Institute of Actuaries of India

CT8 – Financial Economics

Indicative Solution

November 2008

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable

Q.1
(a)

- Semi-strong
- Strong
- Weak

(b)

- by taking higher systematic risk ie by buying stocks with higher beta

- If the investor trades on the basis of inside information

Total Marks: 5

Q.2
(a)

Five measures of investment risks can be:

- Variance of return
- Downward semi-variance of return
- Shortfall probabilities
- Value at risk (VaR)
- Tail value at risk (TailVaR) and expected shortfall

Variance of return

Variance of return is defined as

$$\int_{-\infty}^{\infty} (\mu - x)^2 f(x)dx$$

where:

μ : mean return at the end of chosen period
$f(x)$: probability density function of the return

Downward semi-variance of return

Most investor may not dislike the uncertainty of returns as such, rather they dislike the probability of low returns. The measure that quantifies this view is downward semi-variance of returns. It is defined as:

$$\int_{-\infty}^{\mu} (\mu - x)^2 f(x)dx$$

Shortfall probabilities

Shortfall probability measures the probability of returns falling below a certain level. It is defined as:

L

$$\int_{-\infty} f(x)dx$$

where:

L	: chosen benchmark level
f(x)	: probability density function of the return

Value at risk (VaR)

VaR measures the likelihood of underperformance. It assesses the potential losses on a portfolio over a given future time period with a given degree of confidence.

$VaR(X) = -t$ where $P(X < t) = p$

where:

t	: loss in value
p	: confidence level

Tail value at risk (TailVaR) and expected shortfall

This risk measure can be expressed as the expected shortfall below a certain level.

Expected shortfall, $E[X \mid X < L] = \int_{-\infty}^{L} (L - x)f(x)dx$

where:

L	: benchmark level

If L is chosen as a particular percentile point, then the risk measure is known as **TailVaR**

(b)

Confidence level	:	1 in 400 years = 99.75% ie 2.807
Mean	:	0.1%
Standard Deviation	:	0.09%
Expected default	:	0.1% + 2.807*(0.09%)
	:	0.353%
VaR	**:**	**5000*0.353%**
	:	**17.63 crores**

Total Marks: 12

Q3

(a)

Opportunity Set: Properties such as risk-return combinations of every portfolio available for investment.

Efficient Frontier: A set of efficient portfolios. A portfolio can be described as efficient if for the same expected returns, no other portfolio can be found with lower variance and vice versa.

Optimal Portfolio: The portfolio that maximizes the investor's expected utility. It is at the point on the efficient frontier where indifference curve is tangential to it.

(b)

Equation of the efficient frontier:

Set up a Lagrangian function. This can be:

$$W = (x_1)^2(\sigma_1)^2 + (x_2)^2(\sigma_2)^2 + 2x_1x_2\sigma_1\sigma_2 - \gamma(x_1E_1 + x_2E_2 - Ep) - \mu(x_1 + x_2 - 1)$$

Where:

x_1	:	Proportion of stock 1 in the portfolio
x_2	:	Proportion of stock 2in the portfolio
σ_1	:	Standard deviation of stock 1
σ_1	:	Standard deviation of stock 2
E_1	:	Expected returns of stock 1
E_2	:	Expected returns of stock 2
Ep	:	Expected returns of the portfolio

Therefore, the first order conditions are:

$$\partial W/\partial x_1 = (2\,x_1)(\sigma_1)^2 + 2x_2\sigma_1\sigma_2 - \gamma E_1 - \mu = 0$$

$$\partial W/\partial x_2 = (2\,x_2)(\sigma_2)^2 + 2x_1\sigma_1\sigma_2 - \gamma E_2 - \mu = 0$$

$$\partial W/\partial \gamma = x_1E_1 + x_2E_2 - Ep = 0$$

$$\partial W/\partial \mu = x_1 + x_2 - 1 = 0$$

Therefore;

$$x_1 = (Ep - E_2)/(E_1 - E_2) \qquad \& \qquad x_2 = 1 - x_1 \qquad = (E_1 - E_p)/(E_1 - E_2)$$

Substituting these into the expression of variance of portfolio returns,

$$Vp = [(Ep - E_2)/(E_1 - E_2)]^2(\sigma_1)^2 + [(E_1 - E_p)/(E_1 - E_2)]^2(\sigma_2)^2 + 2[(Ep - E_2)/(E_1 - E_2)][(E_1 - E_p)/((E_1 - E_2)](\sigma_1\sigma_2) \qquad 1$$

$$Vp = [1/(E_1 - E_2)]^2[(Ep - E_2)^2(\sigma_1)^2 + (E_1 - E_p)^2(\sigma_2)^2 + 2[(Ep - E_2)(E_1 - E_p)(\sigma_1\sigma_2)]$$

$$Vp = [1/(E_1 - E_2)]^2[(Ep - E_2)(\sigma_1) + (E_1 - E_p)(\sigma_2)]^2$$

Therefore, standard deviation of the portfolio is

$\sigma_p = [1/(E_1 - E_2)]\ [(Ep - E_2)(\sigma_1) + (E_1 - E_p)(\sigma_2)]$

Thus:

$\sigma_p = AEp + B$ (The equation of efficient frontier)

where:

$A = (\sigma_1 - \sigma_1)/(E_1 - E_2)$ and $B = (\sigma_2 E_1 - \sigma_1 E_2)/(E_1 - E_2)$

(c)

(1) Probability of company exceeding its growth target :

$(33\%)*(100\%) + (33\%)*(60\%) + (33\%)*(0\%) = 53.3\%$

As the person is non-satiated, he will prefer more to less, therefore $U'(x) > 0$

Now $U(x) = 0.9x^2 - 0.4x^3$

Therefore, $U'(x) = 1.8x - 1.2\,x^2$

or $1.2x*(1.5 - x) > 0$

or the **boundary is $x < 1.5$**

(2) Expected utility offered by the job:

(53.3%)*[(0.9*1^2) – (0.4*1^3)] + (1-53.3%)*[(0.9*(50/80)^2) – (0.4*(50/80)^3)] = **0.385**

Total Marks: 14

Q4

(a)

Define two orthogonal factors.

$J_2 = I_2$ (1)

$J_1 = I_1 - x - y*I_2$ (2)

where J_1 is equal to the residuals from the regression of I_1 on I_2, which by definition are independent.

Now express returns on RIL, R stock in terms of J_1 & J_2.

From equations (1) and (2),

$I_1 = J_1 + x + y*J_2$ (3)

From equations (1) and (3),

$R = a + b_1(J_1 + x + y^* J_2) + b_2J_2 + c$

=> $\mathbf{R = (a + b_1x) + b_1 J_1 + (b_1 y + b_2) J_2 + c}$

(b)

Mean-variance theory:

- Expected retuns : 350
- Variances : 350
- Correlation coefficients or covariances : (350*(350-1))/2 = 61,075

Total number of data items : 61,775

Single index model:

- Alphas (α) : 350
- Betas (β) : 350
- Variances of random variables representing the component of historic returns of individual securities specific to the security : 350
- Expected return of the market : 1
- Expected variance of the market returns : 1

Total number of data items : 1,052

(c)

Security market line is an equation relating expected return on any security to the return on the market.

$$E_i - r = \beta_i{*}(E_M - r)$$

where:

E_i	:	expected return on security 'i'
r	:	return on risk-free asset
E_M	:	expected return on market portfolio
β_i	:	beta of the security 'i' defined as $cov(R_i,R_M)/V_M$

Expected returns of the security is:

$E_i - r = \beta_i{*}(E_M - r)$

Now r = 8%
E_M = 13%
$\beta_i = cov(Ri,R_M)/V_M = (\rho_{iM}\sigma_i\sigma_M)/ (\sigma_M)^2$
ρ_{iM} = correlation coefficient between the security and the market = 0.8

Now, variance of the security is, $(\sigma_i)^2 = (1/N)*\sum(x_i)^2 - \underline{x}^2$

N = Number of observations
$\underline{x}$ = mean of the historic returns of the security

Now, $\underline{x}$ = 6.33%

Therefore, $(\sigma_i)^2 = (1/9)*(6.97\%) - (6.33\%)^2 = 0.0037$
$\sigma_i = 0.0611 = 6.11\%$

Similarly $(\sigma_M)^2 = (1/9)*(7.09\%) - (5.00\%)^2 = 0.00538$
$\sigma_M = 0.0733 = 7.33\%$

Therefore, $\beta_i = (0.8)*(0.0611)*(0.0733)/(0.0733)$^2 = 0.67

So, $E_i = r + \beta i*(EM - r) = 0.08 + 0.67*(0.13 - 0.08) = 0.1133 = 11.33\%$

Total Marks:15

Q5

The lognormal model is consistent with the weak form of market efficiency because log returns over non-overlapping time intervals are assumed to be independent in the model.

Thus knowing the past patterns of returns can't help predict future returns.

Wilkie model is not consistent with the weak form of market efficiency. This model can be used to project the equity risk premium – excess expected total return on equities compared to index-linked government bonds.

If market were efficient, we would expect the projected risk premium to remain in a narrow range. Otherwise, excess profits could be earned by trading activities.

The wide variation in the equity risk premium projections through Wilkie model is equivalent to saying that the equity and index-linked government bond markets are not efficiently priced relative to each other.

Total Marks: 5

Q6

a) Assumptions underlying the Black Scholes formula:

- The price of the underlying share follows a geometric Brownian motion.
- The market in the underlying share is complete: that is, all derivative securities have payoffs which can be replicated.
- There are no risk-free arbitrage opportunities.
- The risk-free rate of interest is constant, the same for all maturities and the same for borrowing or lending.
- Unlimited short selling (that is, negative holdings) is allowed.
- There are no taxes or transaction costs.

The underlying asset can be traded continuously and in infinitesimally small numbers of units.

b) Consider two portfolios:

A: one call plus cash of $(K+X)e^{-rT}$
B: one put plus one share
Where;
K = strike price of the call and put options
r = continuously compounding risk free interest rate
T = time to expiration

Value of portfolio A on maturity is
$S_T - K + K + X = S_T + X$, if $S_T > K$ and the call option is exercised
Or, $K + X$ if $S_T < K$ and the call option is not exercised

Value of portfolio B on maturity is
$S_T + X$, if $S_T > K$ and the put option expires worthless
Or, $K - S_T + S_T + X = K + X$ if $S_T < K$ and the put option is exercised

The values of both the portfolios are the same at maturity irrespective of the share price, i.e. max $(S_T+X, K+X)$.

Since, the values of the portfolios are the same at maturity and the options cannot be exercised before maturity, they should have the same value at any time before the options expire.

Therefore,

$$c_t + (K+X)e^{-rT} = p_t + S_t$$

c) The value of portfolio A (from above) is as follows:

$c_t + (K+X)e^{-rT}$
$= 10 + (50 + 5) \times \exp(-10\% \times 0.25)$
= Rs 63.64

The value of portfolio B (from above) is as follows:
$p_t + S_t$
= 10 + 50
= Rs 60.

Therefore, portfolio B is cheaper than portfolio A and both will have exactly the same payoff on maturity. Therefore, there is an arbitrage opportunity available.

To exploit this opportunity, a trader should borrow an amount $(K+X)e^{-rT}$, i.e. Rs 53.64, sell a call option for Rs. 10; buy a put option for Rs. 10 and 1 share for Rs 50. This will leave the trader with Rs 3.64 immediately.

On maturity, if the share price is > Rs 50 (the strike price); the holder of the call option will exercise the option and the put option will expire worthless. The trader will receive Rs 50 (the strike price); deliver the share he is holding to the buyer of the call option and use the strike price (Rs 50) plus the dividend (Rs. 5) to pay off the borrowed amount.

If the share price is less than Rs 50, the call option expires worthless; the trader exercises the put option and gets Rs 50 for the share that he holds. He can then use this plus the dividend amount to pay off the borrowed amount.

Therefore, on maturity the trader will have a pay off of zero and he has made a profit of Rs 3.64 without any risk.

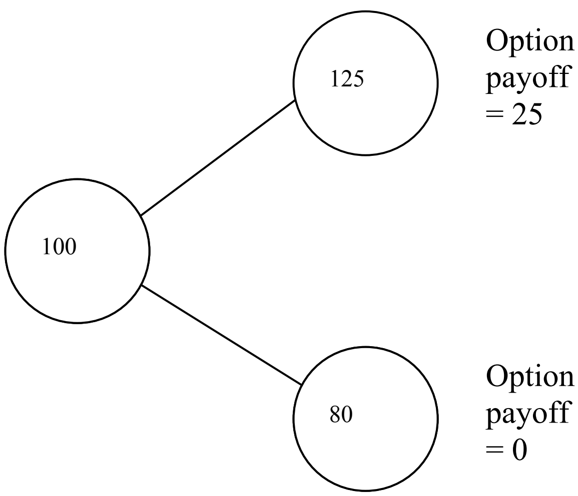
Total Marks: 13

Q 7.

Using the binomial approximation to the continuous time lognormal model, we have

u = exp ($\sigma\sqrt{\Delta t}$) = exp (22.35%×1) = 1.25
d = exp ($-\sigma\sqrt{\Delta t}$) = 0.80

The binomial tree is shown below:



To calculate the risk neutral probabilities, we need the value of e^r, which can be calculated using the bond price.
e^{-r} = 90/100 = 0.9.
Therefore e^r= 1/0.9 = 1.11

Therefore, the risk neutral probability of stock moving up is given by:

$$q = \frac{e^r - d}{u - d} = \frac{1.1 - 0.8}{1.25 - 0.8} = 0.69$$

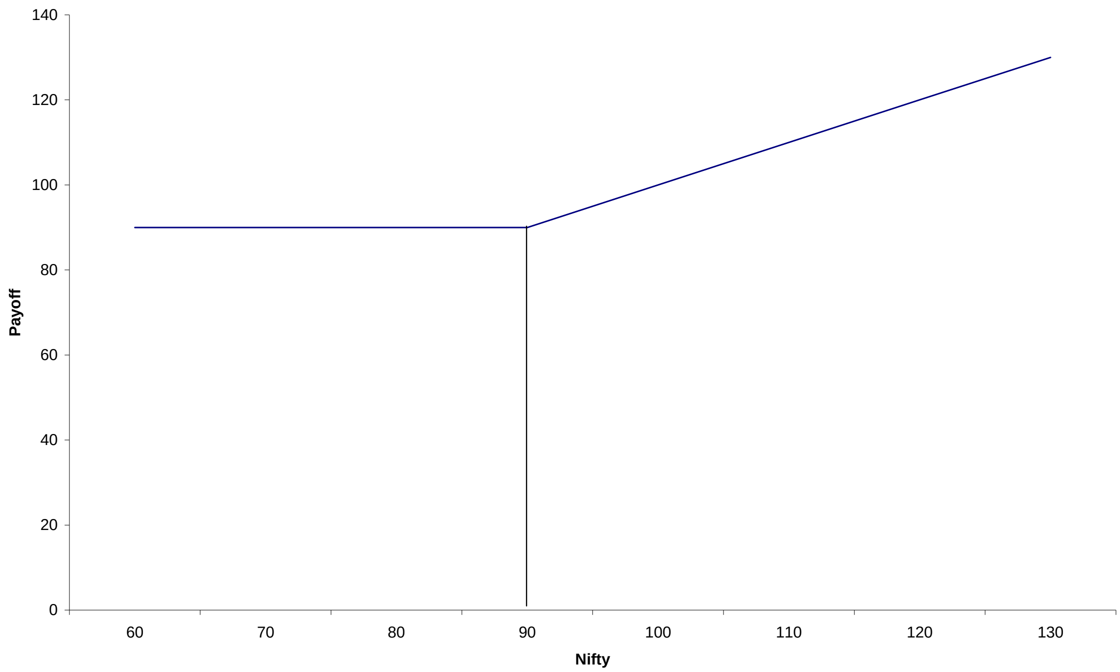
Therefore, price of the call option is:

$c = e^{-r} \times [q \times c_u + (1\text{-}q) \times c_d]$
$= 0.9 \times [\ 0.69 \times 25\ ]$
$= 15.5$

Total Marks: 5

Q 8
a)

The payoff for the policyholder at maturity is as follows:



b) The insurance company can hedge its risk by buying a 5-year put option with a strike price equal to the value of Nifty on the date of investment, i.e. an at the money put option.
c)
We calculate the price of an at-the-money put option. Assume that the value of Nifty is 100. Then the strike price is also 100.

$d1 = \{\log(100/100) + (0.1 + (\frac{1}{2} \times 0.3^2)) \times (5)\} / (0.3 \times \surd 5) = 1.08$
$d2 = 1.08 - (0.3 \times \surd 5) = 0.41$

Price of the put option $p_t = K \times (\exp(-r \times (T-t))) \times \Phi(-d_2) - St \times \Phi(-d_1)$
$\Phi(-d_2) = 0.34$
$\Phi(-d_1) = 0.14$

Therefore, $p_t = 100 \times (\exp(-0.1 \times 5)) \times 0.34 - 100 \times 0.14$
= 6.6 or 6.7 (depending on the number of places after the decimal are considered in the calculations).

Therefore, for every Rs 100 that is invested in the funds, the insurance company should offer Rs 6.6 as the price of the derivative.

d)
The delta of a put option is - $\Phi(- d_1)$
From above, in this case;
$\Phi(- d_1) = 0.14$
Therefore delta = -0.14

This implies that the bank needs a short position in equities to hedge its position.
Face value of the put option is Rs 9 crores.

Therefore face value of Nifty to be sold = 0.14×9cr
= 1.26 cr.

Value of Nifty = 100

Therefore, number of units of Nifty to be sold = 1.26 lakhs

Total Marks: 14

Q 9
a)
- Both models are mean reverting and arbitrage free.
- Vasicek model is more mathematically tractable.
- CIR model has the advantage that the interest rates cannot go negative.
- Parameters for both the models are not time-dependent.

b)
We know that
$F(t,T,S) = \{1/(S-T)\}*\log\{B(t,T)/B(t,S)\}$ for $t<T<S$.

Therefore,
$F(0,3,4) = \{1/(4-3)\}*\{\log(83.53/ 75.58)\} = 10\%$

Other approaches leading to the answer should also be given points.

Total Marks: 5

Q 10
a) Three types of credit risk model:
- Structural models
- Reduced form models
- Intensity-based models

Merton model is a structural model.

b)
Assumptions:
- A corporate entity has issued both equity and debt such that the total value at time t is F(t).

- Corporate does not pay dividends on equity and coupons on the bond, i.e. the bonds are zero coupon.
- L is the redemption amount at time T and it is assumed that the company will be wound up at time T by paying the remainder value to the shareholders

The model:
- If at time T, the value of the company's assets is less than L, there will be a default and the bondholders will be paid F(t), which is less than L and shareholders get nothing.
- This can be regarded as treating the shareholders as holders of an European call options on the company's assets with a strike price L.
- The value of the company's equity can be evaluated as the price of the option.
- The value of the debt at time t is then estimated as F(t) - C(t), where C(t) is the price of the call option.
- The value of the bond can be used to determine the applicable interest rate on the bond. This can be compared with the risk free rate to evaluate the credit spread.

c)
Value of equity is the price of a call option on company's assets.
Current value of underlying S = 4 mn,
Strike price X = 2 mn.
Time T = 5 years
Volatility = 30%
Risk free rate r = 8%

Using Black Scholes formula:

d1 = 1.96
d2 = 1.29

Φ (d1) = 0.975
Φ (d2) = 0.902

Price of the call option = $St \times \Phi(d_1) - K \times (\exp(-r \times (T-t))) \times \Phi(d_2)$
= $34 \times 0.975 - 2 \times \exp(-8\% \times 5) \times 0.902$
= 2.692

Value of the bond = 34– 2.692= 1.308

Therefore, if r1 be the interest on the bond,
$2 \times \exp(-r1 \times 5) = 1.308$;
Therefore $r1 = -1/5 \times \ln(1.308/2)$
= 8.49%
Therefore, the credit spread = 8.49% - 8.00% = 0.49%

Total Marks: 12
(Total 100 Marks)

***********END*********