Institute of Actuaries of India

Subject CT5 – General Insurance, Life & Health Contingencies

November 2011 Examinations

INDICATIVE SOLUTIONS

Question 1

${}_{5|10}q_{[40]+1}$ is the probability that a life aged 41 exact now and at the beginning of the second year of selection will die between the ages of 46 exact and 56 exact.

Value is:

$$(l_{46} - l_{56}) / l_{[40]+1}$$
$$= (9786.9534 - 9515.104) / 9846.5384$$
$$= 0.02761$$

[2]

Question 2:

$\mu_x(t) = \mu_x^{(1)}(t) + \mu_x^{(2)}(t)$; since the two decrements are independent

$= 0.2\mu_x(t) + \mu_x^{(2)}(t)$

$\mu_x^{(2)}(t) = 0.8\mu_x(t)$

$q_x^{(1)} = 1 - p_x^{(1)} = 1 - e^{-\int_0^1 0.2kt^2\,dt} = 1 - e^{-0.2k/3} = 0.04$

$k = 0.6123$

Now, ${}_2q_x^{(2)} = \int_0^2 {}_tp_x \; \mu_x^{(2)}dt = 0.8\int_0^2 {}_tp_x \; \mu_x(t)dt = 0.8\; {}_2q_x = 0.8\,(1 - {}_2p_x)$

But, ${}_2p_x = e^{-\int_0^2 \mu_x(t)dt} = e^{-\int_0^2 kt^2\,dt} = e^{-8k/3} = e^{-(8)(0.6123)/3} = 0.19538$

Hence, ${}_2q_x^{(2)} = 0.8\,(1 - 0.19538) = 0.644$

[6]

Question 3:

i.

Overhead expenses are those that in the short term do not vary with the amount of business written i.e. all those expenses of the life insurance Company which by increasing the number of policies, sum assured etc., do not increase.

Overhead expenses are usually allocated on a per policy basis e.g. if the salary of Chief market officer is Rs.5 million and number of policies are 1 million, the allocation of this cost would be Rs. 5 per policy.

Direct expenses are those which vary with the amount of business written i.e. if we write more volume of business, expenses increases with incremental volume.

ii.

a. Underwriting costs **- Direct**
b. Salary of the Chief Marketing Officer - **Overhead**
c. Salary costs of the New Business department - **Direct**
d. Salary of the Appointed Actuary - **Overhead**

[5]

Question 4

Let *P* be the net premium for the policy payable annually in advance. Then, equation of value becomes:

$P\ddot{a}_{45:\overline{15}|}$ =100000* $(A_{45:\overline{20}|} + v^{20}\ {}_{20}p_{45})$

11.386 *P* =100000 * (0.46998 + 0.41075)

P = *Rs.*7735.20

Net premium reserve at the end of the 13th policy year is

${}_{13}V$ =100000 * $(A_{58:\overline{7}|} + v^{7}\ {}_{7}p_{58}) - P\ddot{a}_{58:\overline{2}|}$

= 100000 * (0.76516+ 0.71209) - 7735.20 * 1.955

=147724.80 - 15122.30
= 132602.50

Death strain at risk per policy = 100000 - 132602.50= - 32602.50

$EDS = 199q_{57}$ x (- 32602.50)

=199 x0.00565 x (-32602.50)
= - 36656.62

ADS= 4x -32602.50= -130410

Mortality Profit = - 36656.62 – (- 130410.00) = Rs. 93753.38

[6]

Question 5

Unit Fund (UF) projection for the next 3 years:

Year	UF at start	Regular Premium	Allocation Charge	UF after charges	UF after growth	FMC	UF at end
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	50,000	10,000	1,000	59,000	64,900	974	63,927
2	63,927	10,000	1,000	72,927	80,219	1,203	79,016
3	79,016	10,000	1,000	88,016	96,817	1,452	95,365

$(3)_t$ = 10% * $(2)_t$

$(4)_t$ = $(1)_t$ + $(2)_t$ - $(3)_t$

$(5)_t$ = $(4)_t$ * (1 + 10%)

$(6)_t$ = $(5)_t$ * 1.5%

$(7)_t$ = $(5)_t$ - $(6)_t$

Non-unit (NU) cash flows per policy for the next 3 years:

Year	Allocation Charge	FMC	Expense	Commission	Interest	Profit	NUR at start of year
	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1	1,000	974	100	500	24	1,398	(4,334)
2	1,000	1,203	100	500	24	1,627	(3,202)
3	1,000	1,452	100	500	24	1,876	(1,770)

$(11)_t$ = 5% * Regular Premium

$(12)_t = [(8)_t - (10)_t - (11)_t] * 6\%$

$(13)_t = (8)_t + (9)_t - (10)_t - (11)_t + (12)_t$

$(14)_t = [-(13)_t + (14)_{t+1} * (1 - q_t)]/(1 + 6\%)$

Economic Liability = Fund Value as on 31 Dec 10 + Non Unit Reserve = 50,000 – 4,334 = 45,666

[7]

Question 6

For the first 10 years, payment is made only if (45) is alive and (30) is not. The probability of this for time t ≤ 10 is ${}_tp_{45} - {}_tp_{30:45}$

The actuarial present value of payment made during the first 10 years is $\int_0^{10} v^t\,({}_tp_{45} - {}_tp_{30:45})dt$

$= \bar{a}_{45:\overline{10}|} - \bar{a}_{30:45:\overline{10}|}$

After 10 years, payment is made if at least one of (30) or (45) is alive. This is a 10-year deferred last-survivor annuity, with (APV) actuarial present value ${}_{10|}\bar{a}_{\overline{30:45}} = {}_{10|}\bar{a}_{30} + {}_{10|}\bar{a}_{45} - {}_{10|}\bar{a}_{30:45}$

Therefore, $S = \bar{a}_{45:\overline{10}|} - \bar{a}_{30:45:\overline{10}|} + {}_{10|}\bar{a}_{30} + {}_{10|}\bar{a}_{45} - {}_{10|}\bar{a}_{30:45}$

Since, $\bar{a}_{45:\overline{10}|} + {}_{10|}\bar{a}_{45} = \bar{a}_{45}$ and $\bar{a}_{30:45:\overline{10}|} + {}_{10|}\bar{a}_{30:45} = \bar{a}_{30:45}$

$S = \bar{a}_{45} + {}_{10|}\bar{a}_{30} - \bar{a}_{30:45}$

[4]

Question 7

(i) Let P be the monthly premium for the contract. Then:
EPV of premiums valued at rate i where $i = 0.06$ is:

$$12P\ddot{a}^{(12)}_{[30]:\overline{35}|} = 12P(\ddot{a}_{[30]:\overline{35}|} - \frac{11}{24}(1 - v^{35}\frac{l_{65}}{l_{[30]}}))$$

$$where\ v^{35}\frac{l_{65}}{l_{[30]}} = 0.13011\times\frac{8821.2612}{9923.7497} = 0.11566$$

$$= 12P(15.152 - \frac{11}{24}(1 - .11566)) = 12P\times 14.74668 = 176.9601P$$

EPV of benefits valued at rate i where i = 0.06 is:

$$= 75,000A_{[30]:\overline{35|}} = 75,000\times 0.14234 = 10,675.5$$

EPV of expenses not subject to inflation and therefore valued at rate i where i = 0.06 is:

$$0.025\times 12P\ddot{a}^{(12)}_{[30]:\overline{35|}} - 0.025P + 250 + 0.5\times 12P$$
$$= 250 + 10.399P$$

EPV of expenses subject to inflation and therefore valued at rate j where $1 + j = \frac{1.06}{1.0192308} = 1.04$ is:

$$75(\ddot{a}_{[30]:\overline{35|}} - 1) + 300A_{[30]:\overline{35|}} = 75\times 18.072 + 300\times 0.26647 = 1435.341$$

Equating EPV of premiums and EPV of benefits and expenses gives:

$$176.9601P = 10{,}675.5 + 250 + 10.399P + 1{,}435.341$$

$$P = 12{,}360.841/166.5611$$

P= Rs.74.21

[7]

Question 8

(i) $$g(T) = \begin{cases} 5,000v^2\bar{a}_{\overline{T_{63}-2|}} & \text{if } T_{63}\ge 2 \quad (\text{or } 5,000(\bar{a}_{\overline{T_{63}|}} - \bar{a}_{\overline{2|}})) \\ 0 & \text{if } T_{63} < 2 \end{cases}$$

(ii)

$$E[g(T)] = (100)(5,000)v^2\ {}_2p_{63}\bar{a}_{65} = (500,000)(0.92456)(0.992617)(14.871 - 0.5)$$
$$= (500,000)(13.1887) = 6,594,350$$

(iii) $$Var[g(T)] = E[g(T)^2] - E[g(T)]^2$$

For Re 1 of annuity:

$$E[g(T)^2] = \int_2^\infty {}_t p_{63} \mu_{63+t} [v^2 \bar{a}_{\overline{t-2|}}]^2 \, dt$$

Let $t = r + 2 \Rightarrow$

$$E[g(T)^2] = \int_0^\infty {}_{r+2} p_{63} \mu_{63+r+2} [v^2 \bar{a}_{\overline{r|}}]^2 \, dr$$

$$= \int_0^\infty {}_r p_{65\,2} p_{63} \mu_{65+r} v^4 \left[\frac{1-v^r}{\delta}\right]^2 dr$$

$$= \frac{{}_2 p_{63} v^4}{\delta^2} \int_0^\infty {}_r p_{65} \mu_{65+r} [1 - 2v^r + v^{2r}] dr$$

$$= \frac{{}_2 p_{63} v^4}{\delta^2} [1 - 2\bar{A}_{65} + {}^2\bar{A}_{65}]$$

where

$$\bar{A}_{65} = (1.04)^{0.5}(1 - d\ddot{a}_{65}) = 1.019804\{1 - \left(\frac{0.04}{1.04}\right)(14.871)\} = 0.436515$$

and ${}^2\bar{A}_{65} = (1.04)({}^2A_{65}) = (1.04)(0.20847) = 0.21681$

$$\therefore E[g(T)^2] = \frac{(0.992617)(0.85480)}{(0.039221)^2}[1 - (2)(0.436515) + (0.21681)] = 189.622$$

$$Var[g(T)] = 189.622 - (13.1887)^2 = 15.680$$

For annuity of 5,000 we need to increase by $5{,}000^2$ and for 100 (independent) lives we need to multiply by 100.

Total variance = $(15.680)(5{,}000^2)(100) = 39{,}200{,}000{,}000 = (197{,}999)^2$

[13]

Question 9

i. On a without-profit contract, both the premiums and benefits under the policy are usually known and guaranteed at the date of policy issue. Neither premiums nor the level of benefits can be increased after the contract is issued.

On a with-profit contract, the premiums and/or the benefits can vary to give an additional benefit to the policyholder in respect of any emerging surplus of assets over liabilities following a valuation. For example, surplus might be used to reduce the premium payable for the same benefit or to increase the sum assured without any additional premium becoming payable.

(i) Let P be the annual premium. Then:

EPV of premiums:

$$P\ddot{a}_{[50]:\overline{10|}} = 7.698P$$

EPV of benefits:

$$\frac{75,000}{(1+b)} \times (1.06)^{1/2} \{q_{[50]}(1+b)v \ +_1|q_{[50]}(1+b)^2 v^2$$

$$+ \ldots +_9|q_{[50]}(1+b)^{10} v^{10}\} + 75,000\,{}_{10}p_{[50]}(1+b)^{10} v^{10}$$

where $b = 0.0192308$

$$= \frac{75,000}{(1+b)} \times (1.06)^{1/2} A^{1}_{[50]:\overline{10|}} @ i' + 75,000 \times {}_{10}p_{[50]} \times \frac{1}{(1+i')^{10}}$$

$$= \frac{75,000}{1.0192308} \times (1.06)^{1/2} \times (.68007 - .64641) + 75,000 \times .64641 = 2,550.091 + 48,480.75$$

$$= 51,030.84$$

where $i' = \dfrac{1.06}{1+b} - 1 = 0.04$

EPV of other expenses:

$$.5 \times P + 350 + 0.05 \times P(\ddot{a}_{[50]:\overline{10|}} - 1) = 0.8349P + 350$$

Equation of value gives $7.698P = 51,030.84 + 0.8349P + 350$

Therefore, P = Rs. 7486.54

[10]

Question 10:

0 The Risk discount rate is the rate used to discount the profit cashflows of a policy. It is chosen equal to the risk-free rate plus a premium which reflects the risks and uncertainties surrounding the cashflows of the policy.

The shareholder expect higher return than the risk free rate to get the compensation for the risks and uncertainties under the profit cash flows.

(ii) The multiple decrement table is as follows:

Age (x)	$(al)_x$	$(ad)^d_x$	$(ad)^w_x$
50	100000	192.17	4995.07
51	94812.76	252.55	4734.16
52	89826.04		

Values for the multiple decrement table are calculated from the following formulas:

$(aq)^d_x = q^d_x * (1 - 0.5 * q^w_x)$
$(ad)^d_x = (al)_x * (aq)^d_x$
$(al)_{x+1} = (al)_x - (ad)^d_x - (ad)^w_x$

Cashflows allowing for risk margin:

	Premium	Expense	Interest	Death Ben	With. Ben	Surv. Ben	Risk Margin	Profit	Prob. IF	Disc. Val. @7% p.a
1	7,500.0	1,000.0	455.0	192.17	280.97	-	75.0	6,406.9	1	5,987.7
2	7,500.0	300.0	504.0	266.37	561.73	9,474.0	75.0	(2,673.1)	0.94813	(2,213.7)

NPV = 5987.7 – 2213.7 = Rs. 3774

Cashflows without risk margin:

	Premium	Expense	Interest	Death Ben	With. Ben	Surv. Ben	Profit	Prob. IF	Disc. Val. @15% p.a
1	7,500.0	1,000.0	455.0	192.17	280.97	-	6,481.9	1	5,637.2
2	7,500.0	300.0	504.0	266.37	561.73	9,474.0	(2,598.1)	0.94813	(1,863.2)

NPV = 5637.2 – 1863.2 = Rs. 3774

Since the discounted value is same for both the cashflows, hence RDR is 15% p.a.

[12]

Question 11:

(i) SMR = $\dfrac{\sum_x E^c_{x,t}\, m_{x,t}}{\sum_x E^c_{x,t}\, {}^s m_{x,t}}$

$$= \frac{\sum_x E^c_{x,t}\, {}^s m_{x,t} \dfrac{m_{x,t}}{{}^s m_{x,t}}}{\sum_x E^c_{x,t}\, {}^s m_{x,t}}$$

which is the weighted average of the age-specific mortality differentials between the population being studied and the standard population.

i.e. $\dfrac{m_{x,t}}{{}^s m_{x,t}}$

weighted by the expected deaths in the population being studied based on standard mortality.

i.e. $E^c_{x,t}\ {}^s m_{x,t}$

(ii) SMR for 2007-2008 is (1.8*10 + 0.9*20)/30 = 1.2
SMR for 2009-2010 is (2*10 + 0.8*20)/30 = 1.2

(iii) (a) CMF = $\frac{\sum_x {}^sE^c_{x,t}\ m_{x,t}}{\sum_x {}^sE^c_{x,t}\ {}^s m_{x,t}}$

$$= \frac{\sum_x {}^sE^c_{x,t}\ {}^s m_{x,t} \frac{m_{x,t}}{{}^s m_{x,t}}}{\sum_x {}^sE^c_{x,t}\ {}^s m_{x,t}}$$

This is simply a weighted average of $\frac{m_{x,t}}{{}^s m_{x,t}}$ weighted by ${}^sE^c_{x,t}\ {}^s m_{x,t}$

The differences between the SMR and CMF figures indicates that the Standard Population A and the observed population have different proportions in the two age ranges. As the CMF<SMR, this indicates that Standard Population A is more heavily weighted to the older age group.

(b) In my opinion, use of the SMR gives better results for comparing the population in each of the two periods. The mortality experience in the two periods is compared using Standard Population A exposed to risk in the CMF calculations and the observed population exposed to risk in the SMR calculations.

Standard Population A appears to have a significantly different composition from the observed population. Therefore, using the Standard Population A exposed to risk in the weight calculations could introduce differences in the results which have nothing to do with underlying mortality differences. Use of the observed population exposed to risk removes this difficulty and results should be more reliable.
(c) I disagree with the committee's conclusion. The SMR figures indicate that the mortality experience has not changed between 2007-2008 and 2009-2010.

[10]

Question 12:

(1 From the multistate model, there is no recovery to the healthy state and the premiums are payable only until the first claim. ${}_tp_x^{00} = {}_tp_x^{\overline{00}} = (0.87)^t$

Therefore, EPV of premiums = P {1 + 0.87v + $(0.87v)^2$ +…} = 5.5789P

Valuing the benefit from the point when the first claim arises, we get the following probabilities:

the first claim payment will be at level 1;
the second claim payment will be at level 1 with probability 0.6 and level 2 with probability 0.3;

the third claim payment will be at level 1 with probability 0.62 = 0.36 and at level 2 with probability 0.6*0.3+ 0.3*0.6 = 0.36 ;
the fourth claim payment will be at level 1 with probability 0.63 = 0.216 and at level 2 with probability 0.6 * 0.3* 0.6 + 0.6 * 0.6 * 0.3 + 0.3* 0.6 * 0.6 = 0.324 .

If the first claim is in n years time, the expected present value will be $50000*0.6*1.06^n*v^n$. With v at 6%, this is 30,000 for all n. Similarly the present value of any level 2 claim will be 50,000, so we can ignore interest in valuing claims.
The EPV of all claims at the point of the first claim payment arising is therefore:
30,000*(1+ 0.6 + 0.36 + 0.216) + 50,000*(0 + 0.3+ 0.36 + 0.324) =114,480

Finally, the probability that the first claim occurs at the end of year 1 is 0.1, at the end of year 2 is (0.87)*(0.1), at the end year 3 is $(0.87)^2$*(0.1) and in general at the end of year n is $(0.87)^{n-1}$*(0.1).

The probability of a claim is therefore 0.1*(1 + 0.87 + 0.87^2 +…) = 0.1/0.13 = 0.76923

EPV of claims = 0.76923 * 114480 = 88,061.45

Equation of value is:
(1 – 0.075) * 5.578947P = 88,061.45
P = Rs. 17,064.43

[10]

Question 13:

EPV of past pensions: $(n/60)(Sal)({}^{z}M_x^{ia} + {}^{z}M_x^{ra})/\ {}^{s}D_x$

EPV of future pensions: $(1/60)(Sal)({}^{z}\bar{R}_x^{ia} + {}^{z}\bar{R}_x^{ra})/\ {}^{s}D_x$

EPV of contributions @1% of salary: $(0.01)(Sal)({}^{s}\bar{N}_x)/\ {}^{s}D_x$

Age	Salary	Past service	${}^{s}D_x$	${}^{z}M_x^{ia}$	${}^{z}M_x^{ra}$	${}^{z}\bar{R}_x^{ia}$	${}^{z}\bar{R}_x^{ra}$	${}^{s}\bar{N}_x$
30	25,000	5	41558	64061	128026	1502811	4164521	680611
35	20,000	6	31816	61843	128026	1187407	3524390	502836

	EPV past pension	EPV future pension	EPV cont. 1% salary
	9,629.46	56,821.51	4,094.34
	11,935.44	49,365.07	3,160.90
Total	**21,564.90**	**106,186.58**	**7,255.24**

Total Liability = 21,564.90 + 106,186.58 = 127,751.48

Contribution rate needed = 127,751.48/7,255.24 = 17.6% of salary

[8]

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