INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

9th November 2010

Subject CT6 – Statistical Methods

Time allowed: Three Hours (10.00 – 13.00 Hrs.)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception***

2. ***Mark allocations are shown in brackets.***

3. ***Attempt all questions, beginning your answer to each question on a separate sheet. <u>However, answers to objective type questions could be written on the same sheet.</u>***

4. ***In addition to this paper you will be provided with graph paper, if required.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) A veteran actuary believes that the claims from a particular type of policy follow the Burr distribution with parameters $\alpha = 2$, $\lambda = 1000$ and $\gamma = 0.75$. As per his recommendation, the insurance company has set a deductible such that 25% of the losses result in no claim to the insurer.

(i) Calculate the size of the deductible. (3)

(ii) An actuarial trainee suspects that the deductible set by the veteran actuary is based on more of surmise than data. She has access to data on 1250 claims (net of deductible). Continuing with the assumption of the Burr distribution for the original claims, she wishes to estimate its parameters from the available data, by using the method of maximum likelihood. Give an expression for the probability density function of the observed data (net of deductible), and the likelihood function that has to be maximized. (4)

(iii) Give an expression for the maximum likelihood estimate (MLE) of the true fraction of the losses that result in no claim to the insurer, in terms of the MLE of the parameters. (2)

[9]

Q. 2) The annual number of claims on a particular risk has the Binomial distribution with maximum claim number 10 and average claim number θ. The prior density of the parameter θ is $\frac{1}{10}\times\frac{(n_1+n_2+1)!}{n_1!n_2!}\times\left(\frac{\theta}{10}\right)^{n_1}\left(1-\frac{\theta}{10}\right)^{n_2}$, where n_1 and n_2 are known positive integers. The number of claims in the years 2007, 2008 and 2009 were X_1, X_2 and X_3, respectively.

(i) Determine the prior mean of θ. (2)

(ii) Determine the maximum likelihood estimator of θ. (2)

(iii) Determine the Bayes estimate of the number of claims in the year 2010, under the squared error loss function. (4)

(iv) Show that the estimator of part (iii) has the form of a credibility estimate, and identify the credibility factor. (2)

(v) Determine the credibility estimator of θ under EBCT Model 1 and compare with the result of part (iii). (6)

[16]

Q. 3) The aggregate claims process for a risk is a compound Poisson process with rate $\lambda = 50$ per annum. Individual claim amounts are Rs. 2500 with probability 0.25, Rs. 5000 with probability 0.5, or Rs. 7500 with probability 0.25. The premium loading is 10%. Let S denote the aggregate annual claim amount.

(i) Calculate the mean and variance of S. (2)

(ii) Using a normal approximation to the distribution of S, calculate the initial surplus required in order that the probability of ruin at the end of the first year is 0.05. (3)

(iii) A reinsurer offers to sell to the insurer proportional reinsurance for 25% of the claims, for premium loading 15%. If this offer is accepted, calculate the modified initial surplus required in order that the probability of ruin at the end of the first year is 0.05. (4)

[9]

Q. 4) The cumulative incurred claims (in thousands of rupees) on a portfolio of insurance policies are as given in the following table.

Accident Year	Development Year			
	0	1	2	3
2006	2,463	2,749	3,529	3,980
2007	3,013	3,278	3,608	
2008	3,321	3,716		
2009	3,953			

The earned premium for the year 2009 is Rs. 6,472,000, while the paid claims are Rs. 1,731,000.

(i) Assuming that the Ultimate Loss Ratio is 88%, calculate the reserve needed for 2009 using the Bornhuetter-Ferguson (basic) method. (8)

(ii) State the assumptions underlying the use of the above method. (3)

[11]

Q. 5) Consider the autoregressive process given by

$$Y_t = \alpha Y_{t-2} + Z_t,$$

Z_t being white noise with mean zero and variance σ^2.

(i) What is the range of values of the real valued parameter α so that the process is stationary? (2)

(ii) Obtain a representation of Y_t as $\sum_{j=0}^{\infty} a_j Z_{t-j}$, by specifying $a_0, a_1, \ldots$ explicitly. (3)

(iii) Using part (b) or otherwise, find an expression for the variance of Y_t in terms of α and σ^2. (2)

(iv) Compare the result of part (iii) with the variance of an AR(1) process and explain any similarity or dissimilarity. (2)

[9]

Q. 6) The sample ACF and PACF values at lags 1 to 10 of a time series of length 500, are as given below.

Lag	1	2	3	4	5	6	7	8	9	10
SACF	-0.7793	0.6180	-0.4824	0.386	-0.341	0.3172	-0.2989	0.2728	-0.2181	0.163
SPACF	-0.7793	0.0275	0.0188	0.0232	-0.084	0.0538	-0.0289	0.0004	0.0616	-0.0301

(i) Determine through a statistical test whether the time series can be regarded as white noise. (5)

(ii) Indicate, with reasons, if an AR(p) or an MA(q) model may be appropriate for this time series, and if so, what could be the model order. (4)

[9]

Q. 7) List six perils that are typically insured against under a household building policy. **[3]**

Q. 8) A claim analyst of a health insurance company examines data on a portfolio of health insurance policies. He plans to use a generalized linear model for the claim amounts, involving the following rating factors.

SA : Sum assured (x), a continuous variable.

AG : Age group, a factor with 10 levels.

OC : Occupation, a factor with 6 levels.

A preliminary analysis produces the following summary for the models considered by the analyst.

Model	Linear predictor	No of parameters	Scaled deviance
SA	$\alpha + \beta x$	2	238.4
SA + AG	?	?	206.7
SA + AG + SA * AG	?	?	178.3
SA * AG + OC	?	?	166.2
SA * AG * OC	?	?	58.9

(i) Complete the table by filling in the cells with question marks. (4)

(ii) On the basis of the scaled deviance, which model should the analyst choose? (3)

(iii) What further considerations should be given before the analyst makes a recommendation about the choice of the model? (2)

[9]

Q. 9) An actuary uses the following algorithm to generate pseudorandom numbers X from the Poisson distribution with mean λ.

Step 1: Input lambda.

Step 2: Set X=0; Z=0.

Step 3: Set Y = Random sample from the uniform U(0,1) distribution.

Step 4: Increment Z by the amount –ln(Y)/lambda.

(ln is the log function).

Step 5: If Z<1, then increment X by 1; GO TO Step 3.

Step 6: Output X.

Step 7: GO TO Step 2 for generating the next value of X.

(i) By analysing the above algorithm, show that it generates the value $X = 0$ with the correct probability. (3)

(ii) If five successively random samples from the uniform distribution, generated in Step 3, happen to be 0.564, 0.505, 0.756, 0.610 and 0.046, and $\lambda = 2$, follow the above algorithm to generate as many samples of X as this information permits. (4)

(iii) Using the uniformly distributed random samples and the value of λ given in part (ii), generate samples (as many as possible) from the Poisson distribution, using the inverse distribution transform method. (5)

[12]

Q. 10) (i) State the individual risk model, with a clear description of the assumptions. (5)

(ii) How is this model different from the collective risk model? (3)

[8]

Q. 11) The owner of a personal computer has to decide whether to sign an Annual Maintenance Contract (AMC) or to pay for repair separately on each occasion of computer fault. The AMC costs Rs. 1000, and provides for an unlimited count of repair services. In the absence of the AMC, the servicing agency charges Rs. 300 for each repair. The owner assumes the probability distribution of the annual number of faults as follows.

Number of faults	0	1	2	3	4	5	More than 5
Probability	0.1	0.1	0.2	0.3	0.2	0.1	0

(i) Form the loss matrix for the owner of the computer in respect of the above decision. (2)

(ii) What is the minimax decision? (1)

(iii) What is the Bayes decision? (2)

[5]

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