INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

18th November 2010

Subject CT8 – Financial Economics

Time allowed: Three Hours (10.00 – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1) *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*

2) *Mark allocations are shown in brackets.*

3) *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*

4) *In addition to this paper you will be provided with graph paper, if required.*

AT THE END OF THE EXAMINATION
Please return your answer book and this question paper to the supervisor separately.

Q. 1) (a) For the situations below, state the form of Efficient Market Hypothesis that exists for each of the situation and justify it.

(i) The price of shares of a particular company increase due to press release by that company of unexpected gains.

(ii) Usual correct tips from your friend for buy / sell of shares

(iii) Active fund manager showing a lower return than passive fund manager over a long period. (6)

(b) Why should the returns net of various costs be considered to test the efficiency of markets? (2)

(c) What are volatility tests? Why are the historical volatility tests described as inconclusive? (8)

[16]

Q. 2) (i) An investor believes that the price of gold increases when the volatility of the equities market is high and decreases when the volatility is low. The investor therefore wishes to model the price of gold X_t, as an Itô process defined by

$$dX_t = V_t\, dB_t + V_t^2\, dt$$

where B_t is standard Brownian motion and V_t is a measure of market volatility calculated from the equity price information available at time t.

Comment briefly on the suitability of this model, mentioning in particular its behavior when V_t is large and when V_t is small. (2)

(ii) (a) State Itô's Lemma. (2)

(b) Use Itô's Lemma to find an expression for dM_t, where $M_t = e^{-2Xt}$. (4)

(c) Deduce that Mt is a martingale. (1)

[9]

Q. 3) (i) What are the equations for the expected returns based on the Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Theory (APT).
Define all symbols used. (4)

(ii) Briefly explain the major differences between these models. (5)

[9]

Q. 4) The expected utility function for a young actuarial student, who wishes to construct a portfolio consisting of a risk-free and a risky asset, is

$$E[U] = r_a - \frac{1}{2}\ \sigma_a^2$$

where r_a and σ_a are the mean and standard deviation of the portfolio rates of return. The risk-free asset has an expected rate of return of 4% p.a. The risky asset has an expected rate of return of 12% p.a. and variance of 10%% p.a.

Determine the portfolio that will maximize the investor's expected utility. **[9]**

Q. 5) Return for a security is are distributed R ~ exp(λ). Derive expressions for variance and downside semi variance. Evaluate the derived expressions for $\lambda = \frac{1}{2}$. **[7]**

Q. 6)

(i) State the updating equation for force of inflation under Wilkie model. Describe all the terms used. (3)

(ii) Consider a two-dimensional table in which each row is one simulation for force of inflation and each column corresponds to a future projection date. All the simulations start from the same starting position. In this context define the following terms.

- Cross-sectional properties
- Longitudinal properties (3)

(iii) Explain the difficulty in using statistical properties from historic data to estimate parameters of Wilkie model? (4)

[10]

Q. 7) You observe that the price of an option on a non dividend paying stock is Rs 10. The volatility implied by the option's price is 25% and the force of interest is 10%. Your friend has recently joined a stock broker who has sophisticated computer based models. He provides you the following estimate of Greeks on the option.

Delta	0.5
Gamma	0.01 Rs^{-1}
Theta	-0.1 paise/day
Vega	2 Rs
Rho	3 Rs

A day later the market index fell by 20% but the stock performed relatively well and the closing price was Rs 105. The volatility is reassessed at 40% and the revised force of interest is 9%.Using Greeks based approximation your friend calculates the reduction in the price of the option to be Rs 2.106.

Calculate the initial price of the stock. **[6]**

Q. 8) Assume that the Black-Scholes framework holds.

A non dividend paying share is currently priced at Rs 100 and the volatility implied by one year term to maturity option on the share is 10%. The force of interest is 5%.

A "long call strip" option trading strategy involves buying a series of call options with rising strike prices for the same term to expiry on the same underlying.

You are a speculator and you want to implement this strategy by buying *four* call options with one year term to maturity on the share. The strike price on these options is 100%, 105%, 110% and 120% of the share's current price.

(i) Calculate the maximum loss you can incur if you implement this strategy? Ignore trading expenses. (6)

(ii) What is the maximum profit you can make through this strategy? (2)

(iii) Calculate the overall profit/loss at maturity given that

- you borrowed money from a friend at 10% per annum simple to execute the strategy,
- you hold all the options to maturity, and
- you know that the share price at maturity is Rs 112.5. (3)

(iv) What opinion concerning the share price would you have, to adopt this strategy? (2)

[13]

Q. 9) You are the head of analytics at an investment bank. You observe that the shape of the yield curve implied by the market price of government bonds follows a humped curve. You choose the following model for instantaneous forward rate *f(t,T)*

$f\,(t,T)$ = Long Rate + (Shape Factor I)$*0.5^{(T\text{-}t)}$ + (Shape Factor II)$*0.5^{2(T\text{-}t)}$

You observe the following prices for government bonds

Term to maturity	Price (100 Nominal)
3 months	98.20745
2 Years	78.08764
10 Years	33.02450

(i) Calculate the missing parameters in the model such that it reproduces the prices of the zero coupon government bonds. (8)

(ii) Prove that the fair price rounded to two decimal places of a 3 year zero coupon government bond based on the above formulation is 68.73 per 100 nominal? (2)

A corporate entity with BBB rating approaches you to calculate the fair price of a 3 year zero coupon bond it wants to issue in the market.

You use the reduced form 4 state JLT model for credit ratings. Your team has calculated the following time homogeneous 1 year transition probability matrix between the four credit ratings AAA, BBB, CCC and D under risk neutral probability measure. D is the state of default.

$$\Pi = \begin{pmatrix} 0.5 & 0.3 & 0.2 & 0 \\ 0.3 & 0.5 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.5 & 0.2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

It is usual to assume recovery of 30% of nominal value of bond on default.

(iii) Calculate the fair price of 100 nominal of this security. (6)

[16]

Q. 10) You have adopted a strategy to hold φ_t units of stock and ψ_t units of cash bond at time $t \geq 0$.

(i) Under what circumstances would this strategy be considered as a self financing trading strategy. Describe all the terms used. (2.5)

Let X denote a derivative payment at time 'U'.

(ii) Define a replicating strategy to replicate the derivative payoff and use principal of no arbitrage to arrive at an appropriate price for this derivative at time 0. (2.5)

[5]

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