Institute of Actuaries of India

Subject CT5 – General Insurance, Life and Health Contingencies

November 2010 Examinations

INDICATIVE SOLUTIONS

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Question-1

- The expected cost of paying benefits usually increases as the life ages and the probability of a claim by death increases.
- In the last policy year the probability of payment is large, since the payment is made to all survivors on maturity and for most contracts the probability of survival is large.

- Level premiums received in the early years of a contract used to be more than enough to pay the benefits that fall due in those early years, but in the later years, and in particular in the last year of an endowment assurance policy, the premiums are too small to pay for the benefits. It is therefore prudent for the premiums that are not required in the early years of the contract to be set aside, or reserved, to fund the shortfall in the later years of the contract.
- If premiums received that were not required to pay benefits were spent by the company, perhaps by distributing to shareholders, then later in the contract the company may not be able to find the money to pay for the excess of the cost of benefits over the premiums received.

[4]

Question 2

(i) Given that $l_x=120-x$ and we know that $d_x = l_x - l_{x+1}$

$d_x = (120-x)-(120-x-1) =1$; for all x

(a)

$$A^{1}_{40:\overline{20|}} = \left(\sum_{0}^{19} v^{t+1} * d_{40+t} \right) / l_{40}$$
$$= \left(\sum_{0}^{19} v^{t+1} \right) / l_{40}$$
$$= a_{\overline{20|}} / l_{40}$$

=13.5903/(120-40) = 0.16988

(b)

$$A_{40:\overline{20|}} = A^{1}_{40:\overline{20|}} + v^{20} * l_{60} / l_{40}$$

=0.16988 + 0.45639*(120 -60) /(120-40) =0.51217

(ii)

$$\bar{A}^{1}_{70:\overline{1|}} = \int_{0}^{1} e^{-\delta_{.075}t} \, {}_tp_{70} \mu_{70+t} dt$$ in the general case

Here, assuming μ is constant for $0 < t < 1$, we get

$\mu = -\ln(p_{70}) = -\ln(1 - .03930) = 0.040093$

${}^{t}p_{70} = \exp(-\mu t) = \exp(-.040093t)$

$\delta_{.075} = \ln(1.075) = 0.07232$

$$A^{1}_{70:\overline{1}|} = \int_0^1 e^{-0.07232t} e^{-0.040093t} (0.040093) dt$$
$$= \frac{-(0.040093)}{(0.07232 + 0.040093)} \left[e^{-(0.07232+0.040093)t} \right]_0^1$$
$$= (-0.35610)(0.89368 - 1) = 0.0379$$

[10]

Question 3

(i)

Denote:

D = death

W = withdrawal (excluding those who fail test)

f = those who fail test

i = injury

To calculate dependent probability of death the below given formula would be used:

$(aq)_x^d = q_x^d \; [1 - 1/2 \, (q_x^w + q_x^i) + 1/3 \, q_x^w \; q_x^i]$

A similar approach will apply for injuries and withdrawal. Treat the test as an "abnormal" mode of exit.

	Independent rate of decrement			Dependent rate of decrement		
Age	Death	Injury	Withdrawal	Death	Injury	withdrawal
18	0.00112	0.05122	0.15	0.001010	0.047353	0.146077
19	0.00117	0.05003	0.1	0.001084	0.047501	0.097442
20	0.00119	0.05003	0.1	0.001103	0.047501	0.097441

Let us assume that there are 10,000 lives at aged 18.

Age	Number of people before test	Number of failures	Number of people who pass the test	Number of Deaths	Number of Injuries	Number of withdrawals
18	10,000	0	10,000	10	474	1461
19	8,055	806	7,249	8	344	706
20	6,191	0	6,191	7	294	603
21	5,287					

(ii)

10,000 entrants give us 5287 employees after 3 years. Hence 946 entrants are needed to get, on an average, 500 employees aged exactly 21.

[10]

Question 4:

The death strain at risk for a policy for year $t + 1$ ($t = 0, 1, 2 ….$) is the excess of the sum payable on death over the end of year provisions and survival payments, if any.

i.e. DSAR for year $t + 1 = S - {}_{t+1}V$.

The expected death strain for year $t + 1$ ($t = 0, 1, 2$) is the amount that the life insurance company expects to pay extra to the end of year provision for the policy.

i.e. EDS for year $t + 1 = q(S - {}_{t+1}V)$

The actual death strain for year $t + 1$ ($t = 0, 1, 2…$) is the observed value at $t+1$ of the death strain random variable

i.e. ADS for year $t + 1 = (S - {}_{t+1}V)$ if the life died in the year t to $t + 1$ = 0 if the life survived to $t + 1$

(ii)

Annual premium for pure endowment with Rs.1,50,000 sum assured given by:

$P^{PE} = (150000/\ddot{a}_{45:\overline{15}|})*(D_{60}/D_{45}) = (150000*882.85/11.386*1677.97) = 6931.97$

Annual premium for term assurance with Rs. 3,00,000 sum assured given by:

$$P^{TA} = P^{EA} - 2P^{PE} = \frac{300000 * A_{45:\overline{15}|}}{\ddot{a}_{45:\overline{15}|}} - 2 * P^{PE}$$

=(300000*0.56206/11.386) – 2*6931.43

= 946.38

Provisions at the end of the third year:

For pure endowment with Rs.150,000 sum assured given by:

$${}_3V^{PE} = 150000 * (D_{60}/D_{48}) - P^{PE} * \ddot{a}_{48:\overline{12}|}$$

= 150000 * 882.85/1484.43 – 6931.43 * 9.613

= 22579.17

For term assurance with Rs.3,00,000 sum assured given by:

$${}_3V^{TA} = {}_3V^{EA} - {}_3V^{PE}$$

= $300000 * A_{48:\overline{12}|} -$ (2*6931.43+ 946.38)* $\ddot{a}_{48:\overline{12}|}$ - 2*22579.17

=300,000* 0.63025 –14809.24*9.613- 45158.34

=1555.44

For temporary immediate annuity paying an annual benefit of Rs.50,000 given by:

$${}_3V^{IA} = 50000 * a_{58:\overline{2}|}$$

$$= 50000 * (\ddot{a}_{58:\overline{3}|} - 1)$$

$$= 50000 * (\ddot{a}_{58} - v^3 \, {}_3p_{58} \ddot{a}_{61} - 1)$$

$$= 50000 * \left(16.356 - (1.04)^{-3} \frac{9802.048}{9864.803} \times 15.254 - 1\right)$$

=94075.82

Death Strain at Risk per policy:
Pure endowment: DSAR = 0 - 22579.17 = - 22579.17

Term assurance: DSAR = 300000 -1555.44= 298444.56

Immediate annuity: DSAR = 0 - (94075.82 + 50000) = 144075.82

Mortality profit = EDS –ADS

For term assurances

EDS = 4985 * q_{47} * 298444.56 = 4985 * 0.001802 * 298444.56 = 2680918.53

ADS = 8 * 298444.56 = 2387556.48

Mortality Profit = 2680918.53 - 2387556.48 = 293362.05

For pure endowment

EDS = 1995 * q_{47} * (- 22579.17) = 1995*0.001802 * (-22579.17) = -81171.89

ADS = 1 * (-22579.17) = - 22579.17

Mortality Profit = - 81171.89 - (-22579.17) = - 58592.72

For immediate annuity
EDS= 995 *q_{57} *(-144075.82) = 995* .001558 * (-144075.82) = - 223347.78
ADS=1*(-144075.82)= -144075.82
mortality profit = -79271.96

Hence, total mortality profit = 293362.05-58592.72 -79271.96
= Rs. 155497.37

[16]

Question 5 (i)

The following are three types of guaranteed reversionary bonuses. The bonuses are usually allocated annually in arrears, following a valuation.

Simple – the rate of bonus each year is a percentage of the initial (basic) sum assured under the policy. The effect is that the sum assured increases linearly over the term of the policy.

Compound – the rate of bonus each year is a percentage of the initial (basic) sum assured and the bonuses previously added. The effect is that the sum assured increases exponentially over the term of the policy.

Super compound – two compound bonus rates are declared each year. The first rate (usually the lower) is applied to the initial (basic) sum assured. The second rate is applied to bonuses previously added. The effect is that the sum assured increases exponentially over the term of the policy. The sum assured usually increases more slowly than under a compound allocation in the earlier years and faster in the later years.

(ii)
Let P be the monthly premium. Then:

$$12P\ddot{a}^{(12)}_{[40]:\overline{25|}} = 155.124P$$

$$\ddot{a}^{(12)}_{[40]:\overline{25|}} = \ddot{a}_{[40]:\overline{25|}} - \frac{11}{24}(1 - {}_{25}p_{[40]}v^{25})$$

$$= 13.290 - \frac{11}{24}\left(1-(1.06)^{-25} \times \frac{8821.2612}{9854.3036}\right) = 12.927$$

EPV of benefits:

$$\frac{100,000}{(1+b)}\times(1.06)^{1/2}\{q_{[40]}(1+b)v \quad +_1|q_{[40]}(1+b)^2v^2$$

$$+....+_{24}|q_{[40]}(1+b)^{25}v^{25}\}+100,000\,{}_{25}p_{[40]}(1+b)^{25}v^{25}$$

where b = 0.0192308

$$=\frac{100,000}{(1+b)}\times(1.06)^{1/2}A^{1}_{[40]:\overline{25|}}\,@\,i' +100,000\times\frac{D_{65}}{D_{[40]}}@\,i'$$

$$=\frac{100,000}{1.0192308}\times(1.06)^{1/2}\times(.38896-.33579)+100,000\times.33579=38949.90$$

where $i' = \dfrac{1.06}{1+b}-1 = 0.04$

EPV of expenses:

$$.875\times12P+175+0.025\times12\times P(\ddot{a}^{(12)}_{[40]:\overline{25|}}-\ddot{a}^{(12)}_{[40]:\overline{1|}})+65[\ddot{a}_{[40]:\overline{25|}}-1]=14.086P+973.85$$

$$\ddot{a}^{(12)}_{[40]:\overline{1|}}=\ddot{a}_{[40]:\overline{1|}}-\frac{11}{24}(1- \quad p_{[40]}v\ )=1-\frac{11}{24}\left(1-(1.06)^{-1}\times\frac{9846.5384}{9854.3036}\right)=0.974$$

EPV of claim expense: [14]

.025 × 38949.9 =973.748

Equation of value gives $155.124P = 38949.9 + 14.086P + 973.85 + 973.75$

Question 6

(i) Gross premium retrospective and prospective reserves will be equal if:

- The mortality and interest rate basis is the same for the retrospective and prospective reserves and is the same as that used to determine the gross premium at the date of issue of the policy.
- The same expenses (excluding the initial expenses) are valued in the retrospective and prospective reserves and also the expenses valued in the retrospective reserves are the same as those used to determine the original gross premium.

- The gross premium valued in the retrospective and prospective reserves is that determined on the original basis using the equivalence principle.

(ii) The prospective reserves at time t are given by

$$S\bar{A}_{x+t} + e\ddot{a}_{x+t}^{(m)} + f\bar{A}_{x+t} - G\ddot{a}_{x+t}^{(m)} \qquad \text{..........(a)}$$

where

S is the sum assured
e is the annual rate of renewal expenses
f is the claim expense
G is the annual rate of gross premium

The retrospective reserve at time t is given by

$$\frac{D_x}{D_{x+t}}\left\{G\ddot{a}_{x:\overline{t|}} - S\bar{A}^{1}_{x:\overline{t|}} - I - e\ddot{a}^{(m)}_{x:\overline{t|}} - fA^{1}_{x:\overline{t|}}\right\} \qquad \text{.....................................(b)}$$

where I is the additional initial expense.

The original gross premium is given by

$$G\ddot{a}_x^{(m)} - S\bar{A}_x - I - e\ddot{a}_x^{(m)} - f\bar{A}_x = 0 \qquad \text{.............. (c)}$$

Add $\frac{D_x}{D_{x+t}}\left\{G\ddot{a}_x^{(m)} - SA_x - I - e\ddot{a}_x^{(m)} - fA_x\right\}$, which is identically 0, to (a).

Combining terms, e.g. $\frac{D_x}{D_{x+t}}G\ddot{a}_x^{(m)} - G\ddot{a}_{x+t}^{(m)} = \frac{D_x}{D_{x+t}}G\ddot{a}^{(m)}_{x:\overline{t|}}$ gives (b), the expression for the retrospective reserve.

[7]

Question 7

(i) Pensioners retiring at normal retirement age
Pensioners retiring before normal retirement age

Pensioners retiring before normal retirement age on grounds of ill-health

(ii)

Class selection – different classes of members experience different mortality rates.

Eg works versus staff. Alternatively ill-health retirements, other early retirements and normal retirements experience different mortality

Temporary Initial Selection – employee turnover rates vary with duration of employment, recent joiners are most likely to leave.

Time Selection – turnover rates may vary with economic conditions.

[6]

Question 8

(i)

Multiple decrement table:

Age	AM92 mortality rates	Independent rate of mortality (80% of AM92)	Independent rate of withdrawal
40	0.000788	0.0006304	0.20
41	0.000962	0.0007696	0.10
42	0.001104	0.0008832	0.10
43	0.001208	0.0009664	0.10

Age	Probability in force at start of year	Dependent rate of mortality	Dependent rate of withdrawal	Probability in force at end of year
40	1	0.000567	0.19994	0.79950
41	0.79950	0.000731	0.09996	0.71899
42	0.71899	0.000839	0.09996	0.64652
43	0.64652	0.000918	0.09995	0.58131

Unit Fund per policy:

Policy Year	Premium	Fund at start of Year	Allocated premium	Interest	FMC	Fund at end of Year
1	10000	0	4500	450	49.500	4900.500
2	10000	4900.5	10000	1490.05	163.906	16226.645
3	10000	16226.645	10000	2622.6645	288.493	28560.816
4	10000	28560.816	10000	3856.0816	424.169	41992.728

Cash flows per policy:

Policy Year	Unallocated Premium	Comm	Exp	Interest on non unit fund	FMC	Extra Death benefit	End of year cashflow
1	5500	1500	2000	100.00	49.500	53.956	2095.544
2	0	300	200	-25.00	163.906	61.248	-422.343
3	0	300	200	-25.00	288.493	59.940	-296.447
4	0	300	200	-25.00	424.169	53.255	-154.086

Policy Year	Probability in force	Discount Factor	Profit Signature	Premium Signature
1	1.000000	0.86956522	1822.2125	10000
2	0.799496	0.75614367	-255.3205	6952.1363
3	0.718992	0.65751623	-140.1452	5436.6151
4	0.646522	0.57175325	-56.95814	4250.9845

Expected PV of Profit – Rs 1369.7887

Expected PV of Premium – Rs 26639.74

Profit Margin – 5.14%

(ii) The expected provisions at the end of year can be calculated using the end of year cashflows figures and decrement tables in (i) above:

${}_3V$ = 154.086/ 1.05 = 146.749

${}_2V$ *1.05 – $(ap)_{42}$ * ${}_3V$ = 296.447 ; ${}_2V$ = 408.004

${}_1V$ *1.05 – $(ap)_{41}$ * ${}_2V$ = 296.447 ; ${}_1V$ = 751.680

These need to be adjusted as the question asks for the values in respect of the beginning of year.

Year 3: 146.749 $(ap)_{42}$ = 131.957

Year 2: 408.004 $(ap)_{41}$ = 366.921

Year 3: 751.680 $(ap)_{40}$ = 600.965

Where, $(ap)_x$ = 1 – dependent rate of mortality – dependent rate of withdrawal

Revised cashflows:

Policy Year	End of year cashflow	Revised cashflow	Probability in force	Discount Factor	Profit Signature
1	2095.544	1494.579	1.000000	0.86956522	1299.634
2	-422.343	0	0.799496	0.75614367	0
3	-296.447	0	0.718992	0.65751623	0
4	-154.086	0	0.646522	0.57175325	0

Expected PV of Profit – Rs 1299.634

Expected PV of Premium – Rs 26639.74

Profit Margin – 4.88%

(iii)

Risks:

- Since there is no surrender penalty in the first few years, the insurance company is exposed to the risk of early surrenders.
- The commission paid is higher than the allocation charges and hence the advisors may collaborate with the customer and select against the insurance company.
- The capital requirement of the insurance company will increase if the allocation charges are reduced. There is a risk of the company not able to raise sufficient capital.

Mitigations:

- The insurance company can introduce surrender penalties subject to it does not affect the marketability of the product.
- The insurance company should reduce the first year commission and increase renewal commission rates.
- The insurance company can restrict the volume of new business written from this product to reduce capital requirement.

[25]

Question 9

(i) A^2_{xx} = ½ A_{xx} = A_x - ½ A_{xx}

= (1-d $\ddot{a}_x$) – ½ (1- d $\ddot{a}_{xx}$)

= ½ - ($\frac{i}{1+i}$) ($\ddot{a}_x$ – ½ $\ddot{a}_{xx}$)

(ii)

Let the Annual Premium be P,

0.95 P $\ddot{a}_{xx}$ = 1, 00,000 A_{xx} + 50,000 A_{xx}^{2} + 1,000

= 1, 00,000 (1- d $\ddot{a}_{xx}$) + 50,000 [½ - d ($\ddot{a}_{x}$ – ½ $\ddot{a}_{xx}$)] + 1000

= 1, 00,000 (1- ($\frac{i}{1+i}$) äxx) + 50,000 [½ - ($\frac{i}{1+i}$) ($\ddot{a}_{x}$ – ½ $\ddot{a}_{xx}$)] + 1000

(iii) (a) Reserve = 1,00,000 $A_{70:70}$ + 50,000 $A_{70\,70}^{\;2}$ - 0.95 P $\ddot{a}_{70:70}$

(b) Reserve = 50,000 A_{70}

[8]

[Total 100 Marks]

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