 and the additional effect of an interaction term over and above the single factor effects (or "marginal" interaction). A set of multipliers in this form can be set as follows:

Factor 1:			A	B
			0.90	-
Factor 2:	X	0.80	0.90	-
	Y	-	-	-
	Z	1.20	0.90	-

In this case fewer parameters are present in the additional interaction term because the presence of the single factor effects makes some of the interaction terms redundant.

iii. Methods for simplifying factors:

a. **Group and summarize data prior to loading**: In this method, the grouping may be decided before the modelling is done based on the distribution of exposure among the various levels of a factor. This requires knowledge of the pattern that is expected. It is now mainly adopted as a method to thin out redundant codes from the data that has little exposure.

b. **Grouping in the modelling package**: Often called a custom factor, this method simply assigns a single parameter to represent the relativity for multiple levels of the factor.

c. **Curve fitting or use of variate**: The levels of a factor are each assigned an x -value and a polynomial (in the examples above a cubic and linear were used) is fitted to the factor. In this case, the parameters in the model are just the parameters from the polynomial itself, excluding the constant term.

d. **Piecewise curve fitting**: The factor levels are broken into sections and a custom factor and/or curve from Methods (b) and (c) is applied to each section. By combining these in different ways, the join at each section boundary can be disjoint or piecewise continuous as the modeller thinks appropriate.

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6. Factors considered in establishing central computer system

Function of the system. Need input from all potential users:

- administration
- accounting
- actuarial / statistical
- individual general insurance companies
- Analysts

For private motor, there is potentially a lot of data to be processed. All of the above users will rely heavily upon the system. Their input initially is important.

Structure or outline of the system. How should the system operate? Will the Individual companies have direct access to the system?

Which computer? What type of software?

Cost of implementation.

Security. How will the system be monitored? Who will have access to the system? Data protection laws should be considered, particularly if the system is to be used by third parties.

The data that will need to be held. For private motor, we should retain all the data for individual policyholders on the system. How to ensure that there is no theft of policy data by rival companies?

How data will be uploaded into the system?

How files will be structured. For example, should the policy and claims records be held on separate files?

How long the data will be stored. Statistical analyses will be necessary (in due course) using the data that develops. Every effort should be made to store data indefinitely.

Capability. The system will need to be able to capture claim features such as multiple payments, nil claims and reopened claims.

The output from the system is important. Again, ask all potential users of the system. Examples of required output are: loss ratios by rating factors, claim frequency, claim severity, may be use in GLM exercise.

Ease of use. Need to consider the computer literacy of the end users.

Implementation of the system. How the system will be developed, and by whom. How it will be tested.

When the system will be ready. training program to general insurance companies to upload the data

Error aversion. What steps will be taken to ensure data accuracy?

Changes to the system. The system must be sufficiently robust so that future developments can be incorporated.

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7.

Background information:

- Any statutory regulation on expense limits and or classification
- Any published / internal policy on expense treatment and allocation
- Volumes of business sold by class, premium income and numbers of policies,
- Expected growth in volumes as per original plan at commencement and as per recently revised plans,
- Similar data of competitors to the extent available from published returns and regulators' reports. (Extent of untapped potential %)
- Summary of expenses for each of the two years split by fixed and variable and by direct and indirect for each class and main risk groups of business.
- Totals of expenses under main heads of account subdivided in a similar way.

Specify problem:

- To identify deficiency in the structure of current expense charge and establish appropriate structure and parameters for use in premium rating of each class of business and main risk groups.

Risks:

- Inadequate expense charges by class, by main risk groups or in total leading to losses or attracting bad risks
- Excessive charge similarly or premium rates not being competitive in certain groups leading to loss of business.
- Improper structure for expense charge leading to losses in some groups or being sensitive to changes in business mix or volumes.
- Unjustified or excessive cross subsidies between classes of business or main risk groups.
- Improper allocation of expenses between classes or main risk groups,

Solution:

- Expenses will be subdivided in to heads of account such as staff salaries, rent, office equipment, and consumables, communication expenses, stamp duties, etc.
- These will again be grouped by nature of functions such as underwriting, policy alterations, claims handling, overheads. etc.
- Allocation of expenses by function and overheads and indirect costs by class/ main risk groups will be as realistic as possible.
- Where allocation is based on a functional costing approach a review of the basis will he made in the light of experience over two years.
- Similarly the basis of allocation between fixed and variable costs will be reviewed in the light of observed correlations for the two years.
- The analysis as above will be used to derive the charges towards claims handling. fixed expenses and variable expenses expressed as per cent of premium.
- Allowance for inflation on fixed expenses may be required.
- The expense charges will be tested and adjusted for robustness and sensitivity for changes in business volumes and mix of business.

Monitoring:

- Volumes of business in terms of numbers of policies and premiums by class of business and main risk groups quarterly- short falls might indicate need for upward revision of expense charges.
- Developments relating to unit costs on heads of account such as salaries, rent, communication charges- deviations from those assumed in expense model will need urgent review.
- Actual experience on expenses and business volumes relating to allocation bases will be reviewed at least annually.

Professionalism:

- Any professional guidance note relevant will be checked for compliance. Where such guidance is not available, a senior professional with adequate experience on the topic may be consulted for opinion.

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8.

i. The x-axis of the exposure curve is the size of the loss in proportion to the size of the risk (or sum insured or the probable maximum loss). As usually there is no sum insured limit in casualty covers, exposure curves cannot be created and used for pricing casualty insurance business.

ii. The assumptions are:

i. the (ground-up) loss frequency is independent of the limit purchased

ii. the (ground-up) severity is independent of the number of losses and of the limit purchased

iii.

Claim band	Number of claims	Total claim amount
1- 100,000	200	12,000,000
100,001 - 200,000	50	7,000,000
200,001 - 500,000	10	3,750,000
More than 500,000	2	1,500,000

Upper Bound	Total claim amount with upper bound as the limit	ILF at the upper bound
100,000	18,200,000	1.00
200,000	21,400,000	1.18
500,000	23,750,000	1.30

Table 8.c.1

Please refer to the formulae in the table below:

	A	B	C	D	E	F	G
1							
2		Claim band	Number of claims	Total claim amount	Upper bound	Total claim limited at the upper bound	ILF at the upper bound
3		1- 100,000	200	12000000	100000	=D3+SUM(C4:C6)*E3	
4		100,001 - 200,000	50	7000000	200000	=SUM(D3:D4)+SUM(C5:C6)*E4	=F4/F3
5		200,001 - 500,000	10	3750000	500000	=SUM(D3:D5)+C6*E5	=F5/F3
6		More than 500,000	2	1500000			
7							

Table 8.c.2

iv. Expected loss cost for limit 200,000 = basic cost * ILF at 200,000
= 2500 * 1.18 = 2,950
Expected loss cost for limit 500,000 = basic cost * ILF at 200,000
= 2500 * 1.30 = 3,250
So, the expected loss cost for the layer 300,000 xs 200,000 = 3250 – 2950 = 300

v. The expected occurrence point for the experience losses was middle of 2010. Policies will be sold through 2011 so the expected occurrence point of the future claims will be end of 2011. Therefore inflation adjustment for 1.5 years will be applied.
Therefore the ILFs above will now be applicable for the limits after applying the above adjustment factor. The new limit at which the ILF is applicable = Old limit * 1.10^1.5 [1.5 marks for correct working in the below table.]

Original Limits	ILFs	New Limits after inflation adjustment
100,000	1.00	115,369
200,000	1.18	230,738
500,000	1.30	576,845

Table 8.e.1

As the pure cost is given at the basic limit of 100,000 and the layer to price is 300,000 xs 200,000, we will require ILFs at 100,000, 200,000 and 500,000. This can be done using linear interpolation:

ILF required at	ILF at the required points (using linear interpolation)	ILF restated with basic limit 100,000
100,000	0.98	1.00
200,000	1.13	1.16
500,000	1.27	1.30

Table 8.e.2

Please refer to the formulae in the table below:

	A	B	C	D	E	F	G	H
1								
2		Original Limits	ILFs	New Limits after inflation adjustment		ILF required at	ILF at the required points (using linear interpolation)	ILF restated with basic limit 100,000
3		100000	1	=B3*1.1^1.5		100000	=C4-(C4-C3)/(D4-D3)*(D4-F3)	1
4		200000	1.18	=B4*1.1^1.5		200000	=C3+(C4-C3)/(D4-D3)*(F4-D3)	=G4/G3
5		500000	1.3	=B5*1.1^1.5		500000	=C4+(C5-C4)/(D5-D4)*(F5-D4)	=G5/G3
6								

Table 8.e.3

The pure claim cost for the layer 300,000 xs 200,000 will be 2800 *(1.30 – 1.60) = 392.
Apart from the assumption required for application of ILF (as per (ii) above), the other assumption is that the inflation impacts all types and size of claims uniformly.

vi. The pure claim cost for the layer 300,000 xs 200,000 increased approximately 30% while the gross claim cost increased approximately 15%. Due to leverage effect of the excess layers, the

impact of inflation on the excess layers is always higher than the effect on the gross claim cost assuming the uniform effect of inflation. Take a very simple example of one claim. If there is only one claim of 400,000, due to 15% inflation it will increase to 460,000. However the excess loss to the layer 300,000 xs 200,000 will increase from 200,000 to 260,000 (30% increase).

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9.

(i) Rating factors

- payroll
- number of employees
- type of industry or occupation
- exposure and claims experience
- location of workforce
- materials handled
- processes involved.

(ii) Employers' liability experience rating

Choice of base period for collecting data

The first step is to consider the base period for collecting the data. The longer the base period, the more credible the data but, equally, the less relevant the data will be.

This might be due to changes in underwriting conditions (such as size of excess, exclusions and type of cover) or due to changes in the employer's type of work, and so on.

A reasonably long period is needed; say five years, to allow for the full development of injury claims.

Looking at each year individually will give some indication of trends.

Collect data

Data must be collected for the exposure and claims relating to the base period

This might be split by class of employee.

Information about the rating factors will also be collected, to see how they have changed over the period of investigation.

The claims data would include the number of claims and claim payment amounts. The number of claims might be split by claim type and peril (to investigate trends). This should also identify many claims from one event, to allow for accumulations of risk.

The claim payment information would consist of claim amounts paid plus outstanding claims. This should allow for reopened claims.

Nil claims should be treated consistently throughout.

Account must be taken of the fact that the latest policy year will not be complete (because renewal is invited before the end of the policy year).

The claim amount must be adjusted to allow for IBNR. This IBNR must be split into accident and industrial disease.

Allowing for accidents will be a reasonably straightforward addition to the claims incurred. For industrial disease this will be very difficult to estimate, but some allowance must be made. Allowing for industrial disease will also affect the pattern of development.

Trends in claim frequency

Any trends in claim frequency should be allowed for, by projecting trends up to the average future claim date. The adjustment for trends might be based upon analysis of employer's experience, analysis of the whole portfolio or industry data, press reports, and so on.

Care should be taken with nil claims when analysing these trends.

Any changes in the rating factors or underwriting conditions could affect the trends in claim frequency.

Large claims

Account must be taken of large claims. These will be truncated at a suitable level and spread over the whole portfolio using a suitable grossing-up factor. The grossing-up factor is derived by analysis of the whole portfolio.

Claim inflation

Claims should be inflated up to the average future claim date by a suitable inflation index. Some claims will be salary-related, so the national average earnings index is an appropriate rate for these claims. For other types of claim, court award inflation might be more appropriate.

Inflation should also be applied to the exposure measure. If claims and exposure both increase by the same amount then the inflation rate is irrelevant.

Calculating the premium

The risk premium will be calculated as a weighted average of the premium based upon the employer's past experience (as calculated above) and the risk premium for the whole portfolio, calculated in the same way for the insurer's whole portfolio.

The size of this weight will depend upon the size of the scheme and commercial considerations.

This risk premium would then be adjusted to allow for expenses, commission, investment income, contingency margins and profit.

The office premium will then be adjusted, taking into account competitors' rates and the insurance cycle.

(iii) Allowing for the deductible

Fit a distribution for the amount of the claim with the deductible

The first step is to find a distribution function for the total amount of claims for all the employees within the scheme, before applying the deductible.

This can be found either by taking the data from part (ii) and fitting a distribution, or by stochastic simulation.

This distribution can then be amended to allow for the deductible, ie fit a distribution for the claim amount after the deductible has been applied.

Fit a distribution for the claim frequency

As for claim amount, a distribution must be fitted for claim frequency. After allowing for the deductible, the claim frequency will fall.

Calculating the premium and other considerations

The premium should use the revised claim amount and revised claim frequency.

Since the premium will fall, the office must consider revising its expense allowance.

The possibility of expenses rising due to the administrative complexity of having the deductible should also be considered.

The insurer is now exposed to large claims, so its profit and contingency loadings should be revised.

The insurer should question whether an aggregate deductible is appropriate for this class, given that employers' liability cover is compulsory.

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