Institute of Actuaries of India

Subject ST7 – General Insurance: Reserving & Capital Modelling

November 2011 Examinations

INDICATIVE SOLUTIONS

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

1. Requirements of a good model

The model being used should be:

- valid
- complete
- adequately documented.

The model chosen should reflect adequately the risk profile of the classes of business being modelled.

The parameter values used should be accurate for the classes of business being modelled, *e.g.* third party fire and theft, comprehensive.

The outputs from the model and the degree of uncertainty surrounding them should be capable of independent verification for reasonableness …

… and should be readily communicable to those to whom advice will be given, *e.g.* pricing actuaries, reserving actuaries, the finance department and senior management.

The model, however, must not be overly complex …

… so that either the results become difficult to interpret and communicate …

… or the model becomes too time consuming or expensive to run.

The model should be sufficiently flexible, *e.g.* to allow for:

- changes in the choice of parameters / parameter values
- changes in the data available
- the required output.

[4]

2 (i) Free reserves are assets less technical reserves and current liabilities

(ii) *Ways to increase the free reserves are to:*

- raise capital (*e.g.* a rights issue, if a proprietary company)
- weaken the valuation basis of assets
- replace any assets which are not recognised for solvency purposes
- weaken the liability valuation basis
- write more profitable business…
- or increase premium rates, assuming minimal withdrawals
- possibly write less business
- control expenses better
- improve investment returns, both income and gains
- pay out less dividends (if a proprietary company)
- possibly make use of equalisation reserves.

[6]

3 Practical Factors

The data must be divided so that the subdivisions are small enough to define distinctive features of the subdivision, *i.e.* to avoid heterogeneity in the data cell …

… but not so small that statistical methods become invalid (because of insufficient data in the subdivisions).

It is therefore necessary to consider how sufficient the available data is in terms of:

- relevance
- credibility
- accuracy.

While internal data is likely to be the most relevant, it might lack credibility, particularly in the tails of distributions.

Industry-wide data is likely to be far more credible, but will probably lack relevance.

It may also contain errors, since the quality of industry-wide data depends heavily on the contributors of the data.

The complexity of the models available may also be a factor, with more sophisticated models being better able to cope with finer subdivisions of data.

Time may also be a constraint, in particular:

- the run time
- the time needed to develop, review and test the models.

Also depends on the level of detail required by the business.

Materiality of the risk

The materiality of the risk will also be a key determinant of the level of granularity. The more material the risk, the more important it is to subdivide the data as far as the available data will allow.

The materiality of the risk may depend on:

- the purpose of the capital modelling exercise
- the level of detail required by the business.

The level of detail may also be affected by any existing (or expected future) regulation.

The level of granularity used in capital modelling is likely to be linked to the level of granularity used in pricing, which in turn will have been affected by factors such as data and materiality.

There is often a compromise between the granularity levels required in theory with that used in practice. It may be appropriate for some approximations to take place, particularly for some of the less material risks (*i.e.* the principle of proportionality applies).

[8]

4 (i) Aggregate Deductible

Introduction of the aggregate deductible means that the sum of the claims to the layer must exceed the deductible before the cedant can make a recovery

..and the amount then payable is the amount that would otherwise have been paid less the aggregate deductible

.. so for a given amount of exposure the aggregate deductible reduces the cedant's expected recovery and increase the cedant's retention.

The extent of the impact of the aggregate deductible depends on:

- the size of the aggregate deductible (for a given exposure in vehicle years)
- the expected number and severity of losses to the layer (for a given exposure in vehicle years)
- e.g. large aggregate deductible relative to expected number/size of losses means lower recoveries for the cedant (and vice versa for a small aggregate deductible)

Stability Clause

With no stability clause, the expected amount of total losses to the layer will increase annually (all else being equal) because of:

- the effect of TPBI inflation on severity of individual losses to the layer (i.e. the conditional expected value of a loss to the layer increases with inflation)

- and the gearing effect of TPBI inflation increasing the frequency of losses to the layer (i.e. probability of a loss to the layer increases with inflation)

A stability clause means the attachment point and/or layer limit are adjusted in line with some specified index (e.g. fixed x% p.a. or a healthcare cost index)

(although if the limit is not indexed the layer reduces in size and if there is a higher layer with attachment point same as first layer limit there will be a gap in cover so this is not recommended (although sometimes used))

..with the intention of maintaining real values to the layer so the layer widens with each application of the index e.g. £1m xs £1m indexed by 2% is £1.02m xs £1.02m

Adding the stability clause has the following expected impact

- The frequency of losses to the layer may drop over time e.g. a claim that starts in the layer may settle below the layer

- For a given loss, its actual attachment point depends on the settlement date (i.e. the attachment point will increase in line with the stability clause index until the loss settles)

- If the deductible is small relative to the expected claims cost without the deductible, the expected claims cost to the layer is simply the cost without deductible less the deductible amount.

- Whereas if the deductible is relatively large then a straight deduction is not correct and claims to the layer can only be estimated using a distribution and probabilities.

- The actual impact of the stability clause depends on the cedant's actual claims experience and on the inflation in TPBI claims relative to the index applied to the layer i.e. inflation could be different to the assumed indexation.

(ii) Reinsurer

+ stability clause ensures alignment of interest by encouraging faster claims settlement (as net retention increases with each year due to the indexation of the attachment point and limit),

+ stability clause gives some protection against expected future inflation in the claims to the layer

+ aggregate deductible reduces exposure to the cedant and allows the reinsurer to use capital elsewhere

+ benefits if the sum of claims to the layer doesn't breach the aggregate deductible or claims settle below the indexed attachment point

− actual claims inflation may outstrip the indexation thereby eroding the benefit of the stability clause over time (likely in practice)
− potential increase in expenses for setting up and managing more complex contracts

− lower premium income with introduction of aggregate deductible
− more volatility in claims cost to the layer relative to the premium charged

Cedant

+ the aggregate deductible reduces reinsurance spend (especially beneficial if reinsurance rates are hard)

+ can use the aggregate deductible to manage risk appetite
+ the aggregate deductible means higher expected profit as ceding less to the reinsurer generally means ceding less profit
+ cedant can manage total exposure to the reinsurer (reinsurer security impacts capital requirement)
+ cedant should be able to negotiate a lower premium because of the stability clause
+ if purpose of reinsurance is taken as providing for claims above those expected an aggregate deductible can remove some claims which are definitely expected to be paid as this is a working layer

− aggregate deductible delays recoveries (cashflow implications)
− greater loss retention, so alternative source(s) of capital required (alternatives may be more costly).
− greater volatility in the retained losses
− retains some inflation risk i.e. if the TPBI inflation is lower than the indexation, then more likely that a claim estimated to settle in the layer settles below the layer

[13]

5 (i) *Case reserving changes*: Changes in claims adequacy are most often a calendar year phenomenon, rather than an accident year or policy year phenomenon.
Can be driven by claims department actions - when a claims department implements a new case reserving procedure, it generally applies to all open cases from all prior accident years, as well as future accident years. This means it affects all accident/policy years at different maturities and to differing extents.
This can also be because of underwriting changes, declining net retentions, or declining policy limits which may be the cause behind declining, average claim sizes on open claims, rather than specific claims department actions
The first step in the process is to perform certain diagnostic tests of the data.
And secondly discussion with the claims department as to the changes they have made in their claims reserving methodology
Based on these tests, we can check whether there have been changes in the level of relative reserve adequacy for claims and adjust the data to put all years onto the current year's basis.

This will avoid any distortion in the claims development patterns.
Alternatively may extrapolate any trends observed in the data

(ii) *Changing net retentions*: This is a policy year or reinsurance underwriting year phenomenon rather than an accident year or calendar year phenomenon.
Changing net retentions affect the net development patterns. Reductions in these values should serve to shorten development patterns, and similarly increases will lengthen the development.
Request detailed information about the insurer's ceded reinsurance program -- including excess of loss and quota share retention by line of business -- for as many years as are available.
May choose to select different development patterns for different groups of accident years, depending on the extent of the changes in retentions and policy limits.
This may be a judgmental determination rather than a statistical construction.

[7]

6 (i) Content will include:

- guidelines for the split of assets, giving ranges for each major asset class
- this will most likely give maximum and minimum ranges
- Indicate mean duration; mean maturity, maximum maturity permitted, etc.

a. Cash (20% - 30%)
b. Fixed interest (50% - 70%)
c. Index linked (20% - 40%)
d. Equity (0% - 20%)
e. Indirect property (up to 10% of the 20% max for equity)
f. No direct property investment
g. Domestic currency only although may be need for non-domestic currency in interest of diversification

Greater than 80% of Fixed interest must be Government guaranteed

The remainder of Fixed interest must be AA or better

Benchmark for investment return – both debt and equities

Limit equities to blue-chip or equivalent

Funds under management should be split broadly equally between at least two different investment managers

For Cash no more than *w*% of funds under management with any one deposit holder.

For other assets no more than *x*% of the insurers total assets should be invested in any one company in aggregate, etc.

The insurer should not hold more than *y*% of any particular issue

The insurer should not hold more than *z*% of any one company's total debt in aggregate (counter-party risk), etc.

Although the question states that there are no regulatory investment restrictions consideration should be given to whether this may change and also to competitive/market/rating agency considerations.

Also, the company has a moderate level of free assets so there is some freedom to invest be in non-matched assets.

Tax implications of different assets should be considered.

Main investment objective is to maximize returns subject to meeting all contractual obligations and recognizing the uncertainties involved.

(ii) Advantages:

- Returns may be higher, in which case the company would improve its overall investment performance
- Equity may be better matched for real, long term liabilities
- If currently low level of equities then to increase holding could be appropriate for diversification

Disadvantages

- Returns may be lower, in which case the company would see worse investment performance
- Equity has a greater volatility of potential returns, which means there is an increase in risk
- Equities too long to match most liabilities
- Capital requirements are likely to be risk-based, which means equity treated as higher risk
- Equity is less liquid, which is disadvantageous in GI business where funds may be needed at short notice to cover unexpectedly high levels of claims
- Equity is less secure, so greater risk of default
- Need to know why stock indices are close to a 52 week low; if because of recession, very low business confidence etc. then stock indices could go even lower as could any new investment in equities

(iii) Advantages:

- Inflation link offers some protection for real liabilities
- Highest level of security
- Long term should be a suitable match
- Lower volatility, so generally lower risk than equity
- Possible lower level of dealing expenses

Disadvantages

- Appropriate term may not be available
- The inflation link may not be suitable (wrong inflation)
- Overall expected returns are lower
- Possibly less liquid / smaller volumes available
- May be expensive if limited supply but high demand

[16]

7 Unearned Premium Reserve (UPR)

Assumption:

- Incidence of risk is uniform over each policy year.
- Policies are sold uniformly over each month
- Expenses of 1% annualized premium are incurred at the outset. Very low, hence DAC can be ignored. Alternatively, Indian regulation does not allow DAC.

Hence use the 24th method to estimate UPR

Month	***Annualized Premium Collected '000***	**UPR Factor**	**UPR**	
Jan-11	4,440	13/24	2,405	¼
Feb-11	5,185	15/24	3,241	¼
Mar-11	5,719	17/24	4,051	¼
Apr-11	7,748	19/24	6,134	¼
May-11	8,970	21/24	7,849	¼
Jun-11	11,970	23/24	11,471	¼
Total	44,032		35,150	½

Outstanding Claim Reserve

On average, claims are settled for 5% less than the amount claimed. The aggregate amounts for claims made but not settled is 2,195. Thus,

OCR = 95% * 2,195 = 2,085

Incurred But Not Reported (IBNR)

On average, claims are reported month after occurrence.

Hence, estimate the cost of IBNR claims as the cost of those claims occurring in the second half of June 2011.

Assumption:

- Claim frequency remains constant
- Average cost per claim remain constant
- Reporting delay remains stable

The average cost of IBNR claims may differ from that of reported claims. Small claims may take longer to be reported due to the relative insignificance of the claim. However, we have no further information on this matter so simply assume they are the same.

Claim frequency

Estimate claim frequency using the claims reported up to end June 2011 (relating on average to occurrence up to mid-June) and policy exposure up to mid-June.

Assuming that policies incept evenly over the month we can calculate the 2011 total exposure as:

Month	**Number of Policies Sold '000**	**Exposure Factor**	**Exposure**
Jan-11	30	11/24	13,750
Feb-11	34	9/24	12,750
Mar-11	38	7/24	11,083
Apr-11	52	5/24	10,833
May-11	60	3/24	7,500
Jun-11	84	1/24	3,500
Total	**298**		**59,417**

Total Exposure is 59,417 policy years

Second half of June exposure is ½ month for policies issued in Jan to May and $3/8^{th}$ of a month (on average) for June Policies

Exposure on which claims would not be reported = (0.5 * 214,000 + 0.375 * 84,000)/12 = 11,542

Therefore, exposure up to middle of June 2011 = 59,417 – 11,542 = 47,875

Claims so far reported = 18,549

$\therefore$ claim frequency = 18,549 / 47,875 = 38.74%

Average Cost per claim

The best estimate cost of these claims is the amounts paid plus the OCR is:

Cost = 4,482 + 2,085 = 6,567

$\therefore$ ACPC = 6,567/18,549 = Rs 354

The number of claims events occurring in the second half of June is 38.74% of the 11,542 policy years exposed = 4,472

Hence IBNR Cost = Number of claims * ACPC

= 4,472 * 0.354 = 1,583

Expenses

Monthly expenses are 1% of issued annualized premium, plus Rs 50 per claim settled

Month	***1% of annualized premium***	***Rs 50 per claim***	***Expenses '000***
Jan-11	44.4	36.8	81.2
Feb-11	51.9	52.2	104.1
Mar-11	57.2	64.9	122.1
Apr-11	77.5	106.25	183.7
May-11	89.7	153.35	243.1
Jun-11	119.7	227.7	347.4
Total	440.3	641.2	1,081.5

Investment Income

Assume that, on average, all cash flows (premium, claims & expenses) occur mid-month.

Net cash flow per month is monthly premium received less claim paid less expenses

Investment income = Net Cashflow x 0.005^{i} where i is investment period

Month	**Premium**	**Claims**	**Expenses**	**Net Cashflow**	**Invest-ment Period**	**Invest-ment Income**
Jan	370	258	81	31	5.5	0.9
Feb	802	355	104	343	4.5	7.8
Mar	1,279	451	122	706	3.5	12.4
Apr	1,924	754	184	987	2.5	12.4
May	2,672	1,079	243	1,350	1.5	10.1
Jun	3,669	1,585	347	1,737	0.5	4.3
Total	10,716	4,482	1,082	5,153		47.9

Revenue Account

Collecting the information calculated earlier gives:

Written Premium		44,032	
Less: Increase in UPR		35,150	
Earned Premium			8,882
Less:	Expenses	1,082	
	Claims Paid	4,482	
	Increase in OCR	2,085	
	Increase in IBNR	1,583	
			9,232
Plus:	Investment Income		48
Contribution to Expenses & Profit			**(302)**

There is an equally valid alternative solution based on premium as a measure of exposure rather than number of policies: this gives an IBNR of 1,566 and contribution to expenses & profit of -285

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8. Data given:

	Years of Development				
Accident Year	0	1	2	3	4
2006	4,200	6,700	7,100	7,300	7,400
2007	5,500	9,000	9,600	9,900	
2008	6,600	10,800	11,600		
2009	7,200	12,300			
2010	7,700				

Calculate volume weighted (sum/sum) cumulative development factors

	1/0	2/1	3/2	4/3
Vol Wtd average	1.651	1.068	1.030	1.014

Backfill triangle using development factors to give fitted cumulative data:

Accident Year	0	1	2	3	4
2006	4,020	6,637	7,088	7,300	7,400
2007	5,452	9,001	9,612	9,900	
2008	6,579	10,862	11,600		
2009	7,450	12,300			
2010	7,700				

Calculate fitted past incremental claims:

Accident Year	0	1	2	3	4
2006	4,020	2,617	451	212	100
2007	5,452	3,549	611	288	
2008	6,579	4,283	738		
2009	7,450	4,850			
2010	7,700				

Incremental actual claims data given:

	Years of Development				
Accident Year	0	1	2	3	4
2006	4,200	2,500	400	200	100
2007	5,500	3,500	600	300	
2008	6,600	4,200	800		
2009	7,200	5,100			
2010	7,700				

Subtract actual claims from fitted claims to give residuals:

Accident Year	0	1	2	3	4
2006	-180	117	51	12	0
2007	-48	49	11	-12	
2008	-21	83	-62		
2009	250	-250			
2010	0				

For one scenario using simplest form of boot strapping by resampling residuals (randomly plus or minus) derived from the residual triangles by random choice. Some values may be chosen more than once. This is on the assumption that the residuals are independent:

Accident Year	0	1	2	3	4
2006	12	-180	250	-180	180
2007	-48	51	-11	180	
2008	-12	51	21		
2009	-21	83			
2010	-48				

Add residuals to fitted incremental data to give pseudo incremental claims triangle

Accident Year	0	1	2	3	4
2006	4,032	2,437	701	32	280
2007	5,403	3,600	600	468	
2008	6,567	4,334	759		
2009	7,429	4,934			
2010	7,652				

Cumulate data to give pseudo cumulative paid claims triangle:

Accident Year	0	1	2	3	4
2006	4,032	6,469	7,170	7,202	7,482
2007	5,403	9,003	9,603	10,071	
2008	6,567	10,901	11,660		
2009	7,429	12,362			
2010	7,652				

Calculate sum/sum year-on-year and cumulative development factors:

Cumulative Paid Loss development factors					Tail
Vol Wtd average	1.653	1.078	1.030	1.039	1.000
Cumulative	1.907	1.153	1.070	1.039	1.000

Multiply cumulative pseudo year-to-date claims by cumulative development factor to give pseudo ultimates and subtract actual year-to-date cumulative paid claims to get the pseudo-reserve:

Predicted Future C Accident Year	Predicted Ultimate claims	Pseudo reserve
2006	7,482	82
2007	10,463	563
2008	12,474	874
2009	14,259	1,959
2010	14,590	6,890
	59,268	10,368

(ii)

The basic assumption is that the residuals are independent and can be replaced anywhere in the triangle.
…this is not the case in general
.. and this is shown for this triangle as the size of the residuals tend to decrease from the 1st column of data to the last column
The residuals are selected, in the current case, from 13 data points in the triangle but are used to fill in 15 data points which does not seem logical
… as the residuals for the last accident year and for the last development year are both zero

The simplistic method generally adopted assumes that alternative ultimate values are being estimated and hence pseudo reserves are calculated by subtracting actual paid claims from pseudo ultimates
It is not clear that this is definitely better than assuming reserves are being simulated by subtracting the pseudo fitted cumulative claims from the pseudo ultimates although it is presumed that this gives too low estimates of variation
However, presumably for the reasons intimated above the generally adopted method gives higher values than other more scientific methods
The simplest way to overcome the problem of residuals reducing across columns is to split the data into 2 or more tranches of sets of columns
.. but this would reduce the population for each tranch to possibly unreasonable levels

A better approach is to use standardised residuals
.. preferably by assuming an underlying method such as the over-dispersed Poisson (ODP) or Thomas Mach method
Such methods depend on the basic chain ladder assumptions e.g. regular development

We are told that there is reason to believe that the claims settlement rates are changing

There is also evidence for this as from the year-on-year development factor diagnostics it can be seen that the development factors for 1/0 and 2/1 are increasing over time:

Accident Year	1/0	2/1	3/2	4/3
2006	1.595	1.060	1.028	1.014
2007	1.636	1.067	1.031	
2008	1.636	1.074		
2009	1.708			
2010				

Any method should not be used as a black box and in this case the mean or expected value of reserves required would have been underestimated
And the range of the reserves about this value may well be overestimated as observed values will be taken as deviations from a mean value rather than primarily as part of a trend

Any stochastic method should be adjusted to allow for known trends
The method as given does not make any allowance for a tail factor although here may not be required as 4/3 ratio was 1.1014 (or any other relevant point re tail factor)

[26]

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