Institute of Actuaries of India

CT5 – General Insurance, Life and Health Contingencies

Indicative Solution

November 2008

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable

1)

a. The conditions are:
   i. The retrospective and prospective reserves are calculated on the same basis.
   ii. The basis is the same used to calculate the premiums used in the reserve calculation.

b. Two reasons are:
   i. The assumptions used for the retrospective calculation (for which the experienced conditions over the duration of the contract up to the valuation date are used) are not generally appropriate for the prospective calculation (for which the assumptions considered suitable for the remainder of the policy term are used).
   ii. The assumptions considered appropriate at the time the premium was calculated may not be appropriate for the retrospective and prospective reserves some years later.

[Total 4 Marks]

2)

i. The expected cost of paying benefits usually increases as the life ages and the probability of a claim by death increases.

ii. Level premiums received in the early years of the contract are more than enough to pay the benefits that fall due in those years, but in the later years, the premiums are too small to pay for the benefits. Its is therefore prudent to for the premiums that are not required in the early years of the contract to be set aside, or reserved, to fund the shortfall in the later years of the contract.

iii. If premiums received that were not required to pay benefits were spent by the company by distributing it to the shareholders, then later in the contract the company may not be able to find money to pay for the excess of the cost of benefits over the premium received.

[Total 3 Marks]

3)

Adverse Selection is any type of selection that leads to an adverse effect on another party.

Example: Selective withdrawal of healthy lives from life insurance policies will result in a higher average mortality experience for the remaining policyholders.

Spurious selection is seen where the heterogeneity in a population leads to incorrect interpretations of mortality or morbidity differences being made.

Example: increasing the strictness of underwriting for life insurance policies over time will lead to a lighter mortality experience. This may be falsely interpreted as an improvement in mortality over time.

[Total 4 Marks]

4)

i.

- Decide on the structure of the product, *eg regular premium* unit-linked endowment assurance
- Build a model to project cashflows for the product
- Choose some model points covering different age, sex, term, level of cover etc.. that represent the target market of the product
- Decide on a risk discount rate and profit criterion
- Set up a profit test basis, probably the best estimate assumptions of all important parameters *e.g. interest rate, mortality rate,* levels of withdrawals, *etc*
- Decide on some "first draft" premium rates for the model points selected
- Profit test the model points using these premium rates
- Vary the premiums until the profit criterion is met and the rates are acceptable in the market
- Carry out the sensitivity test by doing the cashflow projections by varying key parameters, *eg* investment return, mortality, expense inflation, to check that our product design is sufficiently resilient to adverse changes in future experience
- Keep varying premiums and if necessary features of the product design until we have a product that meets the profitability criterion, is marketable, and is resilient to adverse future experience.

ii.

(a) Unit Account:

Policy Year	Premium	Allocated Premium	Cost of Allocation	Fund b/f	Fund before FMC	FMC	Fund c/f
1	50000	48000	45600	0	51072.00	510.72	50,561
2	50000	48000	45600	50561.28	107700.63	1077.01	106,624
3	50000	48000	45600	106623.63	170490.46	1704.90	**168,786**

(b) Non unit Account:

Policy Year	Prob of inforce at beg	Prob of inforce at end	Profit on Allocation	Expense	Commission	Death claims	Non Unit interest	GA on surrender/ Maturity	FMC	Profit	PV of Profit
1	1.00000	0.89900	4400	1000	5000	102.50	-128.00	250	510.72	-1569.78	-1365.03
2	0.89900	0.85315	4400	200	0	155.00	336.00	250	1077.01	5208.01	3540.26
3	0.85315	0.85230	4400	200	0	207.50	336.00	7500	1704.90	-1466.60	-822.70
											1,353

Net present value of profit signature = Rs.1353.00

(c)

- Increase the first year allocation charge and reduce subsequent year allocation charges
- Increase the bid offer spread
- Reduce or remove the Guaranteed addition payout
- Payment of commission may be spread over the duration of the policy rather than paying entire commission in the first year

(d)

- Increase the premium allocation charges
- Increase the fund management charges
- Increase the bid offer spread
- Reduce or remove the guaranteed addition payments
- Reduce the commission payments
- Deduct surrender charges from the account when the contract is surrendered

[Total 19 Marks]

5)

ASSUMPTIONS TO BE MADE

- Decrements in each single-decrement table are uniformly distributed over each year of age.
- Decrements in the multiple-decrement table are uniformly distributed over each year of age

We are given the following multiple decrement table:

Age (x)	No of employees aged x	No of resignations	Number of death between x to x+1
40	20,000	1000	400
41	18,600	100	200
42	18,300		

Step 1: calculate the dependant rates of resignations and deaths by using the following formula:

$(aq)_x^r = (ad)x^r \;/\; (al)_{x;}$ where 'r' stands for resignations;

$(aq)_x^d = (ad)x^d \;/\; (al)_{x;}$ where 'd' stands for deaths

Age		Dependent rate of resignation	Dependent rate of death
40		0.05000	0.02000
41		0.00538	0.01075

Step 2: Calculate the independent rates of resignations and deaths by using the following formula:

$q_x^d = (aq)_x^d \;/\; (1\text{-}0.5^*(aq)_x^r)$ and

$q_x^r = (aq)_x^r \;/\; (1\text{-}0.5^*(aq)_x^d)$

Age	Independent rate of resignation	Independent rate of death (old)	New dependent rate of death
40	0.05051	0.02051	0.01026
41	0.00541	0.01078	0.00539

Step 3: Calculate the new dependant rates of resignations and death by using the following formula:
$(aq)_x^d = q_x^d * (1-0.5*q_x^r)$ and
$(aq)_x^r = q_x^r * (1-0.5*q_x^d)$

Age	Dependent rate of resignation	Dependent rate of death
40	0.05025	0.009997
41	0.00539	0.005376

Step 4: Calculate the new multiple decrement table by multiplying the dependant rate by the number of employees.

New multiple decrement table:

Age	Number of lives	Number of resignations	Number of deaths
40	20,000	1,005	200
41	18,795	101	101
42	18,593		

[Total 9 Marks]

6)

i.

- Age
- Sex
- Geography, including rural/ urban
- Social and economic class
- Year – mortality rates tend to improve or deteriorate over time
- Occupation
- Education
- Genetics
- Policy duration

ii.

Advantages:

- Simple to use; it might be administratively inconvenient for sales and policy servicing teams to use many premium rate tables.
- Lower cost in development and maintenance of premium rates

Disadvantages:

- Using same premium table for different risk classes leaves the Company in a risky position because it could easily lose the low risk lives to a competitor who charges differential premium rates. High risks will be attracted to the company and it will be selected against.
- The company's mortality and morbidity experience may worsen substantially and it may loose competitive position in the market which may lead to lower sales volume.

[Total 9 Marks]

7)

a. The mortality patterns for each group of lives are observed to differ only for the first, say S years. The differences are temporary, producing the phenomenon called Temporary Initial Selection.

S is called the length of select period.

In any investigation the length of select period is determined empirically by considering the statistical significance of the differences in transition rates along each row and the substantive impact of the different possible values of S. Each row would represent the mortality rates for lives who have a common age attained at the time of transition, but different ages at the date of joining the population.

b. $p_{[x]} = ({}_{0.5}p_{[x]})\,({}_{0.5}p_{[x]+0.5})$

$= (1 - {}_{0.5}q_{[x]})\,(1 - {}_{0.5}q_{[x]+0.5})$

$= (1 - 0.25\,q_x)\,(1 - 0.60\,q_x)$

$= (1 - 0.25\,(1 - p_x))\,(1 - 0.60(1 - p_x))$

$= (0.75 + 0.25p_x)\,(0.4 + 0.6p_x)$

$= 0.3 + 0.55\,p_x + 0.15\,(p_x)^2$

[Total 6 Marks]

8.

Require to calculate ${}_{10.5}p_{49.5} = {}_{0.5}p_{49.5} * {}_{10}p_{50}$

$${}_{10}p_{50} = \frac{l_{60}}{l_{50}} = \frac{91732}{96247} = 0.953089$$

a. Assume deaths uniformly distributed so ${}_tp_x.\mu_{x+t}$

Then ${}_{0.5}q_{49.5} = \dfrac{(1-0.5)\,q_{49}}{(1-0.5q_{49})}$

$= 0.001332$

So ${}_{10.5}p_{49.5} = (1 - 0.001332) * 0.953089$
$= 0.95182$

b. Assume that force of mortality is constant across year of age 49 to 50

$$ {}_{0.5}p_{49.5} = e^{-0.5*\mu_{49}} $$

$$ \mu_{49} = -\ln(1-q_{49}) = -\ln(1-0.00266) = 0.002664 $$

$$ {}_{0.5}p_{49.5} = \exp(-0.5*0.002664) = 0.998669 $$

So $ {}_{10.5}p_{49.5} = 0.998669 * 0.953089 = 0.95182 $

[Total 5 Marks]

9):

(i) (a) $ {}_{4}p_{41:44} = (l_{45} * l_{48}) / (l_{41} / l_{44}) $
$= (9801.3123 * 9753.4714) / (9847.051 * 9814.3359)$
$= 0.98918$

(b) $q_{65:62} = 1 - p_{65:62} = 1 - p_{65} * p_{62}$
$= 1 - 0.985757 * 0.989888$
$= 0.02421$

(ii) Case 1:

If life aged 20 is last to die, he must die after attaining age 60 and life aged 25 must have died within 5 years preceding the death of the life aged 20. In this case, expression for present value of the assurance would be;

$$ = \int_{40}^{\infty} v^t ({}_{t-5}p_{25} - {}_{t}p_{25})\ {}_{t}p_{20}\ \mu_{20+t}\ dt $$

$$ = \int_{0}^{\infty} v^{40+s} ({}_{35+s}p_{25} - {}_{40+s}p_{25})\ {}_{40+s}p_{20}\ \mu_{20+40+s}\ ds $$

$$ = v^{40}\ {}_{40}p_{20}\ {}_{35}p_{25} \int_{0}^{\infty} v^s\ {}_{s}p_{60:65}\ \mu_{60+s}\ ds $$

$$ - v^{40}\ {}_{40}p_{20}\ {}_{40}p_{25} \int_{0}^{\infty} v^s\ {}_{s}p_{60:65}\ \mu_{60+s}\ ds $$

$$ = v^{40}\ {}_{40}p_{20}\ {}_{35}p_{25}\ \bar{A}_{\overset{1}{60}:60} - v^{40}\ {}_{40}p_{20}\ {}_{40}p_{25}\ \bar{A}_{\overset{1}{60}:65} $$

Case 2:
There might be two possibilities:

A. If the life aged 25 is last to die, he must die after attaining age 70 and the life aged 20 must have died within preceding 5 years.

B. The life aged 25 must die between ages 65 & 70 and the life aged 20 must have died after attaining age 60 but before the death of life aged 25.

The expression for the present value of the assurance would be;

$$= \int_{40}^{45} v^t \left({}_{40}P_{20} - {}_tP_{20} \right) {}_tP_{25}\ \mu_{25+t}\ dt$$

$$+ \int_{45}^{\infty} v^t \left({}_{t-5}P_{20} - {}_tP_{20} \right) {}_tP_{25}\ \mu_{25+t}\ dt$$

$$= \int_{0}^{5} v^{40+s} \left({}_{40}P_{20} - {}_{40+s}P_{20} \right) {}_{40+s}P_{25}\ \mu_{25+40+s}\ ds$$

$$+ \int_{0}^{\infty} v^{45+r} \left({}_{40+r}P_{20} - {}_{45+r}P_{20} \right) {}_{45+r}P_{25}\ \mu_{25+45+r}\ dr$$

$$= v^{40}\ {}_{40}P_{20:25} \int_{0}^{5} v^s\ {}_sP_{65}\ \mu_{65+s}\ ds$$

$$- v^{40}\ {}_{40}P_{20:25} \int_{0}^{5} v^s\ {}_sP_{60:65}\ \mu_{65+s}\ ds$$

$$+ v^{45}\ {}_{40}P_{20}\ {}_{45}P_{25} \int_{0}^{\infty} v^r\ {}_rP_{60:70}\ \mu_{70+r}\ dr$$

$$- v^{45}{}_{45}P_{20:25} \int_{0}^{\infty} v^r\ {}_rP_{65:70}\ \mu_{70+r}\ dr$$

$$= v^{40}\ {}_{40}P_{20:25}\ \bar{A}_{\overset{1}{65}:\overline{5|}} - v^{40}\ {}_{40}P_{20:25}\ \bar{A}_{60:\overset{1}{65}:\overline{5|}}$$

$$+ v^{45}\ {}_{40}P_{20}\ {}_{45}P_{25}\ \bar{A}_{60:\overset{1}{70}} - v^{45}\ {}_{45}P_{20:25}\ \bar{A}_{65:\overset{1}{70}}$$

Hence the single premium required would be the sum of the components under case 1 & case 2:

$$= \upsilon^{40}\ {}_{40}P_{20}\ {}_{35}P_{25}\ \bar{A}_{\overset{1}{60::60}} - \upsilon^{40}\ {}_{40}P_{20:25}\ \bar{A}_{\overset{1}{60:65}}$$

$$+\upsilon^{40}\ {}_{40}P_{20:25}\ \bar{A}_{\overset{1}{65:\overline{5}|}} - \upsilon^{40}\ {}_{40}P_{20:25}\ \bar{A}_{\overset{1}{60:65:\overline{5}|}}$$

$$+\upsilon^{45}\ {}_{40}P_{20}\ {}_{45}P_{25}\ \bar{A}_{\overset{1}{60:70}} - \upsilon^{45}\ {}_{45}P_{20:25}\ \bar{A}_{\overset{1}{65:70}}$$

$$= \left(\frac{1}{2}\right) \upsilon^{40}\ {}_{40}P_{20}\ {}_{35}P_{25}\ \bar{A}_{60:60}$$

$$+\upsilon^{40}\ {}_{40}P_{20:25}\ \bar{A}_{\overset{1}{65:\overline{5}|}}$$

$$+\upsilon^{45}\ {}_{40}P_{20}\ {}_{45}P_{25}\ \bar{A}_{\overset{1}{60:70}}$$

$$-\upsilon^{40}\ {}_{40}P_{20:25}\ \bar{A}_{60:65}$$

$$Since\ \upsilon^{40}\ {}_{40}P_{20:25}\left(\bar{A}_{\overset{1}{60:65:\overline{5}|}} + \upsilon^{5}\ {}_{5}P_{60:65}\ \bar{A}_{\overset{1}{65:70}}\right)$$

$$= \upsilon^{40}\ {}_{40}P_{20:25}\ (\bar{A}_{\overset{1}{60:65}})$$

$$and\ v^{40}\ {}_{40}p_{20:25}\ \dot{A}_{\overset{1}{60:65}} + v^{40}\ {}_{40}p_{20:25}\ \dot{A}_{\overset{1}{60:65}}$$

$$= v^{40}\ {}_{40}p_{20:25}\ \dot{A}_{60:65}$$

[Total 15 marks]

10)

$$\overline{A^1_{x:n}} = \sum_{t=0}^{n-1}\ {}_{t|}\overline{A^1_{x:1}}$$

$$= \sum_{t=0}^{n-1} v^t\ {}_t p_x\ \overline{A^1_{x:1}}$$

$$\overline{A^1_{x+t:1}} = \int_0^1 v^s\ {}_s p_x\ \mu_{x+t+s}\, ds$$

Assuming uniform distribution of deaths, then ${}_s p_x\ \mu_{x+t+s} = q_{x+t}$

$$\overline{A^1_{x+t:1}} = \int_0^1 v^s q_{x+t}\, ds \quad = q_{x+t}\int_0^1 v^s\, ds$$

$$= q_{x+t} \frac{iv}{\delta}$$

$$\overline{A}^{1}_{x:n} = \sum_{t=0}^{n-1} v^{t}\ {}_{t}p_{x}\ q_{x+t} \frac{iv}{\delta}$$

$$= \frac{i}{\delta} \sum_{t=0}^{n-1} v^{t+1}\ {}_{t}p_{x}\ q_{x+t}$$

$$= \frac{i}{\delta} A^{1}_{x:n}$$

[Total 6 Marks]

11)

V_t = Reserve at the start of year
V_{t+1} = Reserve at the end of the year
i = interest rate
P = Annual premium
q_{x+t} = Probability of death at age x+t

a.

The guaranteed benefit is payable at the end of every month and doubles from the second month therefore the accumulated value of this benefit at the end of the year would be

$$= X(1+i)^{11/12} + 2X(1+i)^{10/12} + \ldots\ldots + (2)^{11}X(1+i)^{0/12}$$

$$= \sum_{j=0}^{11} 2^{j} X (1+i)^{(11-j)/12}$$

Therefore the recursive relationship will be:

$$(V_t + P)(1+i) = q_{x+t} S + \sum_{j=0}^{11} 2^{j} X (1+i)^{(11-j)/12} + p_{x+t} V_{t+1} (2) -$$

b.

The survival benefit of R is payable at the middle of the year, therefore,

$$(V_t + P)(1+i) = q_{x+t} S + p_{x+t-0.5} R(1+i)^{1/2} + p_{x+t} V_{t+1}$$

[Total 5 Marks]

12)

PV of benefits:

$= 0.98\ S\ A_{[35]:\,30} + 0.02\ S\ (IA)^1_{[35]:\,30} + 31 * 0.02\ S\ V^{30}\ {}_{30}p_{[35]}$

PV of Expenses:

$= 300\ A_{[35]:\,30} + 250 + 0.5*P - 150\ V^{30}\ {}_{30}p_{[35]}$

Where,

P = level annual premiums payable at the start of the policy
S = Sum Assured

$(IA)^1_{[35]:\,30} = (IA)_{[35]} - V^{30}\ {}_{30}p_{[35]}\ (\ (IA)_{65} + 30\ A_{65}\ )$

$$= 7.47005 - \frac{689.23}{2507.02}\ (7.89442 + 30* 0.52786) = 0.946137$$

$A_{[35]:\,30} = 0.32187$

Therefore PV of benefits:

$$= 0.98* S * 0.32187 + 0.02S * 0.946137 + 31* 0.02S * \frac{689.23}{2507.02}$$

$= 0.50481\ S$

PV of Expenses:

$$= 300 * 0.32187 + 250 + 0.5*P - 150 * \frac{689.23}{2507.02}$$

$= 305.32 + 0.5P$

a. Level annual premiums of Rs 10,000 are payable in advance which increases by Rs 500 every year from the second policy year.

PV of Premiums = 9500 $\ddot{a}_{[35]:30}$ + 500 $(I\ddot{a})_{[35:]30}$

$$(I\ddot{a})_{[35]:30} = (I\ddot{a})_{[35]} - v^{30}\frac{l_{65}}{l_{[35]}}[30\ddot{a}_{65} + (I\ddot{a})_{65}]$$

$$= 351.937 - \frac{1}{1.04^{30}} * \frac{8821.2612}{9892.9151}[30* 12.276 + 113.911]$$

$$= 219.3731$$

$\ddot{a}_{[35]:30} = 17.631$

Therefore PV of Premiums is:

= 9500 * 17.631 + 500*219.3731

= Rs 2,77,181

Therefore equating PV of benefits and expenses to PV of premiums:

277181 = 0.50481S + 305.32 + 0.5P (P = Rs 10,000)

0.50481S = 271875.68

S = 538570

The sum assured offered would be Rs 5,38,570

b. Level annual premiums of Rs 20,000 payable in advance for the first 25 years and then reduces to Rs 2000 from the 26th policy year.

$$\text{PV of Premiums} = 20000\ \ddot{a}_{[35]:25} + 2000\ v^{25}\frac{l_{60}}{l_{[35]}}\ddot{a}_{60:5}$$

$\ddot{a}_{[35]:25} = 16.029$

$\ddot{a}_{60:5} = 4.550$

Therefore PV of Premiums:

$$= 20000*16.029 + 2000 * \frac{1}{1.04^{25}} * \frac{9287.2614}{9892.9151} *4.550$$

$$= 323784.6$$

Therefore equating PV of benefits and expenses to PV of premiums:

$323784.6 = 0.50481S + 305.32 + 0.5P$ (P = Rs 20,000)

$0.50481S = 313479.3$

$S = 620985$

The sum assured offered would be Rs 6,20,985

[Total 15 Marks]

(Total 100 Marks)

************END***********