INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

16th November 2011

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (15.00 – 18.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception***

2. ***Mark allocations are shown in brackets.***

3. ***Attempt all questions, beginning your answer to each question on a separate sheet. <u>However, answers to objective type questions could be written on the same sheet</u>.***

4. ***In addition to this paper you will be provided with graph paper, if required.***

5. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) **a)** A 9-month loan, repayable by a single payment of ₹ 50,000 is issued at a rate of commercial discount of 18% p.a. What amount was initially lent to the borrower? (2)

b) i. If δ=6%, find $d^{(12)}$

ii. If $d^{(4)}$ = 6%, find $i^{(12)}$ (4)

c) Arrange the following quantities in ascending order of numerical value, giving brief reasons for the order assuming that they all correspond to same effective interest rate:

$$i,\ d,\ i^{(4)},\ \delta$$

(2)

d) Explain in words why the relationship $d = iv$ holds true. (2)

[10]

Q. 2) **a)** A deposit of ₹ 7,049 is accumulated at the following rates of interest:

6% per annum nominal convertible monthly for first two years followed by 1.5% per quarter simple for next two and a half years followed by a rate of discount of 6% per annum convertible monthly for next one and a half years.

Calculate the accumulated amount after 6 years (4)

b) A variable annuity, payable annually in advance for n years, is such that the payment at the start of the year t is t^2. Show that the present value of this annuity can be expressed as

$$\frac{2(I\ddot{a}_{\overline{n}|}) - \ddot{a}_{\overline{n}|} - n^2 v^n}{1-v}$$

(4)

[8]

Q. 3) The assets and liabilities of an insurance company are given below:

Time point (t)	Liability Cash-flow at end of time t	Asset Cash-flow at end of time t
1	x	10
2	2x	10
3	3.5x	0
4	4.5x	0
5	5x	20
6	100	30
8	0	y

The risk free rate of return is 6% per annum effective.

a) Calculate the values of x and y such that first and second conditions of Redington's immunization theory are fulfilled. (8)

b) Calculating the convexities of assets and liabilities explain whether the portfolio is immunized. (4)

[12]

Q. 4) On 1st April, 2009 Mr. X purchased a house and had let it out on rent immediately for ₹ 5000/- per month, payable in advance. The rent is revised upwards at the rate of 5.2% per annum, at the end of each subsequent year. Mr. X incurs a maintenance cost of ₹2000/- per annum payable yearly in advance, which increases at the rate of 2% a year at the beginning of each subsequent year. The maintenance cost was incurred on the day of purchase as well. The property is assumed to be held indefinitely.

a) Assuming a discount rate of 6% per annum, calculate what price Mr. X should have paid to purchase the house. (4)

The inflation indices over 2009-2011 were as per the following table:

Date	01/10/2008	01/04/2009	01/10/2009	01/04/2010	01/10/2010	01/04/2011
Inflation index	99	100	105	104	106	103

As per current tax rules, capital gains tax will be applied on any property sold.

Capital gains tax will be 10% on the capital gains after indexation allowance on the purchase price. For this purpose, the above index with 6 months lag will be considered.

On 1st April, 2011 Mr. X sold the house for a value which assumed that future rental growth would be 5.3% per annum and discount rate would be 6% per annum. Past rentals have increased at the rate of 5.2% per annum and the maintenance cost has increased at the rate of 2% per annum.

b) Calculate what amount he received on sale after paying capital gains tax, if any. (6)

[10]

Q. 5) A company is considering the following two financial instruments to hedge its index linked liabilities:

i) Index-linked government bond with 6 months lag using the Wholesale Price Index;

ii) Interest rate swap offered by an investment bank.

What are the advantages and disadvantages of the two types of instruments?

[5]

Q. 6) **a)** What do you mean by Par Yield? (2)

b) Using the following spot rates, find the par coupon of a four year annual coupon paying bond issued at time t=2.

Year	1	2	3	4	5	6
Spot rates	5%	5.5%	6%	7%	8%	8.5%

Redemption value of the bond is ₹ 100. (4)

[6]

Q. 7) **a)** What do you mean by Arbitrage free contract? (2)

b) A company ABC wants to offer 500 units of convertible stocks to each of its 1000 existing employees. The stocks are convertible only at 3 year's time from the date of issuance, after payment of the dividend then due. The stocks are expected to be issued in 3 months time from now. The stock price on the date of issuance is expected to be ₹ 15 per unit.

The company has decided to give an annual dividend to its shareholders starting exactly after 1 year and 3 months from now. The first dividend amount will be ₹ 0.25 per unit stock. In subsequent years the dividend payment will be compounded at a rate of 2% per annum.

The company wants to hedge its total obligation after 3 years and 3 months using a forward contract.

Assuming risk free rate of interest as 6% per annum effective, calculate

i) The forward price of the contract

ii) The amount the company needs now to enter into the contract.

Assume no employee joins or leaves the company over the next 3 year and 3 month period. (5)

[7]

Q. 8) The fund value and cashflows for a pensions fund from during the calendar years 2008 to 2010 were as follows:

Date	**Cashflow (in ₹)**	**Description of cashflow**	**Fund value 1 day before (in ₹)**
1-January -2008	10,00,000	Initial fund	0
1-April -2008	30,000	interest and dividends on fund	12,00,000
1-January -2009	3,00,000	contribution received from policyholders	11,00,000
1-June -2009	1,00,000	received due to sale of shares	14,00,000
1-September -2009	-2,00,000	paid out to pay maturities	11,00,000
1-March -2010	3,00,000	contribution paid by shareholders	7,30,000
1-January-2011			15,00,000

Calculate the annual effective time weighted rate of return earned on the fund over the period from 1st January 2008 to 31st December 2010.

[4]

Q. 9) **a)** Explain what is meant by discounted payback period of an investment project. (1)

b) An insurance company is considering setting up a branch in a new city. It plans to open the branch in four years' time.

The company has the decision criteria that the cashflows from the project must provide an internal rate of return at least 8% per annum effective.

The following cashflows are generated in the development and operation of the branch (in ₹ Lacs):

Cash Outflows

The insurance company will spend 25.5 per annum for the next four years on development and marketing of insurance products until the branch opens. This outlay is assumed to be a constant continuous payment stream. The rent on the branch building will be 2 per annum paid monthly in advance for ten years starting in four years time. Staff costs are assumed to be 10 in the first year, 10.6 in the second year, rising by 6% per annum each year thereafter. Staff costs are assumed to be incurred at the beginning of each year starting in four years time and assumed to be incurred for ten years.

Cash Inflows

The company expects the sale of products to generate income at a rate of 12 per annum for the first two years after the branch opens rising to 17 per annum in the next three years and to 31 per annum for the following five years. This net income is assumed to be received continuously throughout each year. The company expects to be able to sell the branch operation 14 years from the present time for 120.

Determine whether the decision criterion is fulfilled. (8)

[9]

Q. 10) The force of interest $\delta(t)$ at time t is given by $at + bt^2 - 1$ where a and b are constants. An amount of ₹ 5,000 invested at time t = 0 accumulates to ₹ 6,000 at time t = 3 and ₹ 10,000 at time t = 6.

Determine a and b.

[6]

Q. 11) A loan of ₹ 5,00,000 is repayable in 4 years, payable monthly in arrears with a flat rate of interest charged at 13% p.a. Assuming equal instalments, Calculate

a) The monthly repayment (1)

b) Effective rate of interest per annum (4)

[5]

Q. 12) Person X takes a loan of ₹ 10 Lacs with interest rate charged at the rate of 12% p.a. nominal convertible monthly. He wants to repay it in 10 years in equal monthly instalments, payable in arrears.

a) Calculate the monthly instalment. (2)

b) How much is the total capital and total interest paid in the 5^{th} year? (6)

[8]

Q. 13) Mr. A wants to invest ₹ 5000/- now in a stock which offers an annual return with mean 7% and standard deviation 2.5%. He will invest a further ₹ 10,000/- in exactly one year's time in the same stock at the then available price. The return each year is independent of the return in any other year.

a) Calculate the mean and standard deviation of the accumulated amount after 10 years. (6)

b) Assuming that the accumulated amount follows a normal distribution, find the probability that the accumulated value in (a) will be more than the accumulated value if the amounts were invested in fixed interest securities offering effective return of 7.5% per annum. (4)

[10]

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